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Supervised Nonlinear Control of Hybrid System with Application to HVAC System

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Abstract— In this note, we have proposed a systematic approach for the hybrid system controller designs. We integrate discrete event system (DES) and continuous variable dynamic system (CVDS) theories, while taking both sizes of operation logics as well as the system performance into account, to propose a novel approach for the hybrid system controller design. In order to verify the performance of the proposed controllers, we perform a number of computer simulations of a real-world HVAC hybrid system.

I. INTRODUCTION

Hybrid system is simply a group of time-driven and event-driven components working together to provide multi-object functionalities. The ultimate system dynamics is the resulting of interactions of CVDS and DES. Traditional controller design methodology cannot directly be applied here because hybrid system cannot be modeled by the set of differential equations [7]. Control of hybrid system usually involved multi-layer controller architecture. The operation logics control in the upper layer is to control the on/off sequences of hybrid system that guarantee the controlled hybrid system will complete tasks. The servomechanism control in the lower layer is to control the performance such that will satisfy some set of constrains.

Form level of abstractions, we can imagine the hybrid system evolves as a DES. That is, the system evolutes as between several discrete state (heating, cooling) causing by the occurrence of discrete events. The supervisory controller that feedback event sequence, can do control of the on/off event sequence of hybrid system. When the system operates in some operation mode, the continuous system trajectory (temperature) can be formulated by some set of differential equations. The servo controller that feedback the system trajectory can do control of the servomechanism performance of hybrid system. In fact, DES and CVDS components may affect each other and causing the ultimate hybrid system overly highly complex and difficult to be controlled. So a systematic methodology for the controllers design for hybrid system control is important for the coming hybrid system era and it's also a challenge tasks for their inherent complex nature.

II. SYSTEM MODELING

For the investigation and study logical level behavior of hybrid system, we choose finite state automata [2, 7] as our primary modeling tool in this note. Assuming there are N distinct components each equipped with a sensor and/or actuator. The automaton model of the i^{th} component is a six-tuple.

$$G_i = \{X_i, \Sigma_i, \delta_i, \Gamma_i, x_{i,0}, X_{i,m}\} \tag{1}$$

where X_i is the finite discrete state set, Σ_i is the discrete event set, $\delta_i : X_i \times \Sigma_i \to X_i$ is the transition function which indicates the transition relationship, $x_{i,0}$ is the initial state and $X_{i,m}$ is the marked state set of automaton G_i. Which generate language L_i and mark language $L_{i,m}$.

$$\begin{aligned} L_i &= \{t \in \Sigma_i^* \mid \delta(x_{i,0}, t)!\} \\ L_{i,m} &= \{t \in L_i \mid \delta(x_{i,0}, t) \in X_{i,m}\} \end{aligned} \tag{2}$$

where Σ_i^* is the set of traces or event sequence form from the event set Σ_i and $\delta(x_i, e)!$ means there exist an event e such that bring state from x_i to some other state x_i'. Language L_i is the set of all traces that automaton G_i can perform, which can demonstrate the possible behaviors of the i^{th} equipment. When we model all the individual equipment into automaton, we need to composite them to obtain the overall system automaton model. Composite of two automata is to capture their joint behavior retain each individual behaviors. The formal definition of composite operation as Eq.(3). The overall automaton model of hybrid system can be obtained by consecutively composite all of individual automaton. That is $G = \underset{N}{\|} G_i$, which genetate language L and mark language L_m.

$$G = Ac(X_1 \times X_2, \Sigma_1 \cup \Sigma_2, \delta_{\|2}, \Gamma_{\|2}, (x_{10}, x_{20}), X_{1m} \times X_{2m})$$

$$\delta_{\|2}((x_1, x_2), e) = \begin{cases} (\delta_1(x_1, e), \delta_2(x_2, e)) & \text{if } e \in \Gamma_1(x_1) \cap \Gamma_2(x_2) \\ (\delta_1(x_1, e), x_2) & \text{if } e \in \Gamma_1(x_1) \setminus \Sigma_2 \\ (x_1, \delta_2(x_2, e)) & \text{if } e \in \Gamma_2(x_2) \setminus \Sigma_1 \\ \text{undefined} & \text{otherwise} \end{cases} \tag{3}$$

While system operates well, the ultimate system behavior can be thought as switching system, which can be formulated as

$$\begin{cases} \dot{x} = \sum_k f_k(x, u_k, \Theta_k) \\ \Theta_k = \Omega_k(x) \\ \sum_k \Theta_k = 1 \end{cases} \tag{4}$$

where Θ_k is the operation indicator of the k^{th} operation mode, it is a discrete mapping from system state x to discrete values 0 or 1. It can also interpreter as the occurring of discrete event when system state reaches to some saturation region. Where we assume there are only one operation mode at any instantaneous time point and the system is modeled as the collecting of k subsystems each modeled by differential equation f_k.

III. CONTROLLERS DESIGN

3.1 Supervisory Controller Design

Supervisory control of DES is based on the assumptions that some of the uncontrolled system behavior are illegal and we hope to restrict, by control, it from occurring. That is we hope system behavior become from L to the relative small one K, the system specification. When we model the overall system into automaton G, the procedures of supervisory controller design listed as below.

(a) According to the problem we concerned, labeling each of the illegal state $\overline{x} \in X$ in automaton G.

(b) Removing these illegal states and all the directed traces attached on it, obtain automaton H.

(c) System specification $K = L_H$.

(d) If $\Sigma_{uc} = \Sigma_{uo} = \phi$, see case 1.

(e) If $\Sigma_{uc} \neq \phi, \Sigma_{uo} = \phi$, see case 2.

(f) If $\Sigma_{uc} = \phi, \Sigma_{uo} \neq \phi$, see case 3.

(g) If $\Sigma_{uc} \neq \phi, \Sigma_{uo} = \phi$, see case 4.

Language K is said to be controllable with respect to L and Σ_{uc}, if

$$\overline{K}\Sigma_{uc} \cap L \subseteq \overline{K} \tag{5}$$

and observable with respect to P and Σ_c, if for all $t \in \overline{K}$ and for all $\sigma \in \Sigma_c$, Eq. (5) hold. Where $P : \Sigma^* \to \Sigma_o^*$ is the projection from traces to observable traces.

$$[t\sigma \notin K] \wedge [t\sigma \in L] \Rightarrow P^{-1}[P(t)]\sigma \cap \overline{L} = \phi \tag{6}$$

Case 1

In such case, K is controllable and observable respect to L, Σ_{uc} and P, Σ_c. Design supervisor $S = G \times H$, The feasible event set function of controlled DES S/G becomes $\Gamma_{S/G}(x_G, x_H) = \Gamma(x_G) \cap \Gamma(x_H)$. Same as the control action $S(t)$ is *encoded* into the transition function of H. That is

$$\begin{aligned} L(H \times G) &= L(H) \cap L(G) \\ &= \overline{K} \cap L(G) \\ &= \overline{K} \\ &= L(S/G) \end{aligned} \tag{7}$$

Case 2

In such case, we need to test the controllability of system specification K due to the uncontrollable event effect. If K is controllable, uncontrollable event no effect, we follow the supervisor design approach discussed in case 1. If K is uncontrollable, we know there exist no supervisor to achieve control goals. Instead we can calculate the supremal controllable sublanguage $K^{\uparrow C}$ and infimal prefix-closed controllable superlanguage $K^{\downarrow C}$. Procedures to calculate the supremal controllable sublanguage $K^{\uparrow C}$ listed as below.

(a) Labeling the state x in H such that $\delta(x, e) = \overline{x}$, for any $e \in \Sigma_{uc}$, name these states are supremal intermediate states.

(b) Removing these supremal intermediate states in H and obtain automaton $H^{\uparrow C}$.

(c) The supremal controllable sublanguage $K^{\uparrow C} = L_{H^{\uparrow C}}$.

Procedures to calculate the infimal prefix-closed controllable superlanguage $K^{\downarrow C}$ listed as below.

(a) Labeling the illegal state $\overline{x}$ in automaton G such that $\delta(x, e) = \overline{x}$, for any $x \in X_H$ and $e \in \Sigma_{uc}$, name these illegal states are infimal intermediate states.

(b) Including these infimal intermediate states and all the directed traces attached on it to automaton H and obtain the automaton $H^{\downarrow C}$.

(c) The infimal prefic-closed controllable superlanguage $K^{\downarrow C} = L_{H^{\downarrow C}}$.

Case 3

In such case, we need to test the observability of system specification K, due to the affect of unobservable event. If K is observable, unobservable event no effect, we need to build the observer of automaton H, $\hat{H}$. Observer automaton $\hat{H}$ can be obtain from H after grouping the state of H. Here, observer means the state transition of $\hat{H}$ depends on the occurrence of observable event of H. For any event sequence $t = t'\sigma, (\sigma \in \Sigma_o)$ that $\hat{H}$ observe so far, the event sequences that H may generates are

$$L_t = \{s \mid s = P^{-1}(t')\sigma\} \tag{8}$$

state of $\hat{H}$ is $\hat{\delta}(\hat{x}_0, t)$ and the *possible* states of H is the set

$$X = \{x \mid s \in L_t, \delta_H(x_{H,0}, t)!\} \tag{9}$$

we call $\hat{\delta}(\hat{x}_0, t)$ is a "*super*" state because it can be represented as the set of states of H. $\hat{H}$ can be obtained from H if we grouping the state of H. Grouping of states means if there exist an unobservable event sequence between states in H then all these states were on same group and belong to a particular super state of $\hat{H}$. Control action $S(t)$ in such case becomes

$$S(t) = \bigcup_{x_H \in \hat{\delta}(\hat{x}_0, t)} \Gamma_R(x_H) \tag{10}$$

Eq. (10) means the control action after supervisor observe event sequence t is depends on feasible event set of observer $\hat{\Gamma}(\hat{\delta}(\hat{x}_0, t))$. The feasible event set of observer at some super state, called $\hat{\delta}(\hat{x}_0, t)$, equals to the union of all the feasible event set of H at x_H for any x_H contains in the super state $\hat{\delta}(\hat{x}_0, t)$.

When system specification K is *unobservable*, we know there exist no supervisor can achieve our control goals due to

control policy contradiction problem. Instead, we can calculate the maximum observable sublanguage $K^{\uparrow MO}$, infimal prefix-closed observable superlanguage $K^{\downarrow O}$.
we know that K is unobservable due to there exist some states in H, which have mutual exclusive control policy and which are belongs to same super state in $\hat{H}$. Because we hope to obtain the maximum observable sublanguage from K, we need to remove these contradiction states from H to ensure the language generated thereafter is observable. Procedures to calculate the maximum observable sublanguage $K^{\uparrow MO}$ listed as below.

(a) Labeling each of the control policy contradiction state of H.

(b) Removing these control policy contradiction states in H and obtain automaton $H^{\uparrow MO}$.

(c) The maximum observable sublanguage $K^{\uparrow MO} = L_{H^{\uparrow MO}}$.

Procedures to calculate the infimal prefix-closed observable superlanguage $K^{\downarrow O}$ listed as below.

(d) Labeling states $x \in X_G \setminus X_H$ not in the state space of H and which are causing control policy contradiction due to disabling of some event from occurring.

(e) Include it and all the directed path connected with it to H and obtain $H^{\downarrow o}$.

The infimal prefix-closed observable superlanguage $K^{\downarrow O} = L_{H^{\downarrow O}}$.

3.2 Servo Controller Design

In general, embed in a hybrid system are DES and CVDS. DES usually reflects the state transition or switching of operation modes in hybrid system. State transition in hybrid system causing by the occurring of discrete event at discrete time point, event such as turn on/off of a equipment, detect button pressed or the generate of interrupt signal when system state reach to some particular saturation region. Different operation model has its own dynamics also and the CVDS of hybrid system can be thought as switching between these several dynamics different subsystems as illustrate in Figure 1. Design a hybrid system controller to stable the system is a challenge tasks because the ultimate system dynamics is complex and contains event driven component and time driven component.

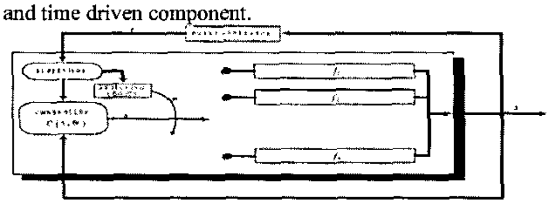

Fig. 1. Hybrid system conceptual diagram

Assume there are k different operation modes within a hybrid system and the operation mode indicator, denoted by Θ_k, is determinate by the DES embed in it. We model the hybrid system as the switching between a set of equations. That is

$$\begin{cases} \dot{x} = \sum_k f_k(x, u_k, \Theta_k) \\ \Theta_k = \Omega_k(x) \\ \sum_k \Theta_k = 1 \end{cases} \tag{11}$$

Eq. (11) means the effect of system state is contributed by the of k subsystems, each of them model by a differential equation $f_k(x, u_k, \Theta_k)$ with a discrete event effect embed in it. Where Θ_k is the indicator of k^{th} operation mode, which can be determinate by function Ω_k, $\Omega: x \to \{0,1\}$ is a discrete mapping from continuous system state to discrete value 0 or 1, used to represent the occurrence of model-changed event. Where value 1 indicates system states reach at or over some saturation region and 0 otherwise. Furthermore, we assume there is only one operation mode at any instantaneous time point.

Stability issue [M.S. Branicky] is important of hybrid system and our goal is to design a controller $C(x, \Theta_\bullet)$ to stable the overall system. Let us define a Lyapunov function candidate $V = \frac{1}{2}x^2$, the time variation is

$$\dot{V} = x\dot{x} = x\left(\sum_k f_k(x, u_k, \Theta_k)\right) \tag{12}$$

while system operate at some operation model, say mode j, rewrite Eq. (12) as

$$\dot{V} = xf_j\left(x, C(x, \Theta_j)\right) \tag{13}$$

design the correspondence j^{th} mode control algorithm $C(x, \Theta_j)$ such that satisfy $\dot{V} < -\gamma x^2$, the j^{th} mode control algorithm then obtain as

$$C(x, \Theta_j) = -\gamma x\left(f_j^{-1}(x, \bullet)\right) \tag{14}$$

from Lyapunov theory, we know system will stable in mode j if we apply the control as Eq. (14). Continuing design control algorithm for the total k possible operation mode $C(x,\Theta_1), C(x,\Theta_2), \cdots, C(x,\Theta_k)$ and choose the control apply to hybrid system as the from

$$C(x, \Theta) = \sum_i C(x, \Theta_i)\Theta_i \tag{15}$$

it is a variable structure controller. From Eq. (15), we know V will decade asymptotically for arbitrary switching between any two operation mode and the overall system will state in the sense of Lyapunov.

IV. Application to HVAC System

We follows the controllers design method proposed in the preceded section apply to a real word HVAC (heating, ventilating and air conditioning system) hybrid system. The main propose of HVAC system is to provide the people working inside building with conditioned air so that they will have a comfortable and safe work environment. Conditioned air means that air is clean and odor-free, and the temperature, humidity, and movement of air are within certain comfortable ranges. The schematic diagram of the conventional HVAC system illustrated as Fig. 2. Where a group of components working together to move heat to where is wanted (heating) or to remove heat from where it is not wanted (cooling) and put is where it is un-subjectionable (ventilation).

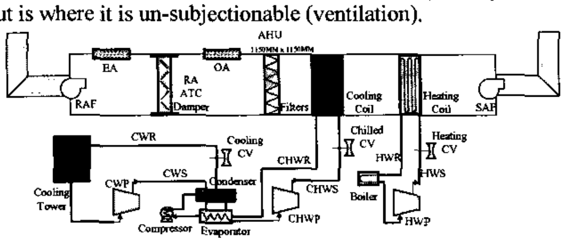

Fig. 2. Schematic diagram of HVAC System

4.1 Supervisory Controller Design

(a) Model each of the individual components into automaton model. The automaton model of component those perform heating functionality show as Fig. 3 and cooling functionality show as Fig. 4. where dashed circle in each automaton model is its initial state. We assume the sensor reading be the uncontrollable event, that is $\Sigma_{uc} = \{\text{Above\_SP}, \text{Below\_SP}\}$.

(b) Composite each individual automaton model to system.

Heating subsystem-

Automaton model of heating subsystem G_H is composite by 7 individual automaton models. That is,

$G_H = G_{RAF} \,||\, G_{SAF} \,||\, G_{TS} \,||\, G_{TC} \,||\, G_{HV} \,||\, G_{HWP} \,||\, G_{LD}$

number of states in G_H is **296.**

Cooling Subsystem-

The cooling subsystem G_C is composite by 10 individual automaton models. That is,

$G_C = G_{RAF} \,||\, G_{SAF} \,||\, G_{TS} \,||\, G_{TC} \,||\, G_{CHV} \,||\, G_{CHWP} \,||\, G_{COV} \,||\, G_{CWP} \,||\, G_{CP} \,||\, G_{LD}$

number of states in G_C is **1280**. Due to space consideration, we will not sketch out heating and cooling subsystem automaton model.

(c) Labeling each of the forbidden states

Heating subsystem-

After composite operation the state space of heating subsystem is 7-pairs as $(x_{RAF}, x_{SAF}, x_{TS}, x_{TC}, x_{HV}, x_{HWP}, x_{LD})$, the *pressure-forbidden*[1] states of G_H are

(•,•,•,•,HVOFF,HWPON,•)

and *precedence-forbidden*[2] states of G_H are

(RAFOFF,•,•,•,HVON,•,•)

(•,SAFOFF,•,•,HVON,•,•)

where symbol • means don't care, same as for *any* possible state.

Cooling subsystem

State in G_C is 10-pairs as $(x_{RAF}, x_{SAF}, x_{TS}, x_{TC}, x_{CHV}, x_{CHWP}, x_{COV}, x_{CWP}, x_{CP}, x_{LD})$ the *pressure-forbidden* states of G_C are

(•,•,•,•,CHVOFF,CHWPON,•,•,•,•)

(•,•,•,•,CHVOFF,•,•,•,CPON,•)

(•,•,•,•,•,•,COVOFF,CWPON,•,•)

(•,•,•,•,•,•,COVOFF,•,CPON,•)

and the *precedence-forbidden* states of G_C are

(RAFOFF,•,•,•,CHVON,•,•,•,•,•)

(•,SAFOFF,•,•,CHVON,•,•,•,•,•)

(•,•,•,•,•,CHWPOFF,•,•,CPON,•)

(•,•,•,•,•,•,•,CWPOFF,CPON,•)

Removing the illegal states.

Heating subsystem

Let automaton H_H be the automaton, which obtain from after removing forbidden states of G_H. Number of states in H_H is 118.

Cooling subsystem

Let automaton H_C be the automaton, which obtain from removing forbidden states of G_C. Number of states in H_C is **163**

System specification.

Now, we must to elect two event sequences $t_H \in L(H_H)$ and $t_C \in L(H_C)$ be our reference control event sequence illustrated for hating and cooling subsystem. The overall HVAC system reference control event sequence show as in Fig. 5.

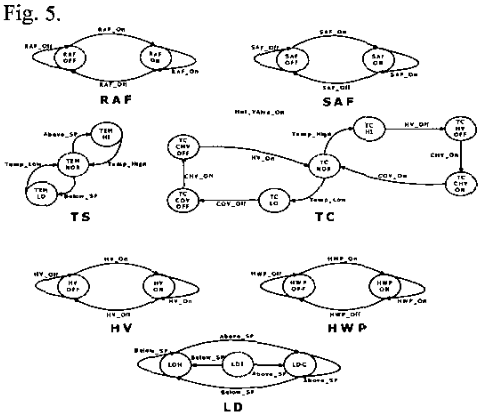

Fig. 3 Automaton model of heating components

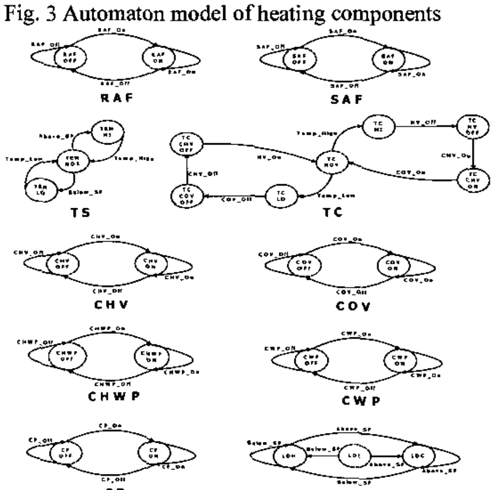

Fig. 4 Automaton model of cooling components

[1] Cause system under high pressure.

[2] Indicate control RAF or SAF inappropriately.

4.2. Servo Controller Design

The single-zone thermal system model show in Fig. 6 is chosen for our analysis [5]. System variables and parameters describe in Table 1. The system equations are derived from the conservation of energy principles and are given by

$$\begin{cases} \rho c_p V_{he}\dfrac{dT_2}{dt} = f\rho c_p(T_1 - T_2) + \Theta_H\eta_H C(x_1,x_2,\Theta_H) - \Theta_C\eta_C C(x_1,x_2,\Theta_C) \\ \rho c_p V_{ts}\dfrac{dT_3}{dt} = f\rho c_p(T_2 - T_3) + Q_L \end{cases}$$

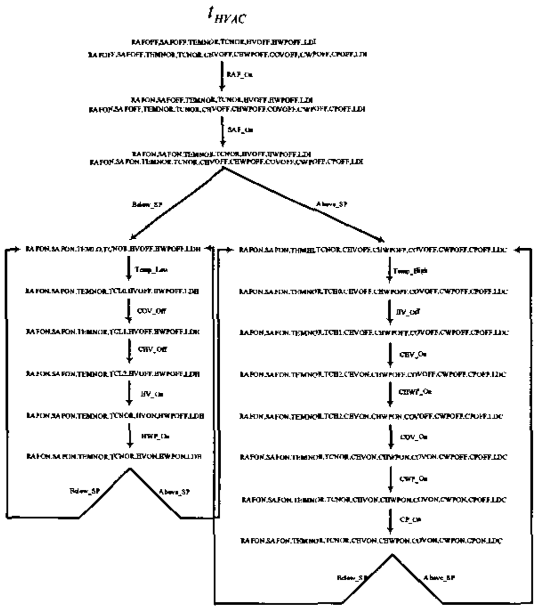

Fig. 5 HVAC Reference Control Sequence

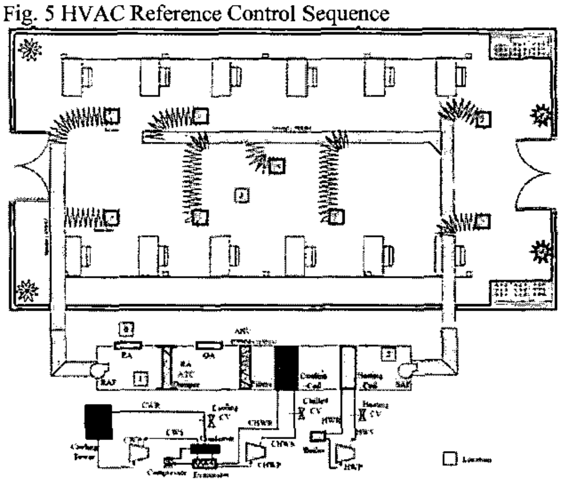

Fig. 6 Single-zone Thermal System Model

Table 1. System Parameters

ρ	air density	1.19kg/m^3
c_p	constant pressure specific heat of air	1005 J/kg °C
V_{he}	Effective heat exchanger volume	1.719 m^3
V_{ts}	Effective thermal space volume	1655.115 m^3
f_o	Volumetric ventilation airflow rate	2 m^3/s
f	Volumetric circulate airflow rate	8.0231 m^3/s
t	time	s
Q_L	Thermal load	84970 W

T_1 is the mixed air temperature which can be obtain from T_o and T_3 as $T_1 = T_3 + (T_0 - T_3)\times\frac{f_o}{f}$, let us denoting system variable $x = [T_2 \quad T_3]^T$ and making the appropriate substitutions of T_1, yield

$$\begin{cases} \dot{x}_1 = \dfrac{1}{V_{he}}\left[(T_o - x_2)f_o + (x_2 - x_1)f + \dfrac{\Theta_H\eta_H C(x_1,x_2,\Theta_H) - \Theta_C\eta_C C(x_1,x_2,\Theta_C)}{\rho c_p}\right] \\ \dot{x}_2 = \dfrac{1}{V_{ts}}\left[(x_1 - x_2)f + \dfrac{Q_L}{\rho c_p}\right] \end{cases}$$

our goal is to control x_2 to the set point x_2^*. Define error $e_2 = x_2 - x_2^*$, rewrite into error dynamics equation $\dot{e}_2 = \frac{1}{V_{ts}}\left[(x_1 - e_2)f - x_2^* f + \frac{Q_L}{\rho c_p}\right]$, now we obtain

$$\begin{cases} \dot{x}_1 = \dfrac{-f}{V_{he}}x_1 + \dfrac{f - f_o}{V_{he}}e_2 + \dfrac{T_o f_o}{V_{he}} + \dfrac{f - f_o}{V_{he}}x_2^* + \dfrac{\Theta_H\eta_H C(x_1,x_2,\Theta_H) - \Theta_C\eta_C C(x_1,x_2,\Theta_C)}{V_{he}\rho c_p} \\ \dot{e}_2 = \dfrac{f}{V_{ts}}x_1 - \dfrac{f}{V_{ts}}e_2 - \dfrac{f x_2^*}{V_{ts}} + \dfrac{Q_L}{V_{ts}\rho c_p} \end{cases}$$

let new variable $x_2 = e_2$, rewrite as

$$\begin{cases} \dot{x}_1 = f(x_1,x_2) + g(x_1,x_2)(\Theta_H\eta_H C(x_1,x_2,\Theta_H) - \Theta_C\eta_C C(x_1,x_2,\Theta_C)) \\ \dot{x}_2 = h(x_1,x_2) \end{cases}$$

where

$$f(x_1,x_2) = \frac{-f}{V_{he}}x_1 + \frac{f - f_o}{V_{he}}x_2 + \frac{T_o f_o}{V_{he}} + \frac{f - f_o}{V_{he}}x_2^*$$

$$g(x_1,x_2) = \frac{1}{V_{he}\rho c_p}$$

$$h(x_1,x_2) = \frac{f}{V_{ts}}x_1 - \frac{f}{V_{ts}}e_2 - \frac{f x_2^*}{V_{ts}} + \frac{Q_L}{V_{ts}\rho c_p}$$

Now, suppose x_2 can be stabilized by a smooth state feedback control when $x_1 = \phi(x_2)$. Define a Lyapunov function $V(x_2)$ that satisfies the inequality

$$\dot{V} = \frac{\partial V}{\partial x_2}\left[h(\phi(x_2), x_2)\right] \le -W(x_2)$$

and define variable $z = x_1 - \phi(x_2)$ as the error of x_1, yield

$$\begin{cases} \dot{z} = \dot{x}_1 - \dot{\phi}(x_2) \\ \dot{x}_2 = h((z + \phi(x_2)), x_2) \end{cases}$$

where $\dot{\phi}(x_2) = \frac{\partial\phi}{\partial x_2}h(x_1,x_2)$, let $v = \dot{x}_1 - \dot{\phi}(x_2)$ for simplicity, yield

$$\begin{cases} \dot{z} = v \\ \dot{x}_2 = h((z + \phi(x_2)), x_2) \end{cases}$$

equation (4.9) describe that x_2 will stable if variable z , by control v , is stable. Define a Lyapunov function $V(x_1,x_2) = V(x_2) + \frac{1}{2}z^2$. Analysis its time variation, yield

$$
\begin{aligned}
\dot{V}(x_1,x_2) &= \frac{\partial V(x_2)}{\partial x_2}\dot{x}_2 + z\dot{z} \\
&= \frac{\partial V(x_2)}{\partial x_2}\left[h\left(\left(z+\phi(x_2)\right),x_2\right)\right] + zv \\
&= \frac{\partial V(x_2)}{\partial x_2}\left[h\left(\phi(x_2),x_2\right)\right] + \frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}z\right) + zv
\end{aligned}
$$

choosing $v = -\frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}\right) - kz$, where $k > 0$, yields

$$\dot{V}(x_1,x_2) \le -W(x_2) - kz^2$$

according to Lyapunov theory, we know trajectories x_1 and x_2 will stable asymptotically. Substituting $v = \dot{x}_1 - \dot{\phi}(x_2)$ and $z = x_1 - \phi(x_2)$, we obtain

$$v + \dot{\phi}(x_2) = f(x_1,x_2) + g(x_1,x_2)\left(\Theta_H \eta_H C(x_1,x_2,\Theta_H) - \Theta_C \eta_C C(x_1,x_2,\Theta_C)\right)$$

apply $v = -\frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}\right) - kz$, yield

$$
\begin{aligned}
&-\frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}\right) - k\left(x_1 - \phi(x_2)\right) + \frac{\partial \phi}{\partial x_2}h(x_1,x_2) \\
&= f(x_1,x_2) + g(x_1,x_2)\left(\Theta_H \eta_H C(x_1,x_2,\Theta_H) - \Theta_C \eta_C C(x_1,x_2,\Theta_C)\right)
\end{aligned}
$$

when $\Theta_H = 1, \Theta_C = 0$, control design as

$$C(x_1,x_2,\Theta_H) = \frac{1}{\eta_H g(x_1,x_2)}\left(-\frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}\right) - k\left(x_1 - \phi(x_2)\right) + \frac{\partial \phi}{\partial x_2}h(x_1,x_2) - f(x_1,x_2)\right)$$

when $\Theta_C = 1, \Theta_H = 0$, control design as

$$C(x_1,x_2,\Theta_C) = \frac{-1}{\eta_C g(x_1,x_2)}\left(-\frac{\partial V(x_2)}{\partial x_2}\left(\frac{\partial h}{\partial x_1}\right) - k\left(x_1 - \phi(x_2)\right) + \frac{\partial \phi}{\partial x_2}h(x_1,x_2) - f(x_1,x_2)\right)$$

V. Simulation Results

The validity of the controllers design for HVAC system proposed in previous section will be verified through simulations. Here we assume the reference set point of $T_3^* = 21.66$ °C and the outdoor temperature be 29.44 °C.

Regulation results

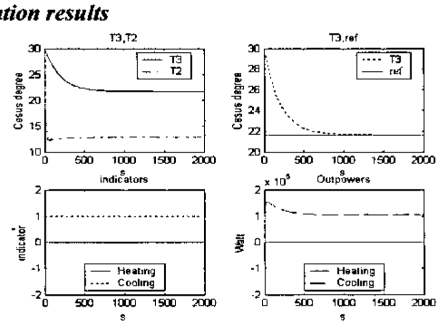

Fig. 7. Cooling performance.

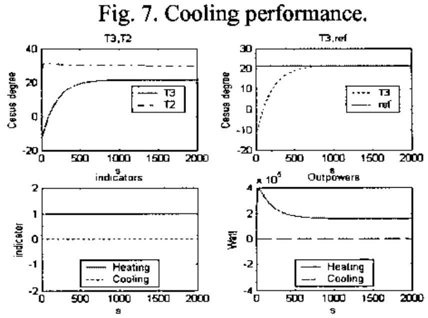

Fig. 8. Heating performance

Tracking results

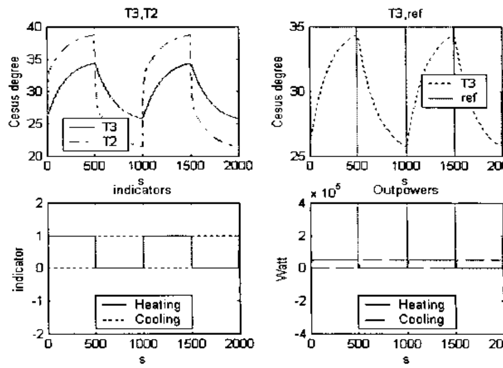

Fig. 9. Switching of operation mode simulation.

VI. Conclusion

In this paper, we integrate DES switching effect into HVAC dynamics and model a HVAC control as a hybrid system control problem. We prove the stability of the ultimated hybird system through Lyapunov stability analysis. The simulations have been sdopted to verify the feasibility not only of the individual operation mode, but also the switching during two operation modes.

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