Calculation the Infinite norm ball of Constant Modulus Receivers by D-K iterations

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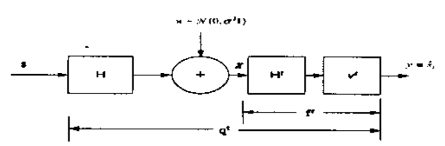
ABSTRACT -

In this paper, we propose a method to calculate the infinite norm ball for the geometrical characterization of the Constant Modulus Algorithm (CMA), and Shavi and Weinstein's blind estimation of linear receivers. The Constant Modulus Algorithm uses the second- and fourth- order moments of a pre-whitened chip-rate received signal. The approach provides a framework within which various blind and nonblind Wiener receivers can be finding an ellipsoid by infinite norm balls of different types. A necessary and sufficient condition for the equivalence among constant modulus, Shalvi-Weistein, zero-forcing, and Wiener receivers is obtained. Including their locations and their relationship with Wiener receivers are provided for the special orthogonal channel and general two-dimensional (2-D) channel-receiver impulse response. It is also shown that the calculation of infinite norm ball via D-K iteration [17] in 2-D, each CM or SW receiver can be obtained from associated with one and only one Wiener receiver for QAM system. Some design examples are simulated to verify our results.

I. Introduction

A. The Problem

Consider the linear estimation problem shown in Fig. 1:



$$x = Hs + w \tag{1}$$

$$y = f'x = q's + f'w \tag{2}$$

where

- $\mathbf{s}$ source vector;
- $\mathbf{H}$ channel impulse response matrix;
- $\mathbf{w}$ additive Gaussian noise;
- x received signal;
- y estimated of s_i (the ith element of $\mathbf{s}$) by a linear estimator $\mathbf{f}$.

The combined channel-receiver response is denoted by $\mathbf{q} = \mathbf{H}'\mathbf{f}$. A receiver is said to have the signal space property if it belongs to the column space of the channel matrix $\mathbf{H}$, i.e., $\mathbf{f} \in C_{\mathbf{H}}$, i.e., $\mathbf{f} = \mathbf{H}\mathbf{v}$ for some $\mathbf{v}$. In other words, the receiver can be viewed as a cascade of a matched filter and a linear combiner as illustrated in Figure 1.

In practical approach, a Constant Modulus (CM) receiver is usually obtained from the stochastic gradient algorithm, leading to CMA. Suppose that we have a sequence of observations from the linear model

$$\mathbf{x}[k] = \mathbf{H}\mathbf{s}[k] + \mathbf{w}[k]. \tag{3}$$

In the CMA with constellation $\mathcal{A}$ and a sequence $\{s_k\}$ of i.i.d. symbols from $\mathcal{A}$ such that $f_{k,I} = \mathrm{Re}[s_k]$ and $f_{k,Q} = \mathrm{Im}[s_k]$ are independent and have the same probability distribution. We assume a receiver in which sampling of the received signal occurs at the symbol rate and tracking of the carrier phase is carried out at the equalizer output, as shown in Figure 2.

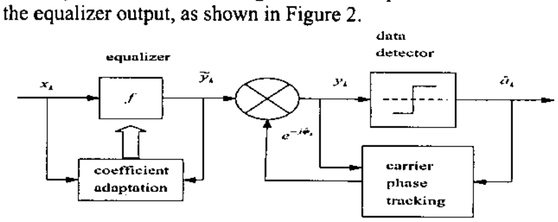


Fig. 2 Block diagram of a blind equalizer with Sato algorithm for a QAM system using the CMA.

If $\tilde{y}_k = f^T \mathbf{x}_k$ is the equalizer filter output, the sample at the decision point is then given by

$$y_k = \tilde{y}_k e^{-j\hat{\varphi}_k}. \tag{4}$$

We let

$$y_{k,I} = \mathrm{Re}[y_k] \text{ and } y_{k,Q} = \mathrm{Im}[y_k] \tag{5}$$

and introduce the Sato cost function for QAM systems,

$$J = E[\Phi(y_{k,I}) + \Phi(y_{k,Q})] \tag{6}$$

where

$$\Phi(v) = \tfrac{1}{2}v^2 - \gamma_S, \text{ and} \tag{7}$$

$$\gamma_S = \frac{E[s_{k,I}^2]}{E[|s_{k,I}|]} = \frac{E[s_{k,Q}^2]}{E[|s_{k,Q}|]}. \tag{8}$$

The blind equalization is based on the cost function

$$J_{CM} = E[(|\tilde{y}_k|^p - \gamma_p)^2] = E\{(|y_k|^p - \gamma_p)^2\} \tag{9}$$

where p is a parameter that usually assumes that the value $p = 1$ or $p = 2$, and the constant $\gamma_p = \frac{E[|s_k|^{2p}]}{E[|s_k|^p]}$. We note that J_{CM} depends on the absolute value of the equalizer output raised to the p-th power, the CMA does not require the knowledge of the carrier phase estimate. The gradient of (9) is given by

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$$\nabla_f J_{CM} = 2pE[(|\widetilde{y}_k|^p - \gamma_p)|\widetilde{y}_k|^{p-2}\widetilde{y}_k \mathbf{x}_k^*] \quad (10)$$

By using (10), we obtain the equalizer coefficient updating law

$$\mathbf{f}_{k+1} = \mathbf{f}_k - \mu(|\widetilde{\mathbf{y}}_k|^p - \gamma_p)|\widetilde{y}_k|^{p-2}\widetilde{y}_k \mathbf{x}_k^* \quad (11)$$

where μ is the step-size, $\mathbf{f}_{k+1}$ the receiver vector at time k and $\widetilde{y}_k$ the corresponding estimate. The CM receiver is limited to the estimation of sub-Gaussian sources $(\kappa_i < 0)$ as pointed out by Godard [4]. This deficiency can be circumvented by imposing a power constraint and searching for both minima and maxima, which will be evident from the connection of the SW receiver with the CM receiver.

Then, we make the following assumptions in our analysis.

A1) Entries of $\mathbf{s}$ are i.i.d. sub-Gaussian random variables with equal probability from the set $\{\pm 1\}$.

A2) Entries of $\mathbf{w}$ are i.i.d. Gaussian random variables with zero mean and variance σ^2.

A3) $\mathbf{s}$ and $\mathbf{w}$ are independent.

A4) $\mathbf{H}$ has full column rank.

A5) All variables are real.

Ericson [1] showed that an optimal receiver designed from any reasonable criterion includes a matched filter as its front-end. The signal space property of the CM receiver was first presented in [2] and then in [3] in more general form. We summarize this property in the following theorem.

Theorem 1 ([3], [10], [11], [15]) *Assume that $C_{\mathbf{H}^\perp}$ is not empty, i.e., $N > M$, and $\sigma^2 > 0$. 1). If one of the sources is sub-Gaussian, then*

$$J_{CM} = \{\arg\min_{\mathbf{f}\in C_{\mathbf{H}}} J_{CM}(\mathbf{f})\}. \quad (12)$$

2). If there is no sub-Gaussian source, then

$$J_{CM} = \{\mathbf{f} \mid \|\mathbf{f}\|_{\mathbf{R}}^2 = \frac{\gamma}{3}, \mathbf{f} \in C_{\mathbf{H}^\perp} \cup C_{\mathbf{H}_g}\}, \quad (13)$$

where $C_{\mathbf{H}_g}$ is the space spanned by the columns of $\mathbf{H}$ corresponding to the Gaussian sources.

The above theorem shows that restricting the receiver to the signal space has no effect on the estimator. It is this property that allows us to analyze the estimator in the joint channel-receiver space. Another criterion developed by Shavi and Weinstein in [6] involves maximizing

$$J_{SW}(\mathbf{f}) \stackrel{\Delta}{=} \{\arg\max_{\mathbf{f}\in C_{\mathbf{H}}, \|\mathbf{f}\|_{\mathbf{R}}=1} |K_4(y)|\} = |\sum_i \kappa_i [\mathbf{f}'\mathbf{H}\mathbf{e}_i]^4|, \quad (14)$$

where

$$|K_4(y)| = |\sum_i \kappa_i q_i^4| \le |\kappa| \quad (15)$$

is the fourth-order cumulant of y. The signal space property of SW receivers for the QAM system is simple.

Theorem 2 ([10], [11]) *Assume that $C_{\mathbf{H}^\perp}$ is not empty, i.e., $N > M$, and $\sigma^2 > 0$. Then*

$$J_{SW} = \{\arg\max_{\mathbf{f}\in C_{\mathbf{H}}, \|\mathbf{f}\|_{\mathbf{R}}^2=1} J_{SW}(\mathbf{f})\}. \quad (16)$$

Because of the signal space properties of Theorem 1 and 2, there is a one-one correspondence between the system parameter space and the signal space where optimal receiver reside. Therefore, the optimization with respect to the receiver coefficient $\mathbf{f}$ is equivalent to that with respect to system parameter $\mathbf{q}$. For both CM and SW receivers, we only need to express the output power $E(y^2) = \|\mathbf{f}\|_{\mathbf{R}}^2$ in terms of $\mathbf{q}$. Because $\mathbf{f} \in C_{\mathbf{H}}$, we have $\mathbf{f} = (\mathbf{H}')^+$, where $(\cdot)^+$ denotes the Moore-Penrose inverse. Hence, from (2)

$$\varepsilon_p^f = E(y^2) = \mathbf{q}'(\mathbf{I}_M + \sigma^2(\mathbf{H}'\mathbf{H})^{-1})\mathbf{q} \stackrel{\Delta}{=} E[|f_p(n)|^2] \quad (17)$$

We summarize the equivalent cost functions in the system parameter space in the following theorem where we have also included the Wiener (MMSE) receivers.

Theorem 3 ([10], [11]) *Let*

$$\Phi = \mathbf{I}_M + \sigma^2(\mathbf{H}'\mathbf{H})^{-1} = \begin{pmatrix} a & \mathbf{b}' \\ \mathbf{b} & \mathbf{C} \end{pmatrix} \quad (18)$$

set $a := [\Phi]_{v,v}$, $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$. *Assume that $\sigma^2 > 0$. Define the CM, SW and MSE (for source i) cost functions in the system parameter space by*

$$\text{CM}: J_{CM}(\mathbf{q}) \stackrel{\Delta}{=} E(y^2 - \gamma)^2 = \sum_i \kappa_i |q_i|^4 + 3\|\mathbf{q}\|_\Phi^4 - 2\gamma\|\mathbf{q}\|_\Phi^2 + \gamma^2, \quad (19)$$

$$SW: J_{SW}(\mathbf{q}) \stackrel{\Delta}{=} |\kappa_4(y)| = |\sum_i \kappa_i |q_i|^4|, \quad (20)$$

$$MSE: J_m^{(i)}(\mathbf{q}) \stackrel{\Delta}{=} E(y \pm s_i)^2 = \|\mathbf{q}\|_\Phi^2 \pm 2\mathbf{q}'\mathbf{e}_i + 1. \quad (21)$$

Let the CM, SW and MMSE receivers in the system space be defined by

$$Q_{CM} = \{\arg\min_{\mathbf{q}} J_{CM}(\mathbf{q})\} \quad (22)$$

$$Q_{SW} = \{\arg\max_{\|\mathbf{q}\|_\Phi^2=1} J_{SW}(\mathbf{q})\} \quad (23)$$

$$Q_w = \{\arg\min_{\mathbf{q}} J_m^{(i)}(\mathbf{q})\} = \{\pm \mathbf{R}^{-1} e_i\}. \quad (24)$$

Then

$$J_{CM} = \{(\mathbf{H}')^+ \mathbf{q}, \mathbf{q} \in Q_{CM}\} \stackrel{\Delta}{=} (\mathbf{H}')^+ Q_{CM} \quad (25)$$

$$J_{SW} = \{(\mathbf{H}')^+ \mathbf{q}, \mathbf{q} \in Q_{SW}\} \stackrel{\Delta}{=} (\mathbf{H}')^+ Q_{SW} \quad (26)$$

$$J_w = \{(\mathbf{H}')^+ \mathbf{q}, \mathbf{q} \in Q_w\} \stackrel{\Delta}{=} (\mathbf{H}')^+ Q_w. \quad (27)$$

Similar proof can be found in [10] and [11] and thus is omitted to save space. Hence the CM cost is affected by the combination of 1). the output power of the receiver, 2). the fourth power of the residue interference $\mathbf{q}_i^4$ from source i, and

3). the 4th-order cumulant κ_i of source i.

In next section, we show some of the preliminary results of blind equalization of CMA/SW and MMSE. The Calculation of Infinite Norm Ball for CMA/SW receivers by D-K iteration will be formed in Section III and some important properties of QAM communication systems will be discussed in next section. Then, we end with important features in conclusion.

2. Preliminary Results

Geometrical Analysis of CM/SW/MMSE Receivers: Global Characterization

Our strategy is to view the optimization in (23) geometrically. The constraint $\|\mathbf{q}\|_{\Phi}^2 = 1$ defines an ellipsoid $\mathcal{E}(\mathbf{q})$

$$\mathcal{E}(\mathbf{q}) = \{\mathbf{q} : \mathbf{q}'\Phi\mathbf{q} = 1\},\ \Phi = \mathbf{I}_M + \sigma^2(\mathbf{H}'\mathbf{H})^{-1}, \quad (28)$$

and the SW receiver is to be found on $\mathcal{E}(\mathbf{q})$. The cost $J_{SW}(\mathbf{q}) = \alpha$ defines an M-1 dimensional manifold $\mathcal{M}(\mathbf{q},\alpha)$,

$$\mathcal{M}(\mathbf{q},\alpha) = \{\mathbf{q} : \sum_i \kappa_i |q_i|^4 = \alpha\}. \quad (29)$$

Recalling (23), we have

$$\mathbf{q}_w^{(i)} = \arg\min_{\mathbf{q}} J_m^{(i)}(\mathbf{q}) \approx \arg\max_{\|\mathbf{q}\|_{\Phi}^2 = 1} |\mathbf{q}'\mathbf{e}_i|. \quad (30)$$

As α increases (or decreases), the above optimization is equivalent to finding tangent points of $\mathcal{M}(\mathbf{q},\alpha)$ with $\mathcal{E}(\mathbf{q})$.

Lemma 1

Let $d^{-1} = [d_{ij}]_{i,j=1}^M$ *and* $\Lambda = diag(\frac{1}{d_{11}}, \cdots, \frac{1}{d_{MM}})$.

Then,

$$Q_w = \{\arg\max_{\|\mathbf{q}\|_{\Phi}^2 = 1} \|\Lambda\mathbf{q}\|_{\infty}\}, \quad (31)$$

where Q_w *is the set of Wiener receiver defined in (24).*

In other words, the Wiener receiver can be viewed as finding the ∞-norm ball of

$$\mathcal{W}(q,\alpha) \overset{\Delta}{=} \{q : \|\Lambda q\|_{\infty} = \alpha\}. \quad (32)$$

Specifically,

$$Q_w' = \{\arg\max_{\|\mathbf{q}\|_{\Phi}^2 = 1} \|\mathbf{K}\mathbf{q}\|_{\infty}\} \subseteq Q_w,\ \mathbf{K} = diag(|\kappa_1|^{1/4}, \cdots, |\kappa_M|^{1/4}). \quad (33)$$

Comparing the CM and SW cost functions in (19) and (20), we see that the SW receiver can be obtained from the minimization of the CM cost under the unit output variance constraint. In Li-Ding ([8], [12]) and Gu-Tong ([10], [16]) established the equivalence between CM and SW receivers for sub-Gaussian i.i.d. sources. Here we give a generalization to deal with heterogeneous sources.

Lemma 2 [10], [11], [15]: *If* $\mathbf{f}^* \in \mathcal{F}_{CM}$ *and* $\kappa_4(\mathbf{x}'\mathbf{f}^*) < 0$, *then* $\frac{1}{\|\mathbf{f}^*\|_{\mathbf{R}}^2}\mathbf{f}^* \in \mathcal{F}_{SW}$. *Conversely, if* $\mathbf{f}^* \in \mathcal{F}_{SW}$ *and* $\kappa_4(\mathbf{x}'\mathbf{f}^*) < 0$, *then* $\gamma^*\mathbf{f}^* \in \mathcal{F}_{CM}$ *for some* γ^*. *If all sources are sub-Gaussian, then* $\mathcal{F}_{SW} \approx \mathcal{F}_{CM}$.

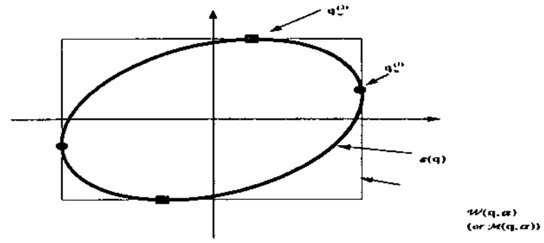


Fig. 3. Wiener receiver as constrained optimization on $\mathbf{e}(\mathbf{q})$ by enlarging $\mathcal{W}(q,\alpha)$ for homogeneous sources.

A. The Noiseless Case

The Homogeneous Sources

When $\sigma^2 = 0$, $\Phi = \mathbf{I}_M$, and $\mathcal{E}(\mathbf{q}) = \{\mathbf{q} : \|\mathbf{q}\|_{\Phi} = 1\}$ defines a unit 2-norm ball. Assume that $|\kappa_i| = 1$,

$$\mathcal{M}(\mathbf{q},\alpha) = \{\mathbf{q} : |\sum_i \kappa_i q_i^4| = \alpha\} = \{\mathbf{q} : \|\mathbf{q}\|_{\infty} = \alpha\} \quad (34)$$

defines a inf-norm ball of radius $\alpha^{1/4}$. Hence, optimization SW criterion is equivalent to enlarging the inf-norm ball $\mathcal{M}(\mathbf{q},\alpha)$ and SW receivers are obtained by finding the largest possible inf-norm ball that has intersections with the 2-norm ball, as shown in Fig.3.

The Heterogeneous Sources

When some sources are super-Gaussian $(\kappa_i > 0)$ and others are sub-Gaussian $(\kappa_i < 0)$, the manifold

$$\mathcal{M}(\mathbf{q},\alpha) = \{\mathbf{q} : \|q_1^4 - q_2^4\| = \alpha\} = \{\mathbf{q} : \|\mathbf{q}\|_{\infty} = \alpha\} \quad (35)$$

is no longer a 4-norm ball, but an inf-norm ball. It is, in q_i^2, a multi-sheet hyperboloid. Figure 4 shows the case when $\kappa_1 = -\kappa_2 = 1$. Again, SW and MMSE receivers are equivalent $Q_{SW} = \{\pm\mathbf{e}_i\}$. When there are zero 4th – order cumulant sources (Gaussian like sources), the manifold $\mathcal{M}(\mathbf{q},\alpha)$ becomes a cylinder and those extrema associated with non-zero cumulant sources are not affected.

Theorem 4 [10], [11], [15] [*Assume that* $\sigma = 0$. *Then*

$Q_{SW} = \{\pm\mathbf{e}_i\}_{i=M_0+1}^{M_0+M_-} \cup \{\pm\mathbf{e}_i\}_{i=M_0+M_-+1}^{M}$.

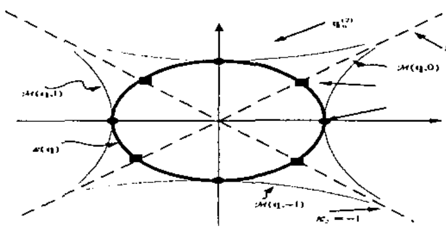


Fig. 4 The noiseless case for heterogeneous sources $\kappa_1 = 1, \kappa_2 = -1$.

B. The Noisy case

Again, SW receivers can be viewed as tangent points of $\mathcal{M}(\mathbf{q},\alpha)$ and $\varepsilon(\mathbf{q})$. Since the SW cost function depends only on the system parameter $\mathbf{q}$ and source cumulants but not on the noise statistics, the manifold $\mathcal{M}(\mathbf{q},\alpha)$ remains the same as that for the noiseless case. The difference lies in the constraint $\varepsilon(\mathbf{q})$ which now is an ellipsoid whose shape and orientation depends on the noise variance σ^2 and the channel matrix $\mathbf{H}$. Figure 4 and 5 illustrate such scenarios for both homogeneous and heterogeneous sources. Since Φ is diagonal if and only if either there is no noise or columns of $\mathbf{H}$ are orthogonal, it suggests that SW receivers coincide with the Wiener receivers if and only if there is no noise or channel matrix has orthogonal columns.

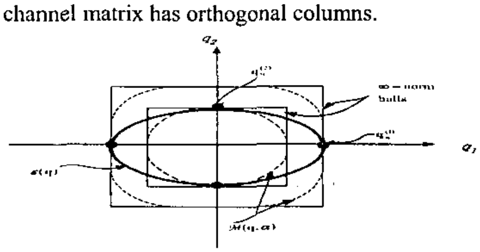


Fig. 5 The noisy homogeneous case with $\kappa_1 = 1, \kappa_2 = 1$.

Theorem 5 [10], [11], [16]] *Let Φ be partitioned as*

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{21}' \\ \Phi_{21} & \Phi_{22} \end{pmatrix}$$

where Φ_{11} *corresponds to zero-kurtosis sources. Then*

$$Q_{SW} \approx \{\pm \mathbf{e}_i\}_{i=M_0+1}^{M_0+M_-} \cup \{\pm \mathbf{e}_i\}_{i=M_0+M_-+1}^{M} \qquad (36)$$

if and only if $\Phi_{21} = 0$ and Φ_{22} *is diagonal.*

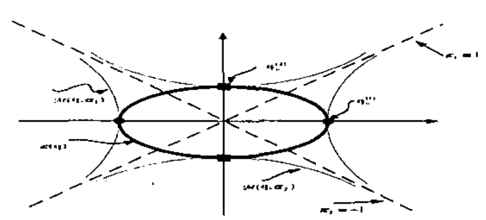


Fig. 6. The noisy heterogeneous case: $\kappa_1 = 1, \kappa_2 = -1$.

3. Calculation of Infinite Norm Ball for CMA/SW receivers by D-K iteration

The structured singular value (μ) approach through D-K iteration [17] can be used to solve the infinite norm ball problem of equation (17) ([20]-[21]). The following robustness conditions will be the basis for the proposed synthesis method.

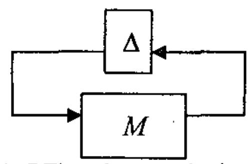


Fig.7 The robust synthesis problem.

Theorem 6 The system in equation (17) shown in Fig.7 achieves the infinite norm ball if and only if (*iff*) the following condition holds: $\inf_{Q\, D\in\underline{D}} \left\|D^{-1}\Phi D\right\|_\infty < 1$ where $\underline{D} := \{diag(d_1,\cdots,d_n) : d_i > 0\}$.

The dependence of Φ on the controller K can be captured through the Youla parameter Q which provides a parameterization of all possible Φ's that can be obtained with a stabilizing controller ([20]-[21]). Hence, the robust synthesis problem can be stated as follows:

$$\inf_{D\in\underline{D}} \inf_{Q\in\ell_\infty} \left\|D^{-1}\Phi(Q)D\right\|_\infty := \gamma^*, \qquad (37)$$

where $\Phi(Q) = H - U * Q * V$ is the standard parameterization of closed-loop system using Youla parameter Q. For each fixed D = diag(d_1, d_2), this problem is a standard ℓ_∞ norm-minimization problem [21]. By introducing a regularizing bound Q of the form: $\|Q\|_\infty \le \alpha$, we define

$$\gamma_{opt} := \inf_{\substack{\text{subject to} \\ \|Q\|_\infty \le \alpha \\ \Phi = H - U*Q*V \\ D\in\Re}} \left\|D^{-1}\Phi_\varepsilon D\right\|_\infty. \qquad (38)$$

Clearly, if the optimization problem (38) has a solution Q_{opt}, then for any $\alpha \ge \|Q_{opt}\|_\infty$ the two problems (37) and (38) are equivalent. Indeed, if we define $A_{\ell_\infty}^R \Phi \mapsto A_{\ell_\infty}(D^{-1}\Phi(Q)D)$, then for the given R, the problem $\inf_{Q\in\ell_\infty}\left\|D^{-1}\Phi(Q)D\right\|_\infty$ is equivalent to the program $\inf \gamma_{opt}$

subject to

$$A_{\ell\infty}^D(\Phi^+ + \Phi^-) \le \gamma$$
$$A(\Phi^+ - \Phi^-) = b, \quad \Phi^+, \Phi^- \in \ell_\infty^{n_w \times n_z}, \qquad (39)$$
$$\Phi^+, \Phi^- \ge 0.$$

The objective function as well as the constraints is linear. A solution based on D-K iteration methods can be obtained ([17], [20], [21]).

4. Design Example with CMA/SW/MMSE of FIR Wiener Receivers for QAM system

According to the results of [8] and [12], QAM communication systems are similar to the PAM communication systems in Fig.1 except that the signal a_n, the channel response $h(t)$, and the noise $w(t)$ are all complex-valued. All Intersymbol Interference (ISI) is removed if the equalizer only retains a phase ambiguity.

$$H(z^{-1})G(z^{-1}) = e^{j\theta}, \theta \in [0, 2\pi]. \tag{40}$$

where $H(z)$ is the transfer function of equalizer, used to recover the transmitted symbols: D_n is estimated by $\hat{D}_n = sign([C(z)]X_n)$. The optimal choice for $C(z)$ in the mean square sense, that is, $C(z)$ is chosen such that

$$\|[C(z)]X_n - D_n\|^2 \tag{41}$$

is minimum, and

$$C(z) = \frac{F^*(z^{-1})}{F(z)F^*(z^{-1}) + \sigma_w^2}. \tag{42}$$

Let $T(z)$ is approximated by an MA filter with input Y_n and output $Z_n = \sum_{k=0,q} b_k Y_{n-k}$. The coefficient $(b_k)_{k=0,q}$ are chosen so as to minimize the CM criterion: $\left\| |Z_n|^2 - 1 \right\|^2$.

The transfer function $H(z)$ is selected such that $\left\| |[H(z)]Y_n|^2 - 1 \right\|^2$ is minimum and for QAM systems

$$H(z) = e^{j\theta}T(z), \text{and } G(z) = \sqrt{\alpha}\prod_{k=1,K}(1 - z_k z^{-1}), \quad |z_k| < 1. \tag{43}$$

Finally, in order to suppress the phase error factor $e^{j\theta}$, the following algorithm is used

$$S_n = Z_n e^{-j\theta_{n-1}}, \tag{44}$$

$$\theta_n = \theta_{n-1} + \mu I_m[S_n(\hat{D}_n - S_n)^*], \tag{45}$$

where $\hat{D}_n = sign[S_n]$ are the decisions at the output of the equalizer.

Definition 3.1: An attainable set S_A is the set of all s that the finite length equalizer θ can attain

$$S_A \overset{\Delta}{=} \{\mathbf{s} : s_n = \sum_{i=1}^{p}\sum_{k=1}^{N} \theta_{i,k} h_{n-k}^{(i)}, \theta_{i,k} \in C\} \subset C^{N+K+1}.$$

Definition 3.2: A hypercone S_n is defined as

$$S_n \overset{\Delta}{=} \{\mathbf{s} : |s_n| > |s_k| \text{ for all } k \neq n\}.$$

Let $\{\mathbf{e}_n\}_{n=0}^{N+K}$ be the standard unit orthogonal basis vectors in $\Re^{N+K+1}$. The following result can be directly inferred from the definition of S_n and S_A.

Lemma 3.1: [8], [12] *S_n has the following properties:*

1) $\mathbf{E}_n \overset{\Delta}{=} \{\mathbf{s} = e^{j\phi}\mathbf{e}_n : \phi \in [0, 2\pi]\}$ is the only set of minima for Godard and Shalvi-Weinstein cost functions in the cone S_n [16].

2) If $\mathbf{s} \in S_A \cap S_n$, then $\alpha\mathbf{s} \in S_n$ for any $\alpha \in C$.

3) If $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)} \in S_n$, then $\alpha_1\mathbf{s}^{(1)} + \alpha_2\mathbf{s}^{(2)} \in S_n$, for all $\alpha_1, \alpha_2 \in C$ such that $\measuredangle(\alpha_1) + \measuredangle(s_n^{(1)}) = \measuredangle(\alpha_2) + \measuredangle(s_n^{(2)})$.

Notice that the Godard and SW cost functions are constant over each set of minima $\mathbf{E}_n$. Therefore, $\{\mathbf{E}_n\}_{n=0}^{N+K}$ are N+K+1 global minimum sets. Since the parameters in QAM systems are complex, the definitions of **h**, **f**, **s**, attainable set T and supersphere $\Phi(r)$ are almost same as those in Section II. For a blind equation in a QAM system, it is obvious that if s is one of the minima. Hence, the minima of equalizer can be divided into connected minimum sets.

Theorem 7 [8], [12]: *For FIR equalizer filters in QAM systems, there is one-to-one correspondence between minimum sets of CMA and SW equalizers.*

Theorem 8. [8], [12]: *Let* S_A *be the attainable set of a finite-length FSE equalizer in noiseless QAM communications systems. If* $\mathbf{E}_n \subset S_A \cap S_n$, *then* $\mathbf{E}_n$ *is the only minimum set in* $S_A \cap S_n$ *while there are no minimum points on its boundary* $S_A \cap S_n$.

Design Example:

The simulink model [17] is used to simulate the behavior of the CMA for 16 QAM systems ($\gamma_p = 1.32, \gamma_p = 1$ for 4 QAM). The whole parameters are setting listed in Table 1. The standard stochastic gradient Godard algorithm (CMA) (11) is used in the simulations. The step-size μ is selected so that the algorithm can achieve the fastest convergence without causing instability.

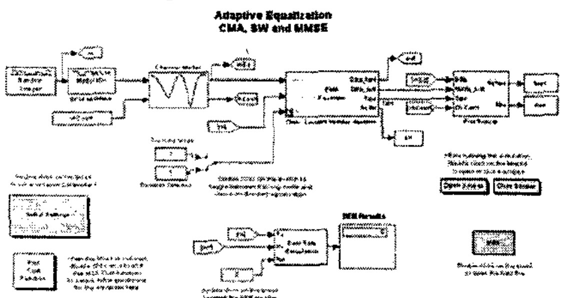


Fig.8. Adaptive Equalization CMA, SW and MMSE via Simulink model [17].

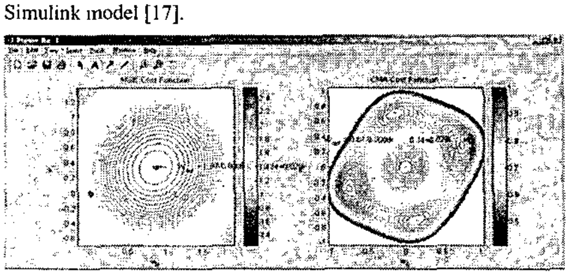
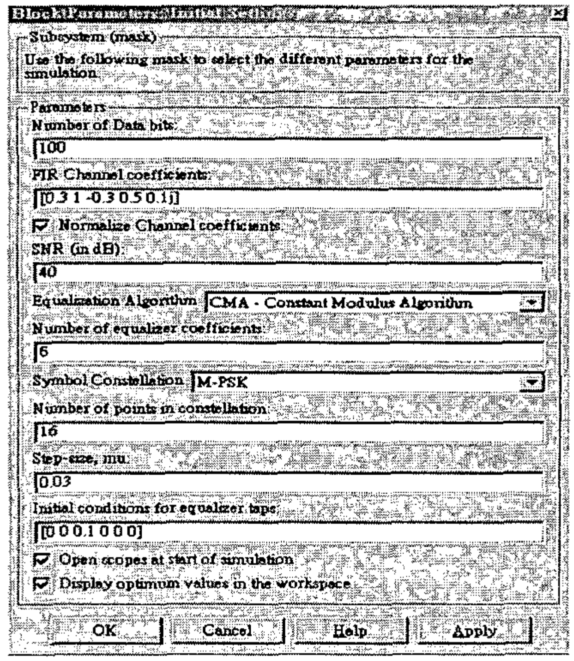


Fig. 9a. The cost function plot for MMSE and CMA.



Block Parameters: Initial Settings

Subsystem (mask)

Use the following mask to select the different parameters for the simulation.

Parameters

Number of Data bits: 100

FIR Channel coefficients: [0.3 1 -0.3 0.5 0.1i]

☑ Normalize Channel coefficients

SNR (in dB): 40

Equalization Algorithm: CMA - Constant Modulus Algorithm

Number of equalizer coefficients: 6

Symbol Constellation: M-PSK

Number of points in constellation: 16

Step-size, mu: 0.03

Initial conditions for equalizer taps: [0 0 0.1 0 0 0]

☑ Open scopes at start of simulation

☑ Display optimum values in the workspace

OK | Cancel | Help | Apply

Table I Values of Parameter setting.

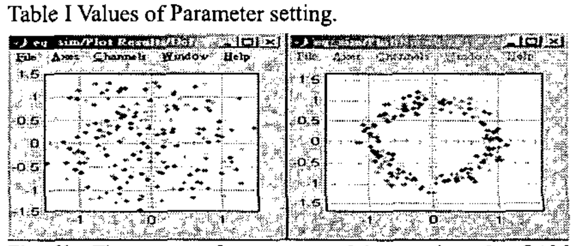


Fig. 9b. The output of constant modulus receiver for QAM system.

The calculation of infinite norm ball solving equation in (38) by D-K iteration of the system model in (40) is about 1.8.

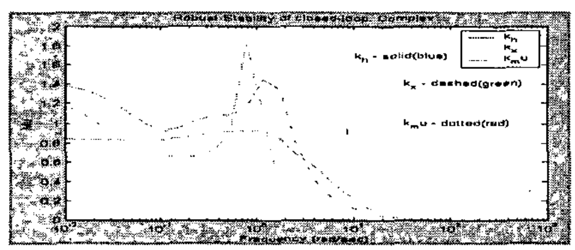


Figure 10. The infinite norm ball by D-K iteration method.

5. Conclusion

In this paper, we presented a geometrical approach to calculate the infinite norm ball value for the analysis of CM, SW and MMSE receivers in the presence of noise for QAM systems. By transforming various blind receiver design problems to expanding (or shrinking) norm balls of different types constrained on an ellipsoid, this approach reveals the connection among CM, SW, and Wiener receivers by finding its infinite-norm ball on SW cost function. When the columns of **H** are orthogonal, we concluded that in the global response space, domain of attraction of SW receivers are the minimum distance decision regions on an ellipsoid determined by the channel condition, the signal power, and the noise variance. The analysis of CM/SW receivers for the 2-D case is significant: not because of its generality but for the insights into the behavior. It allows us to quantitatively assess the performance loss of blind receivers when they are compared with nonblind Wiener receivers. All of the equalizers are verified by the Simulink models and bergulator from Cornell Professor Johnson Jr. [2], and Matlab mu-simulation toolbox to check their results.

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