

# Quasi-3D beam-propagation method for simulating quasi-phase-matched second-order nonlinear interaction

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**Abstract:** An efficient quasi-3D beam propagation method is developed to simulate the parametric interaction in bulk quasi-phase-matched materials. The results predicted by the quasi-3D scheme show that soliton behavior is possible for the parametric converted beam.  
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**OCIS codes:** (190.4410) Nonlinear optics, parametric processes

## 1. Introduction

Coherent mid-IR sources have important applications in fiberoptic chemical sensors, spectroscopy, industrial process monitoring, atmospheric and environmental monitoring, etc. Quasi-phase-matched (QPM) parametric interaction are widely used for IR-generation recently [1], so theoretical models for predicting QPM parametric interaction become increasingly important [2]. However, in the high-efficiency configuration, analytical analysis is difficult, so numerical techniques based on beam propagation method (BPM) had been developed to model nonlinear interaction [3]. In the past, accurate and efficient 2D models that take into account the 1D variation of beam profile had been explored for QPM parametric interaction in waveguide structures [4]. However, in the parametric interaction, the beam profile has 2D variation. Especially, when the propagation beam is not confined in the waveguide, the 2-D models over-estimate the nonlinear conversion. For better simulation, 3D models are necessary, but direct extension of those models to 3D cases is limited because either accuracy or efficiency has to be sacrificed. Therefore, we develop a quasi-3D method to model the QPM parametric interaction in bulk materials without sacrificing neither accuracy nor efficiency. The beam divergence in both transverse directions is considered. Hence this scheme predicts results closer to the experiment than the 2D simulation. With the quasi-3D model, it is also discovered that, with a proper choice of the convergent signal and pump beams, a constant beam size of the idler beam could be maintained in the crystal, with behavior being like soliton propagation.

## 2. Numerical model

The quasi-3D model is based on the 2D iterative finite-difference beam propagation method (IFDBPM) [4]. Applying the phasor notation, the wave equation in the presence of nonlinear polarization is reduced to the following form in the paraxial approximation:

$$2jk_{o1}\bar{n}_1\frac{\partial E_1}{\partial z} = \nabla^2 E_1 + k_{o1}^2 \chi^{(2)} e^{-j\Delta k \cdot z} E_3 E_2^* \quad (1a)$$

$$2jk_{o2}\bar{n}_2\frac{\partial E_2}{\partial z} = \nabla^2 E_2 + k_{o2}^2 \chi^{(2)} e^{-j\Delta k \cdot z} E_3 E_1^* \quad (1b)$$

$$2jk_{o3}\bar{n}_3\frac{\partial E_3}{\partial z} = \nabla^2 E_3 + k_{o3}^2 \chi^{(2)} e^{j\Delta k \cdot z} E_1 E_2 \quad (1c)$$

where  $\bar{n}$ ,  $k_o$ ,  $\chi^{(2)}$  and  $E$  are the refractive index, the wave vector in free space, the nonlinear coefficient, and the slowly varying envelope of the field, respectively. Subscripts, 1, 2, and 3 represent the pump, signal, and idler waves, respectively.  $\Delta k \equiv \bar{n}_{o3}k_{o3} - \bar{n}_1k_{o1} - \bar{n}_2k_{o2}$ . Propagation along the z direction is assumed.

Because the experiments of QPM parametric interaction in bulk materials usually have cylindrically symmetric beam profiles,  $\phi$ -independence could be applied. The transverse Laplacian is then similar to the 2D case, but has singularity at the origin. With some special treatments, the problem of singularity can be solved. Then the quasi-3D

scheme is as follows:

$$\frac{2jk_{o1}\bar{n}_1}{\Delta z}(E_1^{m,s+1(i)} - E_1^{m,s}) = \frac{1}{2} \left[ L_1(E_1^{m,s} + E_1^{m,s+1(i)}) + (F_1^{m,s}E_3^{m,s}E_2^{m,s*} + F_1^{m,s+1}E_3^{m,s+1(i-1)}E_2^{m,s+1(i-1)*}) \right] \quad (2a)$$

$$\frac{2jk_{o2}\bar{n}_2}{\Delta z}(E_2^{m,s+1(i)} - E_2^{m,s}) = \frac{1}{2} \left[ L_2(E_2^{m,s} + E_2^{m,s+1(i)}) + (F_2^{m,s}E_3^{m,s}E_1^{m,s*} + F_2^{m,s+1}E_3^{m,s+1(i-1)}E_1^{m,s+1(i-1)*}) \right] \quad (2b)$$

$$\frac{2jk_{o3}\bar{n}_3}{\Delta z}(E_3^{m,s+1(i)} - E_3^{m,s}) = \frac{1}{2} \left[ L_3(E_3^{m,s} + E_3^{m,s+1(i)}) + (F_3^{m,s}E_1^{m,s}E_2^{m,s*} + F_3^{m,s+1}E_1^{m,s+1(i-1)}E_2^{m,s+1(i-1)*}) \right] \quad (2c)$$

where  $i$  is the iteration count and  $E^{(i)}$  is the  $i$ -th iteration field.  $E^{(0)}$  can be derived by other non-iterative means. The above operators are defined by

$$L_r E_i^{m,s} = \frac{1}{\Delta r^2} \left[ \frac{1}{2m} (E_i^{m+1,s} - E_i^{m-1,s}) + (E_i^{m+1,s} - 2E_i^{m,s} + E_i^{m-1,s}) \right], \quad i=1,2,3$$

$$F_i^{m,s} = k_{oi}^2 \chi^{(2)m,s} e^{-j\Delta k \cdot s \cdot \Delta z}, \quad i=1,2$$

$$F_3^{m,s} = k_{o3}^2 \chi^{(2)m,s} e^{j\Delta k \cdot s \cdot \Delta z}$$

where  $E^{m,s}$  represents the electric field at the point  $(r, z) = (m \cdot \Delta r, s \cdot \Delta z)$ .

### 3. Simulation and discussions

The simulation is performed for difference frequency generation (DFG) in a bulk periodic poled LiNbO<sub>3</sub> (PPLN). The values and conditions for the simulation are listed in Table 1.

Table 1. Values and conditions used in simulation

Wavelength ( $\mu\text{m}$ )	Initial power	Initial beam shape	$d_{33}$	$\Delta x, \Delta r$	$\Delta z$	Crystal length
0.8594 (pump)	0.48 W (pump)	Gaussian (pump)	22 pm/V	1.23 $\mu\text{m}$	1 $\mu\text{m}$	6mm
1.064 (signal)	Varied (signal)	Gaussian (signal)				
4.47 (idler)	0 (idler)	---				

Results predicted by both 2D and quasi-3D schemes are compared to a reported experiment [1]. The comparisons are shown in Fig. 1. As shown in this figure, the quasi-3D scheme predicts the experimental results much better than the 2D scheme.

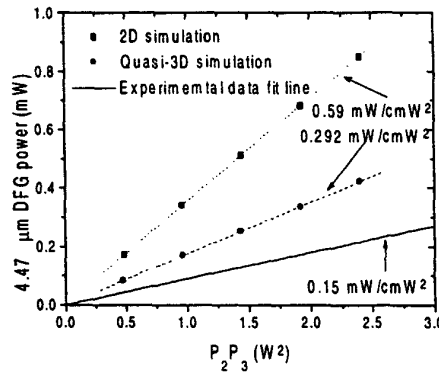


Fig.1 DFG power vs product of pump and signal powers.

The quasi-3D scheme had also predicted that the beam profiles at the exit facet deviate from the Gaussian shape. The physical reason is due to nonlinear conversion of the parametric interaction. This fact indicates that plane-wave approximation and even analytical result based on Gaussian beam for parametric interaction are invalid.

Simulation also reveals that the beam size of the idler wave is influenced by two factors: the convergence or divergence of the pump and the signal waves and the diffraction of the idler beam itself. If both the pump and the signal beams are focused at the end of the crystal, the convergence factor could be stronger than the divergence factor. Then the beam size of the idler beam shrinks as it propagates along the crystal, as shown in Fig. 2. On the other hand, with a proper choice of the convergent signal and pump beams, both factors could exactly cancel out one another to maintain a constant beam size of the idler beam in the crystal and the idler beam then behaves like soliton.

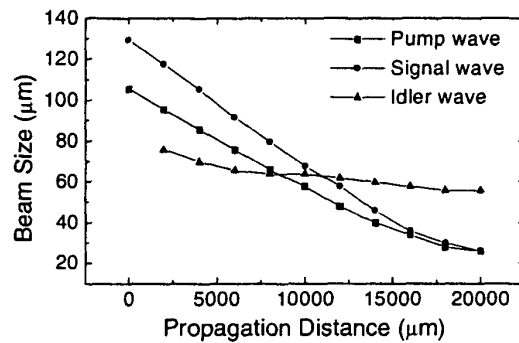


Fig.2 Variation of the beam sizes vs. propagation distance for pump and signal beams focusing position at the end of the nonlinear crystal.

#### 4. References

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