

## Modified Subgridding Algorithm with Multilevel Scheme for Two-Dimensional Finite-Difference Time-Domain Method

CHING-CHENG CHANG and SHYN-KANG JENG

Graduate Institute of Communication Engineering and Department of Electrical Engineering  
National Taiwan University  
No.1, Sec. 4, Roosevelt Rd., Taipei, Taiwan 10617, ROC.  
Email: skjeng@ew.ee.ntu.edu.tw

**Abstract** — Finite-Difference Time-Domain Method (FDTD) has being very popular in solving electromagnetic problems. However, for those that contain small structures or strongly varying fields, very fine meshes have to be used. Since a finer mesh means longer computing time and larger memory, subgridding would be one of the ways to improve the performance of the FDTD method. To expand the power of subgridding, we develop a multilevel subgridding scheme. This new scheme can provide smoother transitions than the conventional one-level subgridding scheme, and achieve better performance.

### 1 Introduction

A subgridding scheme divides the problem into regions with different grid sizes. Usually the grid size in coarse region is  $\lambda/10 \sim \lambda/20$  as in normal FDTD, and this size is scaled by an integer ratio in the fine region. For previous studies on subgridding [1]-[3], there's usually only one subgrid region. If the resolution is not enough, a higher ratio of grid sizes for subgridding would be needed. It will take longer computing time and may cause more error due to more serious mismatch of grid sizes. A modified subgridding algorithm with multilevel scheme can divide grids by a small grid-size ratio between adjacent level to simplify programming and reduce the error owing to the mismatch of grid size. A multilevel subgridding scheme for 2D TM mode had been studied in [4]. A TE version of this scheme is given in this paper.

### 2 Multilevel Subgridding Scheme

A multilevel subgridding scheme is shown in Figs. 1 and 2. Here the ordinary FDTD and subgridding method is as the core process. For a 2-D TE FDTD problem, a coarse grid is divided by a factor of 3, and a second-order homogeneous traveling wave equation is used for spatial interpolation to find out the extra information at the boundary of a subgrid region. During the transition between grids of adjacent levels, linear interpolation in time and bilinear interpolation in space are used. And for updating the E fields at the boundary of a upper grid, the data are sampled at the common E field points located at the boundaries of the lower-level subgrid region.

Further, a tree-like data structure is used in which a flag is added to tell if the grid of a certain level is divided into subgrids or not. As the algorithm shown in Fig. 2, the calculation is designed to be recursive. First the calculation starts at the very lower-left main cell and scans all cells row by row. While scanning all cells, the program will determine if this is a grid with a subgrid. Once the program finds that the cell being dealt with has a subgrid, it starts another FDTD procedure for this subgrid region. If the cell is not subdivided, a normal iteration process of FDTD is applied. Every subgrid region basically is treated as a normal FDTD case, where the process starts at the grid with index  $(i, j) = (0, 0)$  in the local index system and has boundary conditions defined by interaction of

regions in adjacent levels. It also identifies if there is any other subgridding of next level while doing iteration. This process repeats until all cells in the region have been calculated. The control is then returned to the previous level.

### 3 Numerical Example

For verification, a PEC cylinder with a magnetic current filament in free space shown in Fig. 3 is solved. The frequency of the current source is 2.4GHz and the main grid size is 6.25mm. The test region is divided into 141 x 101 main cells with a  $\lambda/20$  mesh size. At the border of the test zone, the second-order Mur's absorbing boundary condition is applied. Four different schemes were tested and compared. One scheme is with uniform coarse grid with cell size  $\Delta=6.25\text{mm}$ , and one is with uniform fine grid in 1/3 main cell size. The other two schemes are with subgrids: one is with a level of subgrid, in which the subgrid size is  $\Delta/3$ ; and the other is with two levels of subgrids as shown in Fig. 4. The size of cells in the second level of the two-level scheme is  $\Delta/9$ .

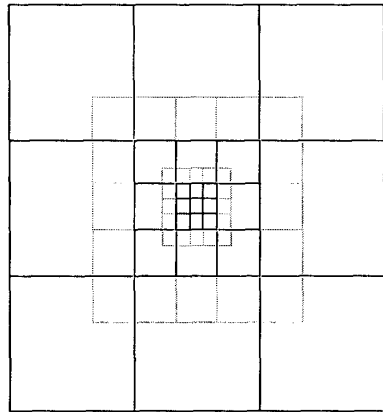
The curves in Fig. 5 report the steady state total field which is the response after about 27 periods around a circle of radius 12.8125cm outside the cylinder. Comparisons of different schemes are given in Table I. The one-level subgridding scheme works well as in other reports. The improvement of this scheme in TE mode is similar to in TM mode version. In Table I, the uniform fine grid can obtain the best results, but the two-level scheme can still improve the accuracy than one-level subgrid and takes only 59.13% of the time in the uniform fine grid scheme. Moreover, for a single round of simulation, the two-level subgridding already offers 9 times the resolution of the main cell, and 3 times the resolution of the uniform fine grids within a comparable short time.

### 4 Conclusion

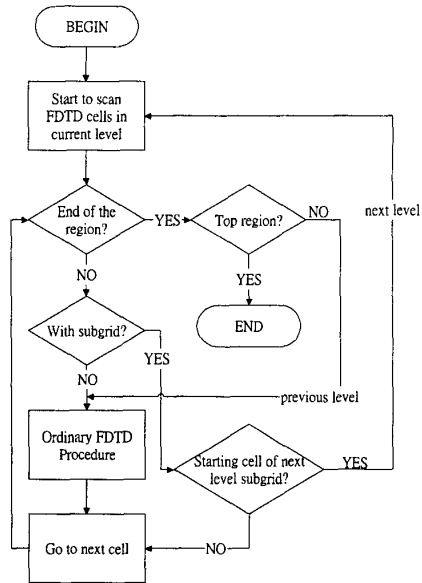
A modified subgridding algorithm with multilevel scheme has been developed. It has been shown that it can improve the resolution of FDTD. Though we only apply it for a 2D case here, it can be easily extended for general 3D problems.

### Reference

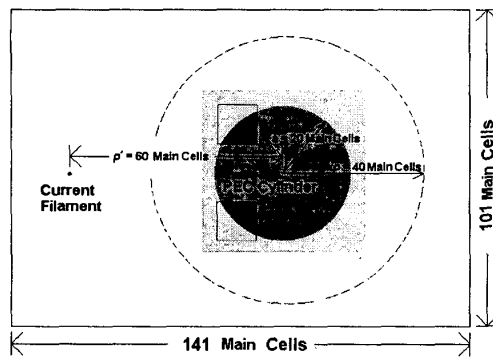
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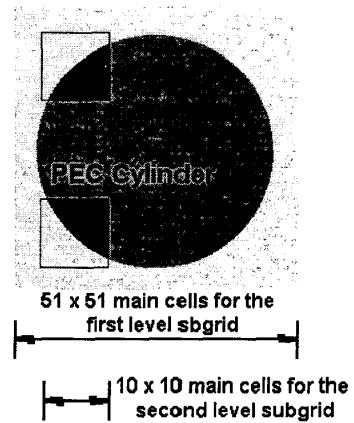
**Fig. 1** Multilevel subgridding



**Fig. 2** The recursive algorithm for multilevel subgridding



**Fig. 3** A PEC cylinder with a current filament



**Fig. 4** Range of subgridding

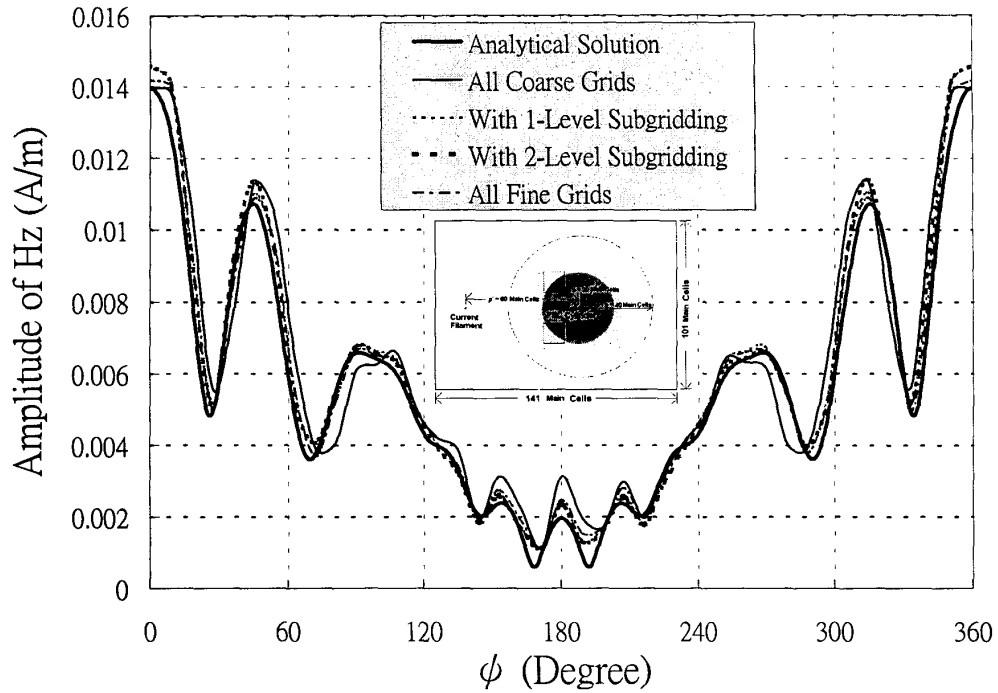


Fig. 5 Total field around a circle outside of the PEC cylinder in Fig. 3

Scheme	Cell size	Time step	CPU time	Cell numbers	Root mean square error
uniform coarse grid	$\Delta=6.25\text{mm}$	$\Delta t=20.833\text{ps}$	140.72s	14241	0.000989435
1-level subgridding	$\Delta, \Delta/3$	$\Delta t, \Delta t/3$	754.18s	37650	0.000553545
2-level subgridding	$\Delta, \Delta/3, \Delta/9$	$\Delta t, \Delta t/3, \Delta t/9$	2044.16s	53850	0.000552623
uniform fine grid	$\Delta/3$	$\Delta t/3$	3456.79s	128169	0.00048024

Table I Performance of different schemes for the problem shown in Fig. 3 with magnetic current filament in TE mode. For every scheme 1500 coarse-grid time steps are requested.