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Robust On-Line Parameter Identification with General Knowledge on Level of Information Noise: Continuous and Discrete Cases

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Abstract

A robust on-line parameter identification problem is posed and solved for systems with general knowledge of the level of the inherent information noise. Both continuous-time and discrete-time cases are considered in this paper. For the former case, the knowledge can be the bound on either the magnitude or the finite-time L^p norm, $p \in [1, \infty)$, of the noise. Whereas for the latter case, it can be the bound on either the magnitude or the finite-index ℓ^p norm, $p \in [1, \infty)$, of the noise. Based on the knowledge, a switching type algorithm is proposed to estimate the parameters of the system from the available input-output data. In spite of the existence of the information noise, this on-line algorithm guarantees that the estimation error is monotonically decreasing, and the parameter estimate is convergent to a steady state value under a mild condition.

1 Introduction

On-line parameter identification has several advantages over the off-line counterpart. The primary one is that it can be used to progressively generate more accurate models, based on which the associated controllers can be continuously refined to improve the performance of the controlled system. Many theoretical and application results have been developed in this context. For example, [1] [2] [7] [8] consider the identification problem with magnitude information, [4] considers the problem with finite-time L^2 norm information, and [3] considers the problem with finite-index ℓ^2 norm information on the level of inherent information noise. However, all the results obtained so far do not consider the problem with finite-time L^p norm or finite-index ℓ^p norm information on the same noised information level, for $p \in [1, \infty)$ and $p \neq 2$. This information is generally available for systems whose nonparametric uncertainties are characterized by bounded induced L^p or ℓ^p norms.

In this paper, a robust on-line parameter identification problem is posed for systems with general knowledge of the level of the inherent information noise. Both continuous-time and discrete-time cases are considered in this paper. For the former case, the knowledge can be the bound on either the magnitude or the finite-time L^p norm, $p \in [1, \infty)$, of the suitably reformulated information noise. Whereas for the latter case, it can be the bound on either the magnitude or the finite-index ℓ^p norm, $p \in [1, \infty)$, of the same noise. The goals of this problem are to on-line find a converging parameter estimate for the system and to reduce monotonically the estimation error. To solve this problem, a concrete switching type algorithm is developed to estimate the parameters by using the available input-output data and bounding information on the noise.

Besides the fact that this paper considers the general knowledge of the level of the information noise, there are several additional differences between the present and previous works. For instance, in [1] [2] [3], complex set-membership identification techniques are used to estimate the admissible parameter set, in which any point can be an admissible estimate of the parameters. When a point estimate is requested, usually the center of the set is chosen as the parameter estimate. However, no guarantee is made that the estimation error is monotonically decreasing. In contrast, the class of algorithms of the present work belong to switching type algorithms (which is called algorithms with dead zone). They are simpler than those based on set-membership identification techniques. Furthermore, in spite of the existence of the information noise, each of the algorithms guarantees that the estimation error is monotonically decreasing, and the parameter estimate is convergent to a steady state value under a mild condition. Note that the monotone and convergence properties of the algorithm are useful to the subsequent controller design and essential in establishing the performance and stability of identification-based adaptive control system.

In summary, the contribution of this paper are as follows. First, it examines and unifies many identification problems in a framework. Second, it develops a switching type parameter identification technique to successfully handle systems with various bounding information on the signal corruption. Finally, it provides a thorough analysis of the performance of the algorithms developed here.

The layout of this paper is as follows. In Section 2, notations and some preliminary results are established. In Section 3, a robust on-line parameter identification problem is formulated precisely and several examples belonging to this problem are examined. In Section 4, concrete algorithms are presented and analyzed for the problem. At the end, Section 5 gives the conclusion.

2 Notations and Preliminaries

Here, some notations and definitions are introduced for subsequent use. Let $\mathbf{I}_0$, $\mathbf{R}_0$, $\mathbf{R}$, and $\mathbf{R}^q$ denote the set of nonnegative integers, nonnegative real numbers, real numbers, and q dimensional real vectors, respectively. Let

$$|v|_p := (\sum_{i=1}^{q} |v_i|^p)^{1/p}$$

denote the p-norm in $\mathbf{R}^q$ space, $p \in [1,\infty)$. For any $t \in \mathbf{R}_0$, $\sigma \in \mathbf{R}_0$, and continuous-time signal vector $x : \mathbf{R}_0 \to \mathbf{R}^q$, two σ-exponentially-weighted L^p norms on x are defined:

$$\|x\|_{L^p,t,\sigma} := \left(\int_0^t e^{-p\sigma(t-\tau)} |x(\tau)|_p^p \, d\tau \right)^{1/p},$$

$$\|x\|_{L^p,\sigma} := \left(\int_0^\infty e^{p\sigma\tau} |x(\tau)|_p^p \, d\tau \right)^{1/p}.$$

Likewise, for any $k \in \mathbf{I}_0$, $\rho \in [1,\infty)$, and discrete-time signal vector $x : \mathbf{I}_0 \to \mathbf{R}^q$, two ρ-exponentially-weighted ℓ^p norms on x are defined:

$$\|x\|_{\ell^p,k,\rho} := \left(\sum_{i=0}^{k} \rho^{-p(k-i)} |x(i)|_p^p \right)^{1/p},$$

$$\|x\|_{\ell^p,\rho} := \left(\sum_{i=0}^{\infty} \rho^{pi} |x(i)|_p^p \right)^{1/p}.$$

Finally, two induced norms on a linear causal system Δ are defined:

$$\|\Delta\|_{L^p,\sigma} := \sup_{\|x\|_{L^p,\sigma} \neq 0} \frac{\|\Delta x\|_{L^p,\sigma}}{\|x\|_{L^p,\sigma}}$$

for continuous-time case, and

$$\|\Delta\|_{\ell^p,\rho} := \sup_{\|x\|_{\ell^p,\rho} \neq 0} \frac{\|\Delta x\|_{\ell^p,\rho}}{\|x\|_{\ell^p,\rho}}$$

for discrete-time case.

In the following, two preliminary results are established for later use in Section 3.

Lemma 1 *For any number $p \in [1,\infty)$, $t \in \mathbf{R}_0$, and $\sigma \in \mathbf{R}_0$, and continuous-time signal vector $x : \mathbf{R}_0 \to \mathbf{R}^q$,*

$$\|\Delta x\|_{L^p,t,\sigma} \le \|x\|_{L^p,t,\sigma}$$

if Δ is a linear causal continuous-time system with $\|\Delta\|_{L^p,\sigma} \le 1$.

Lemma 2 *For any number $p \in [1,\infty)$, $k \in \mathbf{I}_0$, and $\rho \in [1,\infty)$, and discrete-time signal vector $x : \mathbf{I}_0 \to \mathbf{R}^q$,*

$$\|\Delta x\|_{\ell^p,k,\rho} \le \|x\|_{\ell^p,k,\rho}$$

if Δ is a linear causal discrete-time system with $\|\Delta\|_{\ell^p,\rho} \le 1$.

3 Problem Formulation

Consider a class of single-input single-output systems which can be modeled as follows:

$$\begin{cases} c(t) = \phi^T(t)\theta^* + \eta(t), & t \in \mathbf{R}_0, \\ c(k) = \phi^T(k)\theta^* + \eta(k), & k \in \mathbf{I}_0, \end{cases} \tag{1}$$

where c is the observed scalar signal, ϕ is the observed regression vector signal, θ^* is the unknown parameter vector, and η is the unknown signal noise. The problem is to on-line identify the parameter θ^* of a system by using the observed signals c and ϕ. To proceed further, some additional information concerning the signal noise η has to be given.

(IFC) For a continuous-time system, either the bounding information $\alpha_c(t)$ or $\alpha_{L^p}(t)$ with $p \in [1,\infty)$ is known such that

$$|\eta(t)| \le \alpha_c(t) \text{ or } \|\eta\|_{L^p,t,\sigma} \le \alpha_{L^p}(t)$$

at any time instant $t \in \mathbf{R}_0$.

(IFD) For a discrete-time system, either the bounding information $\alpha_d(k)$ or $\alpha_{\ell^p}(k)$ with $p \in [1,\infty)$ is known such that

$$|\eta(k)| \le \alpha_d(k) \text{ or } \|\eta\|_{\ell^p,k,\rho} \le \alpha_{\ell^p}(k)$$

at any time index $k \in \mathbf{I}_0$.

Now we are ready to restate the on-line parameter identification problem precisely as follows.

(PFC) *Problem Formulation for Continuous-Time Systems:*
Given: (i) the observed signals $\{c(\tau), \phi(\tau)\}_{\tau=0}^{t}$, $t \in \mathbf{R}_0$, and (ii) either the bounding information $\alpha_c(t)$ or $\alpha_{L^p}(t)$ described in (IFC);
Find: an on-line algorithm $A_t|_{t=0}^{\infty}$ which maps the given information into a parameter estimate $\hat{\theta}(t)$ such that $\hat{\theta}(t)$ is convergent to a steady state value and the estimation error $e(t) := |\hat{\theta}(t) - \theta^*|_2$ satisfies

$$\begin{cases} \dot{e}(t) < 0 & \text{if } \dot{\hat{\theta}}(t) \neq 0 \\ \dot{e}(t) = 0 & \text{otherwise} \end{cases}. \tag{2}$$

(PFD) *Problem Formulation for Discrete-Time Systems:*
Given: (i) the observed signals $\{c(i), \phi(i)\}_{i=0}^{k}$, $k \in \mathbf{I}_0$, and (ii) either the bounding information $\alpha_d(k)$ or $\alpha_{\ell^p}(k)$ described in (IFD);
Find: an on-line algorithm $A_k|_{k=0}^{\infty}$ which maps the given information into a parameter estimate $\hat{\theta}(k)$ such that $\hat{\theta}(k)$ is convergent to a steady state value and the estimation error $e(k) := |\hat{\theta}(k) - \theta^*|_2$ satisfies

$$\begin{cases} e(k+1) - e(k) < 0 & \text{if } \hat{\theta}(k+1) - \hat{\theta}(k) \neq 0 \\ e(k+1) - e(k) = 0 & \text{otherwise} \end{cases}. \tag{3}$$

In other words, in spite of the existence of the signal noise, the parameter estimate will be settling down to a steady state value and the estimation error is strictly decreasing whenever the parameter estimate updates.

To illustrate how these problems might arise, several examples are examined as follows.
Example 1: (*Identification for continuous-time systems with nonparametric uncertainties*).

The problem is to on-line identify a model of a linear system G. The available input-output data are generated by

$$y = Gu,$$

where u is an applied input and y is the observed output. Suppose the transfer function of G is characterized by a stable factor form as follows:

$$G(s) = \left(\frac{D(s)}{\Lambda(s)} + \Delta_D(s)W_D(s)\right)^{-1}\left(\frac{N(s)}{\Lambda(s)} + \Delta_N(s)W_N(s)\right),$$

where

$$\begin{aligned} D(s) &= s^n + a^*_{n-1}s^{n-1} + \cdots + a^*_0, \\ N(s) &= b^*_m s^m + b^*_{m-1}s^{m-1} + \cdots + b^*_0, \end{aligned}$$

$\Lambda(s)$ is a known Hurwitz polynomial with degree $\deg(\Lambda) \geq n \geq m$, both W_N and W_D are (specified) known causal weighting systems, and

$$\Delta = [\Delta_N, \Delta_D]$$

constitutes an uncertain system with $\|\Delta\|_{L^p,\sigma} \leq 1$ for some $p \in [1,\infty)$ and $\sigma \in \mathbf{R}_0$. After some manipulation, the system equation $y = Gu$ can be rewritten into a linear regression form as (1) with

$$\begin{aligned} c &= F_n y, \\ \eta &= \Delta v, \\ \phi &= [-F_{n-1}y, \ldots, -F_0 y, F_m u, \ldots, F_0 u]^T, \\ \theta^* &= [a^*_{n-1}, \ldots, a^*_0, b^*_m, \ldots, b^*_0]^T, \end{aligned}$$

where

$$v = [W_N u, -W_D y]^T$$

and the transfer function of the system F_k is defined by

$$F_k(s) = \frac{s^k}{\Lambda(s)},\ k = 0, \ldots, n.$$

Because $\{u(\tau), y(\tau)\}^t_{\tau=0}$, $t \in \mathbf{R}_0$, are available and Λ, W_N, and W_D are known, it is clear that the series of the signal triplets $\{c(\tau), \phi(\tau), v(\tau)\}^t_{\tau=0}$, $t \in \mathbf{R}_0$, are available. Furthermore, by Lemma 1,

$$\|\eta\|_{L^p,t,\sigma} \leq \|v\|_{L^p,t,\sigma} := \alpha_{L^p}(t),$$

where $\|v\|_{L^p,t,\sigma}$ can be constructed on-line by the following way:

$$\frac{d}{dt}\|v\|^p_{L^p,t,\sigma} = -p\sigma\|v\|^p_{L^p,t,\sigma} + |v(t)|^p_p,\ \|v\|^p_{L^p,0,\sigma} = 0.$$

Thus, the identification problem considered in this example falls into the class of problems formulated in (PFC) with bounding information $\alpha_{L^p}(t)$.

Example 2: *(Identification for continuous-time systems with disturbances).*

Suppose the available input-output data are generated by

$$y = Gu + d,$$

where

$$G = \sum_{j=1}^{q} a^*_j G_j,$$

for some known systems G_j, $j = 1, \ldots, q$, and d is the unknown disturbance bounded in magnitude by a known signal α_c. It is easy to see that the system equation $y = Gu + d$ can be rewritten into a linear regression form as (1) with

$$\begin{aligned} c &= y, \\ \eta &= d, \\ \phi &= [G_1 u, \ldots, G_q u]^T, \\ \theta^* &= [a^*_1, \ldots, a^*_q]^T, \end{aligned}$$

and, since $\alpha_c(t)$ and $\{c(\tau), \phi(\tau)\}^t_{\tau=0}$, $t \in \mathbf{R}_0$, are available, the identification problem considered in this example falls into the class of problems formulated in (PFC) with bounding information $\alpha_c(t)$.

Example 3: *(Identification for discrete-time systems with nonparametric uncertainties).*

Similar to Example 1, suppose the transfer function of G is characterized by a factor form as follows:

$$G(z) = \frac{N(z) + \Delta_N(z)W_N(z)}{D(z) + \Delta_D(z)W_D(z)},$$

where

$$\begin{aligned} D(z) &= 1 + a^*_1 z^{-1} + \cdots + a^*_n z^{-n}, \\ N(z) &= b^*_1 z^{-1} + \cdots + b^*_m z^{-m}, \end{aligned}$$

both W_N and W_D are (specified) known causal weighting systems, and

$$\Delta = [\Delta_N, \Delta_D]$$

forms an uncertain system with $\|\Delta\|_{\ell^p,\rho} \leq 1$ for some $p \in [1,\infty)$ and $\rho \in [1,\infty)$. After some manipulation, the system equation $y = Gu$ can be rewritten into a linear regression form as (1) with

$$\begin{aligned} c &= y, \\ \eta &= \Delta v, \\ \phi &= [-z^{-1}y, \ldots, -z^{-n}y, z^{-1}u, \ldots, z^{-m}u]^T, \\ \theta^* &= [a^*_1, \ldots, a^*_n, b^*_1, \ldots, b^*_m]^T, \end{aligned}$$

where

$$v = [W_N u, -W_D y]^T.$$

Because $\{u(i), y(i)\}^k_{i=0}$, $k \in \mathbf{I}_0$, are available and both W_N and W_D are known, it is clear that the series of the signal triplets $\{c(i), \phi(i), v(i)\}^k_{i=0}$, $k \in \mathbf{I}_0$, are available. Furthermore, by Lemma 2,

$$\|\eta\|_{\ell^p,k,\rho} \leq \|v\|_{\ell^p,k,\rho} := \alpha_{\ell^p}(k),$$

where $\|v\|_{\ell^p,k,\rho}$ can be constructed on-line by the following way:

$$\|v\|^p_{\ell^p,k,\rho} = \rho^{-p}\|v\|^p_{\ell^p,k-1,\rho} + |v(k)|^p_p,\ \|v\|^p_{\ell^p,0,\rho} = |v(0)|^p_p.$$

Thus, the identification problem considered in this example belongs to the family of problems formulated in (PFD) with bounding information $\alpha_{\ell^p}(k)$.

Example 4: *(Identification for discrete-time systems with disturbances).*

Suppose the available input-output timed data are generated by

$$Dy = Nu + d,$$

where the transfer functions of D and N are defined as those in Example 3, and d is the unknown disturbance bounded in magnitude by a known signal α_d. Let c, ϕ, and θ^* be defined as those in Example 3 and let $\eta = d$. The system equation $Dy = Nu+d$ can be similarly rewritten into a linear regression form as (1). Because $\alpha_d(k)$ and $\{u(i), y(i)\}_{i=0}^{k}$, $k \in \mathbf{I}_0$, are available, the identification problem considered in this example belongs to the family of problems formulated in (PFD) with bounding information $\alpha_d(k)$.

4 Main Results

Note that the signal noise η in (1) can be estimated by

$$\begin{aligned}\hat{\eta} &:= c - \phi^T\hat{\theta}, &(4)\\ &= \eta - \phi^T\tilde{\theta}, &(5)\end{aligned}$$

where

$$\tilde{\theta} := \hat{\theta} - \theta^*.$$

Recall the problems formulated in (PFC) and (PFD), and let the following concrete robust on-line parameter identification algorithms be suggested to treat those corresponding problems. Let $r(\cdot)$ denote an arbitrarily chosen positive function and $\mathrm{sign}(\cdot)$ be defined as

$$\mathrm{sign}(x) := \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}.$$

Algorithm: *(Parameter identification with α_c, $A_t^c|_{t=0}^{\infty}$)*

$$\begin{aligned}\dot{\hat{\theta}}(t) &= r(t)\lambda_c(t)q_c(t),\\ q_c(t) &= \hat{\eta}(t)\phi(t),\\ \lambda_c(t) &= \begin{cases} 1 & \text{if } |\hat{\eta}(t)| > \alpha_c(t) \\ 0 & \text{otherwise}\end{cases}.\end{aligned}$$

Algorithm: *(Parameter identification with α_{L^p}, $AL_t^p|_{t=0}^{\infty}$)*

$$\begin{aligned}\dot{\hat{\theta}}(t) &= r(t)\lambda_{L^p}(t)q_{L^p}(t),\\ q_{L^p}(t) &= \int_0^t e^{-p\sigma(t-\tau)}|\hat{\eta}(\tau)|^{p-1}\mathrm{sign}(\hat{\eta}(\tau))\phi(\tau)\,d\tau,\\ \lambda_{L^p}(t) &= \begin{cases} 1 & \text{if } \|\hat{\eta}\|_{L^p,t,\sigma}^p > \|\hat{\eta}\|_{L^p,t,\sigma}^{p-1}\alpha_{L^p}(t) + \beta_{L^p}(t) \\ 0 & \text{otherwise}\end{cases},\\ \beta_{L^p}(t) &= \int_0^t e^{-p\sigma(t-\tau)}|\hat{\eta}(\tau)|^{p-1}\mathrm{sign}(\hat{\eta}(\tau))\cdot\\ &\quad \phi^T(\tau)(\hat{\theta}(t) - \hat{\theta}(\tau))\,d\tau.\end{aligned}$$

Algorithm: *(Parameter identification with α_d, $A_k^d|_{k=0}^{\infty}$)*

$$\begin{aligned}\hat{\theta}(k+1) &= \hat{\theta}(k) + \lambda_d(k)\xi_d(k)q_d(k),\\ \xi_d(k) &= \frac{2(\hat{\eta}^2(k) - |\hat{\eta}(k)|\alpha_d(k))}{|q_d(k)|_2^2 + r(k)},\\ q_d(k) &= \hat{\eta}(k)\phi(k),\\ \lambda_d(k) &= \begin{cases} 1 & \text{if } |\hat{\eta}(k)| > \alpha_d(k) \\ 0 & \text{otherwise}\end{cases}.\end{aligned}$$

Algorithm: *(Parameter identification with α_{ℓ^p}, $A\ell_k^p|_{k=0}^{\infty}$)*

$$\begin{aligned}\hat{\theta}(k+1) &= \hat{\theta}(k) + \lambda_{\ell^p}(k)\xi_{\ell^p}(k)q_{\ell^p}(k),\\ \xi_{\ell^p}(k) &= \frac{2(\|\hat{\eta}\|_{\ell^p,k,\rho}^p - \|\hat{\eta}\|_{\ell^p,k,\rho}^{p-1}\alpha_{\ell^p}(k) - \beta_{\ell^p}(k))}{|q_{\ell^p}(k)|_2^2 + r(k)},\\ q_{\ell^p}(k) &= \sum_{i=0}^{k}\rho^{-p(k-i)}|\hat{\eta}(i)|^{p-1}\mathrm{sign}(\hat{\eta}(i))\phi(i),\\ \lambda_{\ell^p}(k) &= \begin{cases} 1 & \text{if } \|\hat{\eta}\|_{\ell^p,k,\rho}^p > \|\hat{\eta}\|_{\ell^p,k,\rho}^{p-1}\alpha_{\ell^p}(k) + \beta_{\ell^p}(k) \\ 0 & \text{otherwise}\end{cases},\\ \beta_{\ell^p}(k) &= \sum_{i=0}^{k}\left(\rho^{-p(k-i)}|\hat{\eta}(i)|^{p-1}\mathrm{sign}(\hat{\eta}(i))\cdot\right.\\ &\quad \left.\phi^T(i)(\hat{\theta}(k) - \hat{\theta}(i))\right).\end{aligned}$$

Remark: The algorithms proposed above are indeed on-line algorithms since $q_{L^p}(t)$, $\|\hat{\eta}\|_{L^p,t,\sigma}^p$, $\beta_{L^p}(t)$, $q_{\ell^p}(k)$, $\|\hat{\eta}\|_{\ell^p,k,\rho}^p$, and $\beta_{\ell^p}(k)$ can be implemented by the following ways:

$$\begin{aligned}\dot{q}_{L^p}(t) &= -p\sigma q_{L^p}(t) + |\hat{\eta}(t)|^{p-1}\mathrm{sign}(\hat{\eta}(t))\phi(t),\\ &\quad q_{L^p}(0) = 0,\\ \frac{d}{dt}\|\hat{\eta}\|_{L^p,t,\sigma}^p &= -p\sigma\|\hat{\eta}\|_{L^p,t,\sigma}^p + |\hat{\eta}(t)|^p,\\ &\quad \|\hat{\eta}\|_{L^p,0,\sigma}^p = 0,\\ \dot{\beta}_{L^p}(t) &= -p\sigma\beta_{L^p}(t) + r(t)\lambda_{L^p}(t)|q_{L^p}(t)|_2^2,\\ &\quad \beta_{L^p}(0) = 0,\\ q_{\ell^p}(k) &= \rho^{-p}q_{\ell^p}(k-1) + |\hat{\eta}(k)|^{p-1}\mathrm{sign}(\hat{\eta}(k))\phi(k),\\ &\quad q_{\ell^p}(0) = |\hat{\eta}(0)|^{p-1}\mathrm{sign}(\hat{\eta}(0))\phi(0),\\ \|\hat{\eta}\|_{\ell^p,k,\rho}^p &= \rho^{-p}\|\hat{\eta}\|_{\ell^p,k-1,\rho}^p + |\hat{\eta}(k)|^p,\\ &\quad \|\hat{\eta}\|_{\ell^p,0,\rho}^p = |\hat{\eta}(0)|^p,\\ \beta_{\ell^p}(k) &= \rho^{-p}\beta_{\ell^p}(k-1)\\ &\quad +\rho^{-p}\lambda_{\ell^p}(k-1)\xi_{\ell^p}(k-1)|q_{\ell^p}(k-1)|_2^2,\\ &\quad \beta_{\ell^p}(0) = 0.\end{aligned}$$

Remark: If $\lambda_c = 1$, $\lambda_{L^p} = 1$, $\lambda_d = 1$, or $\lambda_{\ell^p} = 1$, it can be shown that the estimation errors must be nonzero. As is shown in the following theorems, the parameter estimates are updated to reduce the estimation errors.

Theorem 1 *When the algorithms $A_t^c|_{t=0}^{\infty}$ and $AL_t^p|_{t=0}^{\infty}$ are applied to system (1), $p \in [1, \infty)$, the following properties will hold.*

(a) *The estimation error $e(t) = |\hat{\theta}(t) - \theta^*|_2$ satisfies (2) for all $t \in \mathbf{R}_0$, and $\lim_{t\to\infty} e(t)$ exists.*

(b) *If $\liminf_{t\geq 0} r(t) > 0$, and r, $\hat{\eta}$, α_c, $\|\hat{\eta}\|_{L^p,t,\sigma}$, α_{L^p}, and β_{L^p} are uniformly continuous, then, given $\epsilon > 0$, there exist $T_c > 0$ and $T_{L^p} > 0$ such that*

$$|\hat{\eta}(t)| < \alpha_c(t) + \epsilon, \ \forall\ t \geq T_c,$$

and

$$\|\hat{\eta}\|_{L^p,t,\sigma}^p < \|\hat{\eta}\|_{L^p,t,\sigma}^{p-1}\alpha_{L^p}(t) + \beta_{L^p}(t) + \epsilon, \ \forall\ t \geq T_{L^p},$$

respectively, for the algorithms $A_t^c|_{t=0}^{\infty}$ and $AL_t^p|_{t=0}^{\infty}$.

(c) *Let the* consistent parameter set Θ_c^c *be defined as:*

$$\Theta_c^c = \begin{cases} \{\theta : |c(t) - \phi^T(t)\theta| \leq \alpha_c(t), \ \forall\ t \in \mathbf{R}_0\}, \\ \{\theta : \|c - \phi^T\theta\|_{L^p,t,\sigma} \leq \alpha_{L^p}(t), \ \forall\ t \in \mathbf{R}_0\},\end{cases}$$

then, for any $\theta \in \Theta_c^c$, the error $e(t) = |\hat{\theta}(t) - \theta|_2$ satisfies (2) for all $t \in \mathbf{R}_0$, and $\lim_{t\to\infty} e(t)$ exists.

(d) $\lim_{t\to\infty}\hat{\theta}(t)$ *exists if there exist a positive integer* j *and some* $\theta^1,\ldots,\theta^j \in \Theta_c^c$ *such that* $\bigcap_{i=1}^{j}\{v : |v-\theta^i|_2 = c_i\}$ *consists of only isolated points for any* $c_i \in \mathbf{R}_0$.

Theorem 2 *When the algorithms* $A_k^d|_{k=0}^{\infty}$ *and* $A\ell_k^p|_{k=0}^{\infty}$ *are applied to system (1),* $p \in [1,\infty)$, *the following properties will hold.*

(a) *The estimation error* $e(k) = |\hat{\theta}(k) - \theta^*|_2$ *satisfies (3) for all* $k \in \mathrm{I}_0$, *and* $\lim_{k\to\infty} e(k)$ *exists.*

(b) *If* $\liminf_{k\geq 0} r(k) > 0$, *and* r, q_d, *and* q_{ℓ^p} *are uniformly bounded, then, given* $\epsilon > 0$, *there exist integers* $T_d > 0$ *and* $T_{\ell^p} > 0$ *such that*

$$|\hat{\eta}(k)| < \alpha_d(k) + \epsilon, \ \forall\ k \geq T_d,$$

and

$$\|\hat{\eta}\|_{\ell^p,k,\rho}^p < \|\hat{\eta}\|_{\ell^p,k,\rho}^{p-1}\alpha_{\ell^p}(k) + \beta_{\ell^p}(k) + \epsilon, \ \forall\ k \geq T_{\ell^p},$$

respectively, for the algorithms $A_k^d|_{k=0}^{\infty}$ *and* $A\ell_k^p|_{k=0}^{\infty}$.

(c) *Let the* consistent parameter set Θ_c^d *be defined as:*

$$\Theta_c^d = \begin{cases} \{\theta : |c(k) - \phi^T(k)\theta| \leq \alpha_d(k), \ \forall\ k \in \mathrm{I}_0\}, \\ \{\theta : \|c - \phi^T\theta\|_{\ell^p,k,\rho} \leq \alpha_{\ell^p}(k), \ \forall\ k \in \mathrm{I}_0\}, \end{cases}$$

then, for any $\theta \in \Theta_c^d$, *the error* $e(k) = |\hat{\theta}(k) - \theta|_2$ *satisfies (3) for all* $k \in \mathrm{I}_0$, *and* $\lim_{k\to\infty} e(k)$ *exists.*

(d) $\lim_{k\to\infty}\hat{\theta}(k)$ *exists if the conditions in (b) are satisfied and there exist a positive integer* j *and some* $\theta^1,\ldots,\theta^j \in \Theta_c^d$ *such that* $\bigcap_{i=1}^{j}\{v : |v-\theta^i|_2 = c_i\}$ *consists of only isolated points for any* $c_i \in \mathbf{R}_0$.

Remark: The conditions about Θ_c^c in (d) of Theorem 1 or Θ_c^d in (d) of Theorem 2 are almost always satisfied. For example, consider $\theta \in \mathbf{R}^3$, if there exist θ^1, θ^2, and $\theta^3 \in \Theta_c^c$ such that θ^1, θ^2, and θ^3 are not on a line, then $\bigcap_{i=1}^{3}\{v : |v-\theta^i|_2 = c_i\}$ consists at most two isolated points for any $c_i \in \mathbf{R}_0$.

Remark: The results of (b) in Theorem 1 or Theorem 2 mean that the steady state performances of the algorithms $A_t^c|_{t=0}^{\infty}$, $AL_t^p|_{t=0}^{\infty}$, $A_k^d|_{k=0}^{\infty}$, and $A\ell_k^p|_{k=0}^{\infty}$ depend on the bounds $\alpha_c(t)$, $\alpha_{L^p}(t)$, $\alpha_d(k)$, and $\alpha_{\ell^p}(k)$, respectively. Therefore, if these bounds are smaller because the noise level is lower, then the steady state performance is better.

Remark: If in addition a compact convex set Θ_0 is given in advance such that $\theta^* \in \Theta_0$, a projection operator can be applied to the algorithms to ensure $\hat{\theta} \in \Theta_0$ while retaining the other properties of the algorithms. Furthermore, the modified algorithms can be successfully applied to identify the parameter of systems with both nonparametric uncertainties and disturbances [5] [6].

5 Conclusion

A robust on-line parameter identification problem is formulated and solved for systems with general knowledge of the level of the suitably reformulated information noise. A switching type algorithm is developed for this problem, which guarantees that the estimation error is decreasing strictly whenever the parameter estimate updates, and the parameter estimate is convergent to a steady state value under a mild condition. Because the estimation error is guaranteed to decrease monotonically and a large class of applied inputs are admissible, the on-line parameter identification algorithm has a great potential to be incorporated in a robust adaptive control system as a robust estimator.

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