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Control System Design for the PenduLIM: a Novel Integrated Architecture of Inverted Pendulum and Linear Induction Motor

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***Abstract*— We propose an integrated control architecture for the "PenduLIM" which consists of an inverted pendulum (IP) mounted on a linear induction motor (LIM). According to this innovative architecture, the IP is swung up and stabilized to its upright unstable equilibria as well as the displacement is regulated to zero, by exerting horizontal thrust from the LIM. In order to cope with this highly nonlinear and unstable system, the IP is controlled via a passivity-based energy controller coincides with a model-reference adaptive controller while the LIM is controlled via a thrust controller with secondary resistance adaptation. Then, by feeding the IP control law as command of LIM servo control subsystem, the overall closed-loop system is globally asymptotically stable (A.S.) in the sense of arbitrary initial displacement and angle. Finally, the success of proposed control scheme is demonstrated by numerical simulations.**

***Index Terms*— PenduLIM, linear induction motor, inverted pendulum, passivity, homoclinic, model reference adaptive control, secondary resistance adaptation, servo control**

List of Acronyms

A.S. asymptotically stable
DOF degree of freedom
FOC field-oriented control
HES Hall effect sensor
IM induction motor
IP inverted pendulum
LIM linear induction motor
MRACmodel reference adaptive control
PPC pole-placement control
SIMO single-input multi-output

I. Introduction

UNDERACTUATED mechanical control systems provide a challenging research area of increasing interest in both application and theory.In this paper, we will examine a class of underactuated mechanical systems and address problems in both nonlinear control design and integration. This research work will propose a passivity-based swing-up control and a novel adaptive control systems to deal with a special case of such underactuated nonlinear system—the PenduLIM.

Since the late 1970s, the dynamics and control of IPs have attracted worldwide attention. Owing to their nonlinear and unstable nature, pendulums have maintained their usefulness and been used to illustrate many of the ideas emerging in the field of nonlinear control. Åström and Furuta [4] swang up an IP using energy-based approach with an insightful discussion about swing-up behaviors. On the other hand, Spong and Praly [33] separated the problem into a swing-up control and a balance control strategy. And this method has been tested successfully on many typical underactuated mechanical systems such as Acrobot, Pendubot, three-link mechanical robot and reaction wheel pendulum [34]. Many other research have applied the neural network or fuzzy theories to control the inverted pendulum systems [2], [16], [20], [21], [37].

On the other hand, due to the highly nonlinear dynamics, the control of induction motors has become the benchmark of nonlinear control theories and has been extensively explored over the past decade. Recent years, their linear version— the LIMs have also been widely explored by both academics and industries in related fields. LIMs are widely used in different applications, especially in high-speed ground transportation [13], [24]. In this thesis, we will develop a new control system for the novel PenduLIM, which is composed of an IP mounted on an LIM. Undoubtedly, the overall control is an integration of the following two subsystems:

In the IP control subsystem, a passivity-based swing-up control coincides with an model reference adaptive control (MRAC) stabilizing control will be integrated through a switching law. With this parameter-insensitive control, the IP can be swung up and stabilized to its upright position almost globally except for starting from the downward position. In addition, the displacement of cart will also be regulated to zero.

As for the LIM part, a nonlinear adaptive servo[1] control with unknown secondary resistance will be proposed. In particular, this adaptive observer-based controller is capable of achieving high performance thrust as well as velocity tracking, and only the primary currents and linear speed are assumed available for the design.

By taking the advantage of this well-controlled actuator, the overall PenduLIM will work cooperatively to asymptotically erect the IP via the LIM. Most important of all, our analysis and synthesis are rigorous and systematic. The simulation as well as the experimental results will be presented to demonstrate the performance of the hereby developed integrated

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[1]A servo motor is a class of electric motors used in feedback control of a mechanical device for closed loop systems.

control system.

II. Inverted Pendulum Control

The inverted pendulum control we adopt can be found in [6], and the architecture are shown in Fig. 1.

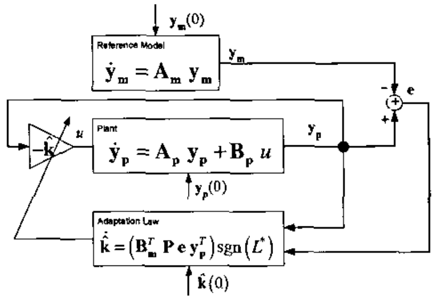


Fig. 1: Architecture of MRAC for IP stabilization.

III. Linear Induction Motor Control

A. Model Development

The steady-state analysis of an LIM is usually done by a classical equivalent circuit. In this thesis, we will follow this convention. Since LIMs are just cut open and rolled flat from the RIMs, to formulate the dynamic model of an LIM, we consider the following assumptions to simplify the analysis.

M1. Three phases are balanced.

M2. The magnetic circuit is unsaturated.

M3. End-effect is negligible.

Please note the Assumption M3 is adequate since many techniques have already been developed to eliminate such nonlinear physical phenomenon. For example, Fujii *et al.* proposed a simple end-effect compensator mounted on the primary part [10].

Then, from the model derivation of a RIM with some proven modification, we can obtain the field-oriented control (FOC) model of a LIM as follows:

Corollary 3.1 (Mathematical Model of a LIM): By introducing some modification to the RIM model, we obtain the LIM model as follows:

$$\begin{aligned} L_o\,\dot{I}_d &= -L_M\,R_s\,I_d - \beta_1\,I_d + R_s\,\psi_d + \beta_2\,v_p\,\psi_q + \beta_3\,V_d \\ L_o\,\dot{I}_q &= -L_M\,R_s\,I_q - \beta_1\,I_q - \beta_2\,v_p\,\psi_d + R_s\,\psi_q + \beta_3\,V_q \\ L_s\dot{\psi}_d &= -R_s\,\psi_d + L_M\,R_s\,I_d - \beta_2\,v_p\,\psi_q \\ L_s\dot{\psi}_q &= -R_s\,\psi_q + L_M\,R_s\,I_q + \beta_2\,v_p\,\psi_d \\ F_e &= k_F\,(\psi_d\,I_q - \psi_q\,I_d) \end{aligned} \tag{1}$$

B. Servo Control System Development

In this section, we will present a control on thrust and velocity control for a LIM. To cope with uncertain secondary resistance R_s, an adaptive observer will be designed as well. Although, the following derivations is copious, but it does provide a systematical approach to solve a complex nonlinear control problem with LIM.

1) Problem Formulation: Given the mathematical model of LIM in (1), the control objective is now to design a nonlinear adaptive thrust servo controller with robustness to variation of secondary resistance which guarantees the asymptotic stability of the closed-loop system.

In addition, the following assumptions are required to clarify the synthesis of our control system.

M4. All the parameters of the motor except the secondary resistance R_s and fluxes (ψ_d, ψ_q) are known.

M5. The primary currents (I_d, I_q) and velocity v_p are measurable.

M6. The secondary resistance R_s is unknown but its upper and lower bounds are known, i.e.,

$$0 < \underline{R_s} < R_s < \overline{R_s}$$

M7. The desired force should be bounded continuously differentiable, i.e. $F' \in BC^1$.

M8. By considering Coulomb friction, viscous friction, and mechanical load effect, the load force F_L can be expressed as:

$$F_L = \mu_0\,\mathrm{sgm}_k\,(v_p) + \mu_1\,v_p + \mu_2\,\dot{v}_p \tag{2}$$

$$= F_e - M_p\,\dot{v}_p - D\,v_p \tag{3}$$

where the positive constants k, μ_0, μ_1, μ_2, D are known and the sigmoidal function $\mathrm{sgm}_k\,(v_p)$ is defined as:

$$\mathrm{sgm}_k\,(v_p) \triangleq \frac{1-\exp\left(-k\,v_p\right)}{1+\exp\left(-k\,v_p\right)}$$

M9. The secondary resistance (R_s) and the equivalent secondary self-inductance (L_s) should satisfy $\frac{R_s}{L_s} - 1 > 0$.

The control scheme is shown in Fig. 2.

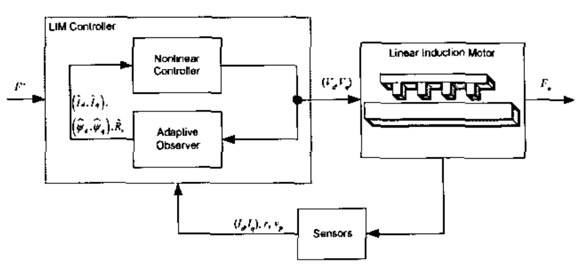


Fig. 2: Block diagram of LIM thrust control.

Theorem 3.2 (Adaptive LIM thrust control): Consider a LIM whose dynamics are governed by (1) under the Assumptions M1–M9. Then the output electrical thrust F_e will approach to the desired thrust F' asymptotically by the following controller:

$$
\begin{aligned}
u_1 &= \frac{L_o}{\beta_3}\Big(-\frac{1}{L_o}\hat{R}_s\,\hat{\psi}_d - \frac{\beta_2}{L_o}\,v_p\,\hat{\psi}_q + \dot{I}_d{}' \\
&\qquad + \frac{L_M}{L_o}\,\hat{R}_s\,I_d' + \frac{\beta_1}{L_o}I_d' - k_1\,e_1 + v_1\Big)\\
u_2 &= \frac{L_o}{\beta_3}\Big(-\frac{1}{L_o}\hat{R}_s\,\hat{\psi}_q + \frac{\beta_2}{L_o}\,v_p\,\hat{\psi}_d + \dot{I}_q{}' \\
&\qquad + \frac{L_M}{L_o}\,\hat{R}_s\,I_q' + \frac{\beta_1}{L_o}I_q' - k_1\,e_2 + v_2\Big)
\end{aligned}
$$

for some constant $k_1 > 0$, where subject to the error dynamics:

$$
\begin{aligned}
L_o\,\dot{\tilde{I}}_d &= -k_0\,\tilde{I}_d - L_M\,\tilde{R}_s\,I_d + R_s\,\tilde{\psi}_d + \tilde{R}_s\,\hat{\psi}_d + \beta_2\,v_p\,\tilde{\psi}_q - u_{o1} - u_{c1}\\
L_o\,\dot{\tilde{I}}_q &= -k_0\,\tilde{I}_q - L_M\,\tilde{R}_s\,I_q + R_s\,\tilde{\psi}_q + \tilde{R}_s\,\hat{\psi}_q - \beta_2\,v_p\,\tilde{\psi}_d - u_{o2} - u_{c2}\\
L_s\,\dot{\tilde{\psi}}_d &= -R_s\,\tilde{\psi}_d - \tilde{R}_s\,\hat{\psi}_d + L_M\,\tilde{R}_s\,I_d - \beta_2\,v_p\,\tilde{\psi}_q - u_{o3} - u_{c3}\\
L_s\,\dot{\tilde{\psi}}_q &= -R_s\,\tilde{\psi}_q - \tilde{R}_s\,\hat{\psi}_q + L_M\,\tilde{R}_s\,I_q + \beta_2\,v_p\,\tilde{\psi}_d - u_{o4} - u_{c4}
\end{aligned}
\tag{4}
$$

with

$$
\begin{aligned}
v_1 &= \frac{1}{L_o\,L_s}\,\hat{R}_s\,\left(L_o\,\tilde{I}_d - \zeta_d\right)\\
v_2 &= \frac{1}{L_o\,L_s}\,\hat{R}_s\,\left(L_o\,\tilde{I}_q - \zeta_q\right)
\end{aligned}
$$

and the auxiliary control inputs u_{ci}, u_{oi}, $i = 1,2,3,4$ and ζ_d, ζ_q, η_d and η_q are designed according to

$$
\begin{aligned}
u_{o1} &= -\frac{\beta_2}{L_s}\,v_p\,L_o\,\tilde{I}_q - \frac{\hat{R}_s}{L_s}\,\left(L_o\,\tilde{I}_d - \zeta_d\right)\\
u_{o2} &= \frac{\beta_2}{L_s}\,v_p\,L_o\,\tilde{I}_d - \frac{\hat{R}_s}{L_s}\,\left(L_o\,\tilde{I}_q - \zeta_q\right)\\
u_{o3} &= -k_0\,\tilde{I}_d - \frac{\beta_2}{L_s}\,v_p\,\tilde{I}_q - u_{o1} - u_{c1}\\
u_{o4} &= -k_0\,\tilde{I}_q + \frac{\beta_2}{L_s}\,v_p\,\tilde{I}_d - u_{o2} - u_{c2}
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
u_{c1} &= -L_o\,u_{c3}, u_{c2} = -L_o\,u_{c4}\\
u_{c3} &= -g_1\,\frac{\beta_2}{L_s\,L_o}\,v_p\,e_2, u_{c4} = g_1\,\frac{\beta_2}{L_s\,L_o}\,v_p\,e_1\\
\dot{\eta}_d &= -\frac{1}{L_s}\,\left(\tilde{I}_d + \frac{g_1}{L_o}\,e_1\right), \dot{\eta}_q = -\frac{1}{L_s}\,\left(\tilde{I}_q + \frac{g_1}{L_o}\,e_2\right)\\
\dot{\zeta}_d &= \frac{1}{L_s}\,\left(\tilde{I}_d + \frac{g_1}{L_o}\,e_1 + \beta_2\,v_p\,\tilde{I}_q\right) - u_{c3}\\
\dot{\zeta}_q &= \frac{1}{L_s}\,\left(\tilde{I}_q + \frac{g_1}{L_o}\,e_2 - \beta_2\,v_p\,\tilde{I}_d\right) - u_{c4}
\end{aligned}
\tag{6}
$$

with the following parameter adaptation law:

$$
\dot{\hat{R}}_s = \begin{cases} \Gamma_s\,\left(\Omega_r + \frac{1}{2}\,g_2\,\gamma^2\,a^2\,\left(\underline{R_s} - \hat{R}_s\right)\right), & \text{if } \hat{R}_s > \overline{R_s} + \delta_1\\ \delta_2, & \text{if } \hat{R}_s = \overline{R_s} + \delta_1 \end{cases}
$$

subject to

$$
\hat{R}_s(0) > \overline{R_s} + \delta_1
$$

for some constants δ_1, δ_2, g_2, $\gamma > 0$. ■

Proof: The detailed development and proofs can be found in [6]. ■

IV. The PenduLIM

We have developed controllers for the IP as well as for its actuator subsystem, the LIM, as proposed in Sections II and III, respectively. In this chapter, we mount the IP on the LIM to formulate our innovative integrated control architecture of the *PenduLIM*, and the apparatus is shown in Fig. 3. On the other hand, by integrating the previously designed controls, we have the overall automatic control system for the PenduLIM, and the functional block diagram is drawn in Fig. 4. Technically, the stability and robustness will be analyzed in Sections IV-A and IV-B, respectively.

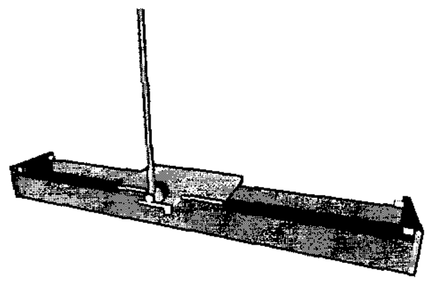


Fig. 3: The apparatus of PenduLIM (perspective view).

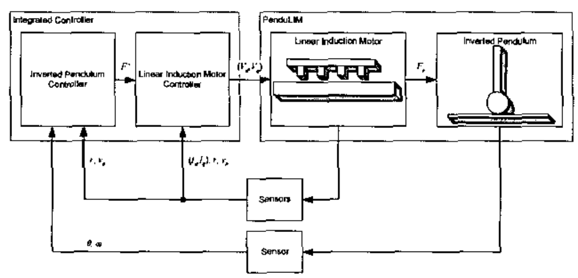


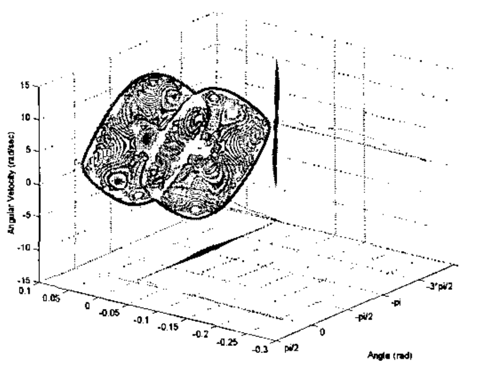
Fig. 4: Integrated control architecture of the PenduLIM.

A. Stability Analysis

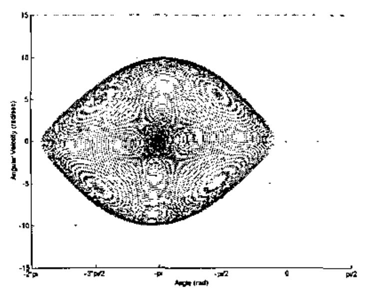
First, I have to emphasize this integration of IP and LIM is valid. Considering the block diagram shown in Fig. 4, we fed the IP control as thrust command of LIM thrust controller. The reason is that the desired thrust trajectory generated by IP controller satisfies the command Assumption M7 required by LIM controller which has been proven in [6]. To show this integration indeed achieves the desired control objectives, i.e., regulate all the states to 0 from downward position, is shown in Fig. 5.

Second, one may doubt the stability of the switching rule. This unstable transition cannot happen because:

- The switching only occurs *once* during the operation. During the stabilizing procedure, θ will not overshoot to θ_t with the adaptive control proposed in Sec. II. The



(a) 3D view of controlled PenduLIM with its projections.



(b) The projection of trajectory on θ-$\dot{\theta}$ plane forms the "homoclinic orbit".

Fig. 5: System trajectory of controlled PenduLIM with its projections, and the brighter line represents the later trajectory.

major advantage of this integrated IP control is that no high-frequent switching between the swing-up control and stabilizing control will arise.

- The switching moment is indeed Lebesgue measure[2] zero compared to the whole trajectory.

In conclusion, this switching strategy will not cause instability of the PenduLIM.

Third, this integrated controlled system is *globally* asymptotically stable. Recall that the IP cannot be swung up if starting from downward position, i.e., $\mathrm{x_p}(0) \triangleq \begin{bmatrix} r(0) & \theta(0) & \dot{r}(0) & \dot{\theta}(0) \end{bmatrix}^T = \begin{bmatrix} 0 & \pi & 0 & 0 \end{bmatrix}^T$, as early proved in Sec. II.However, this plight can be overcome if taking the the LIM subsystem into consideration. Despite the inital command generated by the IP control is zero, the transient response[3] of the LIM system makes the inital states $\dot{r} \equiv v_p \neq 0$. As a consequence, this incident makes the IP immediately leave the forbidden equilibria, and thus the globally asymptotical stability is achieved (shown in Fig. 5).

B. Robust Analysis

To enhance the robustness of the PenduLIM against external disturbances, it is useful to increase the control gain g_1 of LIM control system. However, this adjustment requires more power from the LIM. In case the PenduLIM is knocked down, the control will switch back to swing-up mode and then balance the IP again from the the aids of switching law provided in Sec. II.

As a consequence, we summarize the integrated control architecture for the PenduLIM in the following theorem.

Theorem 4.1 (Automatic erection of the PenduLIM): Consider a PenduLIM which consists of an IP mounted on an LIM. To automatically swing up and stabilize the IP to its upright by exerting horizontal thrust from the LIM, we can utilize the integrated control architecture as shown in Fig. 4. Specifically, by feeding the IP control law (proposed in Sec. II) as command oto the LIM servo control system (proposed in Theorem 3.2), the overall closed-loop system is globally asymptotically stable (A.S.) in the sense of arbitrary initial displacement and angle. ■

V. Simulation & Experiment Results

A. Experiment Apparatus

We have built a PenduLIM equipment in the Advanced Control Laboratory (ACL) in the Department of Electrical Engineering at the National Taiwan University. And, the simulation and experiments are carried out using this PenduLIM

B. Software Simulation

The software we adopt for simulation is MATLAB® 6.1 together with SIMULINK® 4.0.[4] MATLAB® which integrates computation, visualization, and programming into an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. On the other hand, SIMULINK® is a graphical package suitable for fast prototyping and for testing our controllers. The combination of these tools constructs our numerical simulation environment.

However, before performing simulations of the proposed integrated control architecture, it is important to verify the effectiveness of its subsystem—high performance LIM servo control in advance.

1) PenduLIM Control: Having verified the high performance LIM servo control, it is quite straightforward to validate the PenduLIM control in this stage. Most important of all, we not only build the IP control system but also the LIM servo control system into our integrated controlled PenduLIM. Hence, two different initial conditions are simulated to demonstrate the effectiveness of the proposed integrated control architecture.

[2]For more detailed information about the theory of measure, please refer to [1].

[3]Despite the command generated by the IP control is zero, the LIM control system has to adapt to R_s, and thus its response must not be zero.

[4]MATLAB® is a language of technical computing while SIMULINK® is an interactive tool for modelling, simulating, and analyzing dynamic, multi-domain systems. Both are registered trademarks of the MathWorks Inc. Please visit `http://www.mathworks.com/` for more information.

r	θ	$\dot{r}$	$\dot{\theta}$	others
0.3	180.0°	0.0	0.0	0.0

(a) Initial values of PenduLIM.

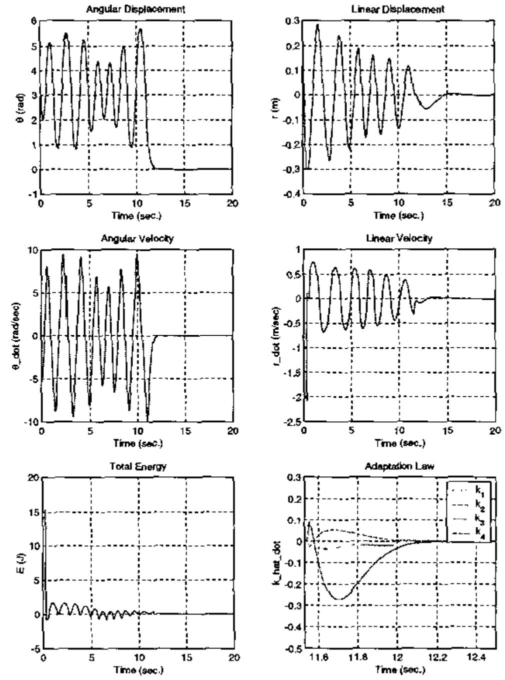
Thrust Controller Gains			
g_1	g_2	γ	k_1
0.5	10.0	1.0	1.0
Observer Gains			
k_0	Γ_s	δ_1	δ_2
1.0×10^{-2}	1.0×10^{-3}	1.0×10^{-2}	1.0
Thrust Controller Initial Values			
$\psi'_d(0), \psi'_q(0)$		others	
$(0.8, 0.6)$		0.0	

(b) Designed parameters and initial values of LIM control system.

Swing-Up Controller Gains			
k_r	k_v	k_E	k_{dr}
0.1	0.01	0.001163	0.0025
Stabilizing Controller Gains			
$\mathbf{\Gamma}$	Ω_0	Ω_1	θ_t
$0.1\mathbf{I}$	3.0	2.0	15°
$\mathbf{k}(0)$			
$k_1(0)$	$k_2(0)$	$k_3(0)$	$k_4(0)$
$k_1^* + 5.0$	$k_2^* + 10.0$	$k_3^* + 10.0$	$k_4^* + 10.0$

(c) Designed parameters of IP control system, where $\mathbf{k}^*$ is the static feedback gain computed from PPC [6].

TABLE I: Various designed gains and initial values of the PenduLIM control system.



(a) Signals of IP.

Fig. 6: Simulation of controlled PenduLIM when swinging up from $(r(0), \theta(0)) = (0.3\ \mathrm{m}, 180°)$ and then stabilized to its upright.

a) $(r(0), \theta(0)) = (0.3\ \mathrm{m}, 180°)$*:* Initially, the pendulum is rested at the downward configuration, and the primary part of the LIM is set at 0.3 m away from the middle of the track. After turning on our integrated control system, the IP takes about 12 sec to automatically swing up to $\theta = \theta_t = 15°$. Soon after that time, the displacement of the primary is regulated to zero. Since the control switches to stabilize the IP to the upright configuration. Therefore, the objective of automatic erection is successfully achieved.

The simulation parameters of the PenduLIM control system is given in Table I. Please note that we perturb the initial guess of dynamic feedback gain $\mathbf{k}(0)$ which leads to the adaptation process of $\mathbf{k}$ as shown in Fig 6.

VI. Conclusions

We have proposed an integrated control architecture for the "PenduLIM" which consists of an underactuated inverted pendulum (IP) mounted on a linear induction motor (LIM). According to this innovative architecture, the IP is swung up and stabilized to its upright unstable equilibria as well as the displacement is regulated to zero, by exerting horizontal thrust from the LIM. In order to cope with this highly nonlinear and unstable system, the IP is controlled via a passivity-based energy controller coincides with a model-reference adaptive controller while the LIM is controlled via a thrust controller with secondary resistance adaptation. Then, by feeding the IP control law as command of LIM servo control subsystem, the overall closed loop system is globally asymptotically stable (A.S.) in the sense of arbitrary initial displacement and angle. Finally, the success of proposed control scheme is demonstrated by numerical simulations and a part of experiments.

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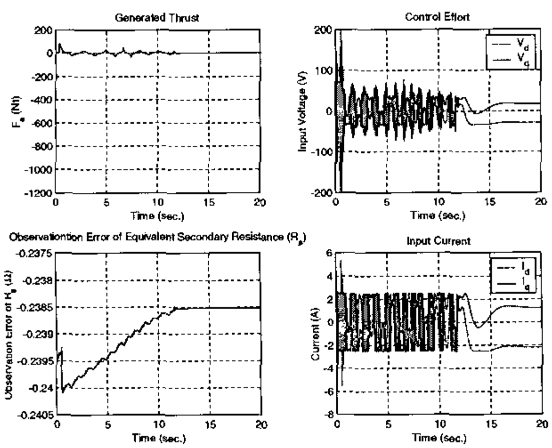

(b) Signals of LIM.

Fig. 6: *(continued).*

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