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Hybrid System Based Adaptive Control for the Nonlinear HVAC System

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Abstract— **An adaptive controller is designed for the nonlinear MIMO heating, ventilating, and air conditioning (HVAC) system. The designed controller has the ability to adapt to the slowly time varying load change of thermal space and maintains good tracking performance for temperature and humidity ratio. Moreover, we integrate a supervisory switching logic into this system to adjust the CO_2 concentration of thermal space and the whole closed loop system is modelled as a hybrid system. The obtained control system shows robustness and effectiveness and simulation results are provided to illustrate the control performance.**

I. INTRODUCTION

Occupant comfort and energy efficiency are two primary goals of control strategies for the heating, ventilating, and air conditioning (HVAC) systems. As indicated in [1], HVAC systems for buildings are major consumers of electrical energy through the world. Temperature, humidity ratio and CO_2 concentration are the quantitative indices of comfort in a room. To control such systems efficiently and effectively with dynamic interactions and disturbances so as to conserve energy while maintaining the desired thermal comfort level requires more than a conventional methodology. Classical HVAC control techniques such as the ON/OFF controllers (thermostats) and the proportional-integral-derivative (PID) controllers are still very popular because of their low cost. However in the long run, these controllers are expensive since they operate at a very low-energy efficiency. With increasing complexity of modern HVAC systems, how to control and optimize the operation with guaranteed performance, stability and reliability becomes a challenging issue. In fact, the air conditioning process is highly nonlinear and the interaction between temperature and humidity is significant. A nonlinear HVAC model which includes dynamics of temperature and humidity ratio is proposed in [2] where an observer to estimate the thermal load and moisture load is designed. The controller with load estimator can achieve the desired performance while the value of loads are not the designed one. In that paper, the load dynamics are assumed to be simple ones, that is, the loads are assumed to be constants. In [3], feedback linearization technique is applied to the same model. In [4], the actuator's dynamics are considered and the feedback linearization approach is adopted to design the controller. The thermal load is treated as a measurable disturbance and is compensated by the feedback controller. But the humidity ratio of thermal space is not controlled since the authors chose different output function. In [5], a decentralized nonlinear adaptive controller consists of a fuzzy feedback controller and a frequency-domain adaptive compensator designed in Fourier space is proposed. The control scheme is capable of dealing with the varying thermal loads and is with short setting.

In this paper, we design an adaptive controller for the nonlinear MIMO HVAC system which is modelled to have some unknown parameters, to achieve robust and good heating, ventilating, and air conditioning performance. These slowly time varying unknown parameters are the thermal heat load and moisture load. In most of the related literatures, the values of the loads are treated as constants. Therefore, the controller proposed here should be more robust, more practical, and still achieving satisfactory performance. In particular, the system with the adaptive controller can operate effectively in the presence of dynamic interactions and disturbances while maintaining the desired thermal comfort level. Adaptive control for nonlinear systems has received a significant research attention and has evolved as a powerful methodology for nonlinear systems with parametric uncertainties. Feedback linearization approach proposed in [6] enhances the robustness to the failure of exact linearization. Many applications of nonlinear adaptive control have been presented in the past few years. In [7], the nonlinear adaptive controller based on feedback linearization is applied to the electro-hydraulic servomechanism. The SISO nonlinear system has a strong relative degree of two and trivial zero dynamics. In [8], adaptive control for nonlinear system are combined with the neural network approach and the nonlinear systems are assumed to have the normal form. This approach has the advantage with increasing computation speed and no need to analyze the complex nonlinear behavior of the system. The HVAC system considered in our paper is two-input, two-output, and has no relative degree. We employ the dynamic extension algorithm to synthesize the feedback controller for the system and then design an adaptive controller to track the desired temperature and humidity ratio.

The remainder of this paper is organized as follows: section II introduces operation process and the hybrid dynamic model of the HVAC system. In section III, feedback linearization via dynamic extension is applied and a non-adaptive controller is designed to achieve the asymptotical stability for the tracking error. Then, we introduce some parametric uncertainties to replace the ideal model and derive the adaptive controller for the MIMO nonlinear plant. We also integrate a switching scheme to adjust the CO_2 concentration. Section IV shows the simulation results and section V concludes this paper.

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II. Hybrid System Model for the HVAC System and Problem Formulation

A. HVAC Model with Temperature and Humidity Ratio

Air in the room is assumed to have an uniform temperature distribution and heat loss between components is neglected. Here we employ the model proposed in [2] which includes both temperature and humidity ratio. The system operates as shown in Fig. 1. Outdoor air enters the system at temperature T_0 and with volumetric flow rate $f_r(t)$. Air with temperature T_0 and flow rate $f_r(t)$ passes through the heat exchanger where an amount of heat is exchanged with the air. Since we have the assumption of perfect mixing, the air temperature inside and exiting the heat exchanger is $T_2(t)$, which represents the supply air temperature. After being cooled or heated in the heat exchanger, the air with temperature T_2 passes into the thermal space with the help of fan and air temperature of the thermal space is $T_3(t)$. The heat load in the room is denoted as Q_o. To save energy, typically 25% of the air is drawn out of the thermal space through the help of fan, and 75% of the air is recirculated to be mixed with the fresh air from outdoor.

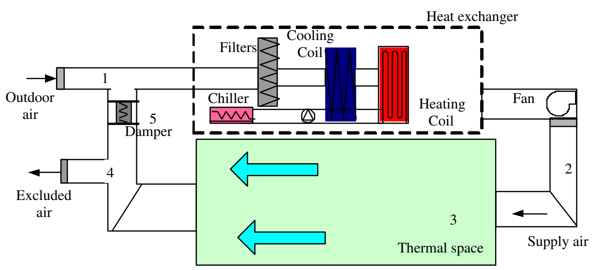

Fig. 1. Schematic layout of the HVAC system

Here, we assume the system is operating either with the cooling process and or with the heating process, but both models will be the same except the signs which can be either positive or negative. Control inputs to the system are the pumping rate (gpm) of cold water from chiller to heat exchanger and the air flow rate (f_r) using the variable speed fan. Notations and parameter values used in the dynamic model and simulations here are the same as those in [2] and are given in Table I.

From energy conservation principle, the dynamic equations of the HVAC system are given by

$$
\begin{aligned}
\dot{T}_3 &= \frac{60 f_r}{V_s}(T_2 - T_3) - \frac{60 h_{fg} f_r}{c_p V_s}(W_s - W_3) \\
&\quad + \frac{1}{(1-\mu)\rho_a c_p V_s}(Q_o - h_{fg} M_o) \\
\dot{W}_3 &= \frac{60 f_r}{V_s}(W_s - W_3) + \frac{M_o}{\rho_a V_s} \\
\dot{T}_2 &= \frac{60 f_r}{V_{he}}(T_3 - T_2) + \frac{60(1-\mu) f_r}{V_{he}}(T_0 - T_3) \\
&\quad - \frac{60 h_w f_r}{c_p V_{he}}((1-\mu)W_0 + \mu W_3 - W_s) - 6000 \frac{\text{gpm}}{\rho_a c_p V_{he}}.
\end{aligned}
$$

TABLE I

HVAC System Variables and Parameters

c_p	Specific heat of air at constant pressure 0.24 (btu/lb$\cdot^\circ F$)
ρ_a	Air mass density 0.074 (lb/ft^3)
V_{he}	Volume of heat exchanger 60.75 (ft^3)
V_s	Volume of thermal space 58464 (ft^3)
W_0	Humidity ratio of outdoor air 0.018 (lb/lb)
W_s	Humidity ratio of supply air 0.0070 (lb/lb)
W_3	Humidity ratio of thermal space (lb/lb)
T_0	Temperature of outdoor 85 ($^\circ F$)
T_2	Temperature of supply air ($^\circ F$)
T_3	Temperature of thermal space ($^\circ F$)
M_o	Moisture load 166.06(lb/hr)
Q_o	Sensible heat load 289897.52(btu/hr)
h_w	Enthalpy of liquid water (J/lb)
h_{fg}	Enthalpy of water vapor (J/lb)
f_r	Volumetric flow rate of air (cfm=ft^3/min)
f_{r0}	Initial of volumetric flow rate of air 4250 (cfm)
gpm	Flow rate of chilled water (gal/min)
μ	Recirculation rate of air in system (75%)
C_s	CO_2 concentration of thermal space (ppm)

and the state equations can be described as

$$
\begin{aligned}
\dot{x}_1 &= u_1\alpha_1(x_3 - x_1) - u_1\alpha_2(W_s - x_2) \\
&\quad + \alpha_3(Q_o - h_{fg}M_o) \\
\dot{x}_2 &= u_1\alpha_1(W_s - x_2) + \alpha_4 M_o \\
\dot{x}_3 &= u_1\beta_1(x_1 - x_3) + (1-\mu)u_1\beta_1(T_0 - x_1) \\
&\quad - u_1\beta_3((1-\mu)W_0 + \mu x_2 - W_s) - 6000 u_2 \beta_2 \\
y &= [h_1(x) \quad h_2(x)]^T = [x_1 \quad x_2]^T
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
& u_1 = f_r,\ u_2 = \text{gpm}, x_1 = T_3,\ x_2 = W_3,\ x_3 = T_2 \\
& \alpha_1 = \frac{60}{V_s},\ \alpha_2 = \frac{60 h_{fg}}{c_p V_s},\ \alpha_3 = \frac{1}{(1-\mu)\rho_a c_p V_s},\ \alpha_4 = \frac{60}{\rho_a V_s} \\
& \beta_1 = \frac{60}{V_{he}},\ \beta_2 = \frac{1}{\rho_a c_p V_{he}},\ \beta_3 = \frac{60 h_w}{c_p V_{he}}
\end{aligned}
$$

This is a bilinear system which is homogeneous in the state. Heat load Q_o and moisture load M_o are usually not measurable and hence will be regarded as unknown parameters. In [2], the authors develop an observer for the system to obtain the estimation of Q_o and M_o. Now, consider the actuator dynamics for the control inputs, namely the valve dynamic as in [4]

$$
u_1 = \frac{k_1}{1 + \tau_1 s} z_1, \qquad u_2 = \frac{k_2}{1 + \tau_2 s} z_2
$$

with $u = [u_1 \ u_2]^T$ be the control signal applied to the plant and $z = [z_1 \ z_2]^T$ the input signals applied to the actuator. Hence, we derive an augmented state space model with the new state vector $x := [x_1 \ x_2 \ x_3 \ u_1 \ u_2]^T := [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$, and input signal $z = [z_1 \ z_2]^T$, so that the system model now becomes

$$
\begin{aligned}
\dot{x} &= f(x)+g(x)z = f(x)+g_1(x)z_1+g_2(x)z_2 \\
&= \begin{bmatrix} a_1(x) \\ a_2(x) \\ a_3(x) \\ a_4(x) \\ a_5(x) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_1}{\tau_1} & 0 \\ 0 & \frac{k_2}{\tau_2} \end{bmatrix} z \\
y &= h(x) = [x_1 \quad x_2]^T
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
a_1(x) &= [\alpha_1(x_3-x_1)-\alpha_2(W_s-x_2)]u_1+\alpha_3(Q_0-h_{fg}M_0) \\
&:= \gamma_1 u_1+\alpha_3(Q_0-h_{fg}M_0) \\
a_2(x) &= \alpha_1(W_s-x_2)u_1+\alpha_4 M_0 \\
&:= \gamma_2 u_1+\alpha_4 M_0 \\
a_3(x) &= [\beta_1(x_1-x_3)+(1-\mu)\beta_1(T_0-x_1)]u_1 \\
&\quad +[-\beta_3((1-\mu)W_0+\mu x_2-W_s)]u_1-[6000\beta_2]u_2 \\
&:= \gamma_3 u_1+\gamma_4 u_2 \\
a_4(x) &= -\frac{u_1}{\tau_1} \quad \text{and} \quad a_5(x) = -\frac{u_2}{\tau_2}
\end{aligned}
$$

B. Hybrid System Model for the HVAC System

The quantitative indices of comfort in the room are temperature, humidity ratio, and CO_2 concentration. In fact, the circulated air which contains too much CO_2 will affect the work efficiency of people. Now, we consider the CO_2 model and then propose a hybrid system model for the HVAC system. From the mass balance equation, the average CO_2 concentration C_s in the room can be represented as

$$\dot{C}_s = \frac{C_q}{V_s}+(1-\mu)(C_i-C_o)$$

where C_q is the amount of CO_2 generated in the room (normally from people entering the room), C_i is the CO_2 concentration of inlet air, C_o is the CO_2 concentration of air leaving the room, and $(1-\mu)$ is the air exchange rate. Assume that the air in the room is well-mixed and C_i is a constant, then $C_o = C_s$ and the equation can be written as

$$\dot{C}'_s = \frac{C_q}{V_s}-(1-\mu)C'_s \tag{3}$$

where $C'_s = C_i - C_s$. We can use nonlinear control of $(1-\mu)$ to adjust the CO_2 concentration at all operating points. The value of C_q is dependent on the number and the physical state of people in the room and there are some reference data in the ASHRAE standard. Instead of controlling C_s to a desired value, we define three levels of CO_2 concentration in the room and adjust the value of μ according to the level at which C_s is located. That is, classify the CO_2 concentration into three intervals {CO_2Low, CO_2Med, CO_2High} and use the supervisory controller S to decide the corresponding value of $\mu \in \{\mu_1,\ \mu_2,\ \mu_3\}$. Thus, the CO_2 concentration is modelled and adjusted by discrete event systems theory. The HVAC system can be modelled as a hybrid system [9] which contains continuous states x and discrete states μ. The continuous dynamics in this system is as in (2) and the discrete dynamics is defined with the supervisory switching logic which will be designed in next section. Thus we have the hybrid system model for the HVAC system as follows:

$$
\begin{aligned}
&\text{Continuous dynamics:} \\
&\quad \dot{x} = f(x,\mu)+gz, \quad \mu \in \{\mu_1,\ \mu_2,\ \mu_3\} \\
&\quad y = h(x) \\
&\text{Discrete dynamics:} \\
&\quad \mu = \mu_i, \quad i \in \{1,2,3\} \text{ assigned by } S
\end{aligned} \tag{4}
$$

We will use the continuous adaptive state feedback controller to control the value of y and the discrete event controller S to decide the value of μ and thus the CO_2 concentration C_s will be adjusted. Our control object is to track the desired temperature and humidity ratio, and construct a supervisor to keep the CO_2 concentration low.

III. Supervisory Adaptive Control for the Nonlinear MIMO HVAC System

In this section, we apply the feedback linearization technique [10] to the nonlinear HVAC system. Assume the values of loads Q_0 and M_0 are pre-specified constants and we design the non-adaptive dynamic state feedback controller for the HVAC system. Then, the adaptive controller is designed to cope with the slowly time varying unknown M_o, Q_o. We also integrate a supervisory switching mechanism which aim to keep the CO_2concentration in the HVAC system low.

A. Feedback Linearization via Dynamic Extension for the HVAC System

To reduce the nonlinear system to an aggregate of independent single-input, single-output channels, which is called the *noninteracting control problem*, we differentiate the outputs $y_i(t)$ until the inputs appear. The intuitive concept of *relative degree* is the smallest number of times that the output have to differentiate such that at least one of the inputs appears in $y_i^{(r_i)}$. For the 2-input, 2-output systems of the form (2), the relative degree is defined as $r = \{r_1, r_2\}$ which satisfies (i) $L_{g_j}h_i = L_{g_j}L_f h_i = \ldots = L_{g_j}L_f^{(r_i-2)}h_i = 0$, for all $j = 1,2$, $i = 1,2$, and (ii) the *decoupling matrix*

$$A(x) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1 & L_{g_2}L_f^{r_1-1}h_1 \\ L_{g_1}L_f^{r_2-1}h_2 & L_{g_2}L_f^{r_2-1}h_2 \end{bmatrix} \tag{5}$$

is nonsingular near the equilibrium point $x = x^e$. Note that $L_f h_i$ stands for the Lie derivative of h_i with respect to f. The definition of relative degree will lead to

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1}h_1 \\ L_f^{r_2}h_2 \end{bmatrix} + A(X)\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

and thus the feedback control law

$$z = -A(x)^{-1}\begin{bmatrix} L_f^{r_1}h_1 \\ L_f^{r_2}h_2 \end{bmatrix} + A(x)^{-1}v \tag{6}$$

will yield the closed-loop decoupled, linear system

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Given the desired output $y_d = [y_{1d},\ y_{2d}]^T$ and define the *output error* as $e = [e_1,\ e_2]^T := [(y_1 - y_{1d}),\ (y_2 - y_{2d})]^T$. Then, we can design the control

$$v_i = y_{id}^{(r_i)} - c_{i1} e_i^{(r_i-1)} - \cdots - c_{ir_i} e_i \tag{7}$$

such that the error equation becomes

$$e_i^{(r_i)} + c_{i1} e_i^{(r_i-1)} + \cdots + c_{ir_i} e_i = 0$$

The coefficients $c_{ij},\ j = 1,2,\ldots,r_i$, are chosen so that $s^{r_i} + c_{i1}s^{r_i-1} + \cdots + c_{ir_i}$ is a Hurwitz polynomial to meet the desired performance specification such as transient response or steady state error.

For the HVAC system (2), we have $r = \{2,2\}$, but the matrix

$$A(x) = \begin{bmatrix} \gamma_1 k_1/\tau_1 & 0 \\ \gamma_2 k_1/\tau_1 & 0 \end{bmatrix}$$

is singular. Thus, the system has no relative degree. To achieve the relative degree and noninteracting control, we resort to the *dynamic extension* algorithm [10] to incorporate the dynamic state feedback into this system. Set $z_1 = \psi_1$, $\dot{\psi}_1 = \zeta_1$, and $z_2 = \zeta_2$. Define the new augmented state as $\tilde{x} = [x,\ z_1]^T \in \mathbb{R}^6$ and the composed system will be

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{f}(\tilde{x}) + \tilde{g}_1(\tilde{x})\zeta_1 + \tilde{g}_2(\tilde{x})\zeta_2 \\ y &= h(x) \end{aligned} \tag{8}$$

where $\tilde{f} = [\tilde{a}_1,\ \tilde{a}_2,\ \ldots,\ \tilde{a}_6]^T$ is equal to f except the term $\tilde{a}_4 = a_4 + \frac{k_1}{\tau_1} z_1$ and $\tilde{a}_6 = 0$. Moreover, the vector field $\tilde{g} = [\tilde{g}_1\ \ \tilde{g}_2]$, where $\tilde{g}_1 = [0,\ 0,\ 0,\ 0,\ 0,\ 1]^T$ and $\tilde{g}_2 = [0,\ 0,\ 0,\ 0,\ \frac{k_2}{\tau_2},\ 0]^T$. After calculation, we find the relative degree now becomes $\tilde{r} = \{\tilde{r}_1, \tilde{r}_2\} = \{3,3\}$ so that

$$\begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \end{bmatrix} = \begin{bmatrix} L_{\tilde{f}}^3 h_1 \\ L_{\tilde{f}}^3 h_2 \end{bmatrix} + B(\tilde{x}) \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} := C(\tilde{x}) + B(\tilde{x})\zeta \tag{9}$$

and the nonsingular decoupling matrix becomes,

$$B(\tilde{x}) = \begin{bmatrix} L_{\tilde{g}_1} L_{\tilde{f}}^2 h_1 & L_{\tilde{g}_2} L_{\tilde{f}}^2 h_1 \\ L_{\tilde{g}_1} L_{\tilde{f}}^2 h_2 & L_{\tilde{g}_2} L_{\tilde{f}}^2 h_2 \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1 k_1}{\tau_1} & \frac{(\alpha_1 \gamma_4 x_4 k_2)}{\tau_2} \\ \frac{\gamma_2 k_1}{\tau_1} & 0 \end{bmatrix}$$

Thus, design the control $\zeta = -B^{-1}C + B^{-1}v$ with $v = [v_1,\ v_2]^T$ as discussed in (7), the output error will converge to zero and the system will be asymptotically stable.

Remark 1: Since the relative degree $\tilde{r}_1 + \tilde{r}_2 = 6$ is equal to the number of states, the system has trivial zero dynamics and thus the internal stability and boundedness of states are guaranteed. If the system has relative degree $\tilde{r}_1 + \tilde{r}_2 = k < 6$, we can obtain the internal dynamics by constructing the local coordinate transform with the function ϕ_i such that $L_{\tilde{g}_j}\phi_i = 0$ for $j = 1,2$ and $k+1 \leq i \leq 6$. The existence of the functions ϕ_i is guaranteed by the Frobenius theorem and the fact that constant vector fields $\tilde{g}_1,\ \tilde{g}_2$ are always involutive.

B. Adaptive Control for the HVAC system

The drawback of the feedback linearization approach is that the linearizing control law is based on exact cancellation of the nonlinear terms. If there is any uncertainty in the knowledge of the nonlinear functions $\tilde{f}$, $\tilde{g}$, the cancellation is not exact and the resulting input-output equation is not linear in practice. The value of heat thermal load Q_o and that of moisture load M_o are not measurable and are hence difficult to estimate. In this section, we will use adaptive control techniques [6] to solve this problem.

Define the values of M_0 and moisture load Q_0 as unknown parameters $\Theta^* := [\theta_1^* = M_o,\ \theta_2^* = Q_o]^T$. The system is linear with respect to the vector field $\tilde{f}$ and the unknown constant parameter vector θ^*, and thus we can rewrite (8) as

$$\begin{aligned} \dot{\tilde{x}} &= \sum_{i=1}^{2} \theta_i^* f_i(\tilde{x}) + f_0 + \tilde{g}\zeta \\ y_i &= h_i(x), \quad i = 1,2. \end{aligned} \tag{10}$$

The estimates of Θ^* is denoted as Θ and thus the estimate of the vector field $\tilde{f}$ is defined as $\hat{f} = \theta_1 f_1 + \theta_2 f_2 + f_0$. Thus, the Lie derivatives in our feedback control are replaced by the estimations as the follows:

$$\begin{aligned} L_{\hat{f}^3} h_i &:= \sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} \frac{\partial}{\partial \tilde{x}} \left[\frac{\partial}{\partial \tilde{x}} \left(\frac{\partial h_i}{\partial \tilde{x}} f_j \right) f_k \right] f_l \theta_j \theta_k \theta_l \\ L_{\tilde{g}} L_{\hat{f}}^2 h_i &:= \sum_{l=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} \frac{\partial}{\partial \tilde{x}} \left[\frac{\partial}{\partial \tilde{x}} \left(\frac{\partial h_i}{\partial \tilde{x}} f_j \right) f_k \right] \tilde{g}_l \theta_j \theta_k \end{aligned}$$

The ideal linearizing control law is replaced by

$$\widehat{\zeta} = \widehat{B}^{-1} \left(- \begin{bmatrix} L_{\hat{f}}^3 h_1 \\ L_{\hat{f}}^3 h_2 \end{bmatrix} + \widehat{v} \right) \tag{11}$$

where $\widehat{B}$ is the estimate of B and the implemented tracking law $\widehat{v}$ is of the following form

$$\widehat{v}_i = y_{id}^{(3)} + c_1(y_{id}^{(2)} - L_{\hat{f}}^2 h_i) + c_2(y_{id}^{(1)} - L_{\hat{f}} h_i) + c_3(y_{id} - y_i), \tag{12}$$

$i = 1,2$. Since in (10) f_1 and f_2 are not dependent on $\tilde{x}$, the terms $(\theta_i\theta_j,\ \theta_i\theta_j\theta_k)$ will not appear in $L_{\hat{f}}^2 h_i,\ L_{\hat{f}}^3 h_i,\ L_{\tilde{g}_j} L_{\hat{f}}^2 h_i$, and thus the parameter vector we need to estimate is $\Theta = [\theta_1,\ \theta_2]^T \in \mathbb{R}^2$. Substitute the control $\widehat{\zeta}$ into the system and define the parameter error $\Phi := \Theta^* - \Theta$, then the error equation with the feedback control will become

$$e^{(3)} + c_1 e^{(2)} + c_2 e^{(1)} + c_3 e = \Xi\Phi \tag{13}$$

where $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{21} & \Xi_{22} \end{bmatrix} = \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ and the terms on the right hand side are the mismatch between the linearizing law and the actual linearizing law as well as that between the tracking control v and the actual tracking control $\hat{v}$. By computation we have

$$\Xi_i \Phi = \left([L_{\tilde{f}}^3 h_i - L_{\hat{f}}^3 h_i] + c_1 [L_{\tilde{f}}^2 h_i - L_{\hat{f}}^2 h_i] + c_2 [L_{\tilde{f}} h_i - L_{\hat{f}} h_i] \right) \tag{14}$$

and $\Xi_{11} = \alpha_3 h_{fg}(-\lambda_1 + \alpha_1 \tilde{a}_4 + c_1\alpha_1 u_1 - c_2) + \alpha_4(\lambda_2 + \alpha_2 \tilde{a}_4 + c_1 \alpha_2 u_1)$, $\Xi_{12} = \alpha_3(\lambda_1 - \alpha_1 \tilde{a}_4 - c_1 \alpha_1 u_1 + c_2)$, $\Xi_{21} =$

$\alpha_4\left[(\alpha_1 u_1)^2 - 2\alpha_1\tilde{a}_4 - c_1\alpha_1 u_1 + c_2\right]$, $\Xi_{22} = 0$, and $\lambda_1 = \alpha_1\left[u_1^2(\alpha_1 + \mu\beta_1) - \tilde{a}_4\right]$, $\lambda_2 = -2\alpha_1\alpha_2 u_1^2 - \mu\beta_3\alpha_1 u_1^2 + \alpha_2\tilde{a}_4$.

For the case of relative degree 3, we define the *augmented error*

$$e_{i_{aug}} = b_1 e_i^{(2)} + b_2 e_i^{(1)} + b_3 e_i \tag{15}$$

such that the transfer function

$$(b_1 s^2 + b_2 s + b_3)/(s^3 + c_1 s^2 + c_2 s + c_3) \tag{16}$$

is strictly positive real (SPR). Let

$$e_{i_{aug}} = e_i + (\Theta^T L^{-1}\Xi_i^T - L^{-1}\Theta^T\Xi_i^T), \quad i = 1,2 \tag{17}$$

where the polynomial

$$L(s) = s^{(3)} + c_1 s^{(2)} + c_2 s^{(1)} + c_3$$

is chosen to be Hurwitz and note that $e_i = L^{-1}(s)\Phi^T\Xi_i^T$. From the fact that Θ^* is a constant vector, we can obtain the error equation for adaptation, i.e., $e_{iaug} = \Phi^T L^{-1}\Xi_i^T$ and define $L^{-1}\Xi_i^T = \xi_i^T \in \mathbb{R}^{2\times 1}$, then

$$e_{aug} = \left[\begin{array}{c} e_{1_{aug}} \\ e_{2_{aug}} \end{array}\right] = \left[\begin{array}{c} \xi_1 \\ \xi_2 \end{array}\right]\Phi := \xi\Phi \tag{18}$$

Hence, we can use the normalized gradient adaptive law

$$\dot{\Theta} = \dot{\Phi} = \frac{\Gamma}{(1 + \xi\xi^T)}(-\xi^T e_{aug}) \tag{19}$$

where Γ is the adaption gain. Since e_{aug} is chosen so that (16) is SPR, the estimated matrix $\widehat{B}^{-1} = B^{-1}$ is away from singularity and the estimated signals are all bounded, the stability analysis for this adaptive control will yield bounded state $\tilde{x}$ and $y \to y_d$ as $t \to \infty$ [6].

Remark 2: There might be some problem when applying adaptive control with dynamic extension. The problem occurs if the true decoupling matrix $B(\tilde{x})$ is singular but its estimate $\widehat{B}(\tilde{x})$ is nonsingular during adaptation. In our case, this will not happen since the unknown parameters only appear in the vector fields $\tilde{a}_1, \tilde{a}_2$ and are not coupled with the state variables. This greatly simplify the computation and the estimated matrix $\widehat{B} = B$ contains no estimate parameters.

C. Supervisory Switching Control for Ventilation and Stability Analysis

Now we start to design the supervisor to decide the recirculation rate μ such that the CO_2 concentration indoor will be adjusted. Note that in [2], the recirculation rate $\mu = 75\%$. Define three classes of the CO_2 concentration as {CO_2High, CO_2Med, CO_2Low} where CO_2High means $C_s \geq 1600$ *ppm*, CO_2Med means $C_s \in (1200\ ppm, 1600\ ppm)$ and CO_2Low means $C_s \leq 1200\ ppm$. The ranges can be defined by requirement. We construct a supervisor S with the following switching logic:

$$S: \left\{\begin{array}{lll} \text{CO}_2\text{High and } y - y_d < \varepsilon & \Rightarrow & \mu = \mu_3 = 60\% \\ \text{CO}_2\text{Med and } y - y_d < \varepsilon & \Rightarrow & \mu = \mu_2 = 65\% \\ \text{CO}_2\text{Low and } y - y_d < \varepsilon & \Rightarrow & \mu = \mu_1 = 75\% \\ \text{Otherwise} & & \text{No switching} \end{array}\right. \tag{20}$$

where ε is a pre-specified small positive constant. Thus, the hybrid dynamical system (4) is modelled with the continuous states $\tilde{x}$ and discrete states μ with the continuous state feedback controller ζ and the discrete state controller S. The whole hybrid system model and the processing procedure can be clearly represented by the *hybrid automaton* [11] shown in Fig. 2.

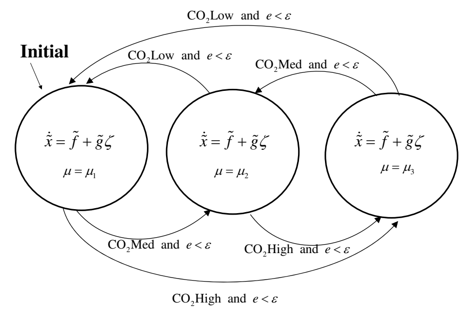

Fig. 2. Hybrid automaton of the HVAC system

Stability issue of this hybrid system will be discussed in the following. Here the stability means that the continuous states are stable and the discrete state will not switch infinite times in finite time interval. Since the value of C_s is continuous and slowly time varying, and our switching will only occur when the tracking error of the current system satisfy the specification, the *zeno* phenomenon [11] will not occur, that is, the existence of the *dwell time* guarantees that infinitely switching in finite time will never occur.

As discussed in [12], switching between stable systems may lead to unstable phenomenon. Since the continuous dynamical system (8) with adaptive controller ζ is stable with $\mu = \mu_i$, $i = 1,2,3$, in (20), respectively, the possible scenario that leads the system to unstable is that the value of μ switches sequentially before the continuous states achieves the temperature and humidity tracking and thus oscillation or divergence might occur. The unstable situation caused from consecutive switching is not allowed in the switching mechanism (20) since the supervisor is designed with the priority rule that the switching can only start when the tracking error converges. Hence, we can conclude that the whole system will be stable with the supervisory switching logic. The simplicity of the stability analysis is attributed to our switching logic and the CO_2 concentration C_s is not coupled with our continuous state dynamic equation $\dot{\tilde{x}} = \tilde{f} + \tilde{g}\zeta$.

IV. Simulation Results

The equilibrium conditions of the HVAC system are $T_3^e = 71°F$, $W_3^e = 0.0088$ lb/lb, $T_0^e = 85°F$, $W_0^e = 0.018$ lb/lb, $W_s^e = 0.0070$ lb/lb, $u_1^e = 17000$ cfm, $u_2^e = 58$ gpm, $M_0^e = 166.06$ lb/hr, and $Q_0^e = 289897.52$ btu/hr. The initial values are $T_2^e = 55°F$ and $\zeta_1 = 15000$, $\zeta_2 = 40$. Figure 3 shows the first output response of the feedback controller with the design load values Q_0^e, M_0^e, and the non-design thermal load with values $Q_o = 350000$ btu/hr and $M_o = 196$ lb/lb. It is clear that the controller does not have good tracking performance when the system works upon the environment of non-design load values. The adaptive controller shows its robustness and the transient response is satisfactory compared with [2].

Suppose the values of Q_o, M_o are changed from 166 to 176 and from 290000 to 310000, respectively, as shown in the upper side of Fig. 4. In real world, the load changes are in a time scale of hours, here we use these values to show our controller performance. From the bottom of Fig. 4, the proposed adaptive state feedback controller has shown that it can response to the time varying load change. In Fig. 5, the CO_2 concentration changes from CO_2Low to CO_2Med at $t = 1800$ (sec) and thus the supervisor switches the recirculation rate from μ_1 to μ_2 at $t = 1800$ according to the switching logic. The time response of temperature and humidity ratio shown in Fig. 5 provides the tracking performance with switchings.

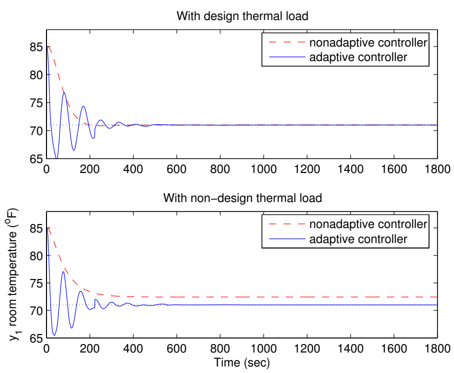

Fig. 3. First output response to design and non-design loads

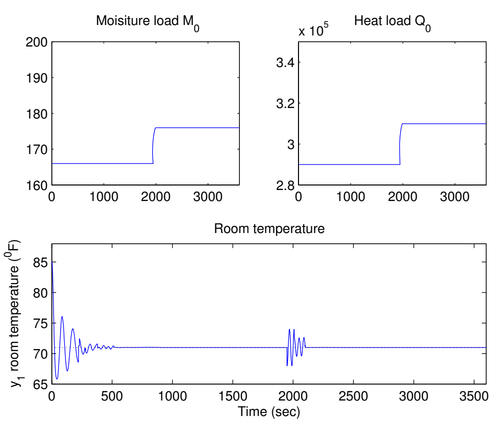

Fig. 4. First output response to time varying Q_o, M_o

V. CONCLUSIONS

In this paper, we propose a hybrid system model for the HVAC system and apply the feedback linearization technique with dynamic extension to design the continuous adaptive control for the nonlinear MIMO system. Values of thermal loads Q_o and M_o may be time varying or not be known

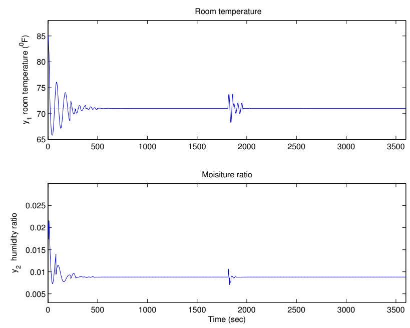

Fig. 5. Output response of the HVAC system with CO_2 supervisor.

exactly, and hence adaptive controller is a good choice for HVAC system. Besides, we construct a discrete supervisory controller to adjust the recirculation rate based on the CO_2 concentration and discuss stability of the whole system by hybrid system theory. The adaptive controller tracks the desired temperature and humidity ratio to keep the comfort of thermal space and the CO_2 supervisor improves the air quality. Computer simulations have proved that such a adaptive controller is superior to the non-adaptive controller due to the ability of adaption to load changes and system perturbation.

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