ADAPTIVE FUZZY SLIDING MODE CONTROLLER DESIGN FOR NONLINEAR MIMO SYSTEMS

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Abstract

This paper will address the problem of controlling an unknown MIMO nonlinear affined system. A fuzzy sliding mode controller (FSMC) is used to approximate the equivalent control by using an on-line fuzzy adaptation scheme, and then the hitting control is appended to show that the proposed FSMC can result in a closed-loop system, which is stable. This scheme also provides the designers flexibility to design and to implement fuzzy rule base without domain experts and without mathematical model. The robust adaptive scheme is shown to be able to guarantee that the output tracking error can converge to a residual set ultimately by a two-dimensional inverted pendulum system.

Keywords
Sliding mode, adaptive fuzzy system, MIMO, nonlinear.

1 Introduction

Sliding mode control (SMC), or variable structure control, employs a discontinuous control law to drive state trajectory toward a specified sliding surface and maintain its motion along the sliding surface in the state space [1, 2, 3]. It has successfully been applied to a wide variety of systems having uncertainties and invariance to unknown disturbance [4]. However, the control chattering problem caused by the discontinuity of the control action is undesirable in most applications [5].

Recently, integrating fuzzy set theory and SMC into fuzzy controller design have acquired superior performance [4, 6, 7, 8]. This approach retains the positive property of SMC but alleviates the chattering, and the fuzzy control rules can be determined systematically by the reaching condition of the SMC. Therefore, the major difficulty of the method is to simultaneously guarantee the stability of the fuzzy control system as well as to obtain a suitable equivalent control system if the system dynamic model is unknown in advance. Furthermore, it is usually difficult to directly extend the design approach into multiple-input multiple-output (MIMO) systems when the system dimension increases and the coupling is unknown.

This paper will address the problem of controlling an unknown MIMO nonlinear affined system. First, a FSMC is used to approximate the equivalent control by using an on-line fuzzy adaptation scheme, and then the hitting control is appended to show that the proposed FSMC can result in a closed-loop system, which is stable.

2 Problem formulation

Consider a MIMO nonlinear system whose equations of motion can be governed by

$$\boldsymbol{y}^{(r)} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{y} = [y_1, \cdots, y_m]^T$ and $\boldsymbol{y}^{(r)} \equiv [y_1^{(r_1)}, \cdots, y_m^{(r_m)}]^T$ denote the output vector and its derivative, respectively, $\boldsymbol{r} = [r_1, \cdots, r_m]$ with $\sum_{i=1}^{m} r_i = n$ is defined as the system relative degree, $\boldsymbol{u} = [u_1, \cdots, u_m]^T$ is the input, $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T = [y_1, \cdots, y_1^{(r_1-1)}, \cdots, y_m^{(r_m-1)}]^T$ is the state vector, $\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})]^T$, $\boldsymbol{G}(\boldsymbol{x}) = [\boldsymbol{g}_1(\boldsymbol{x}), \cdots, \boldsymbol{g}_m(\boldsymbol{x})]$, $f_i(\boldsymbol{x})$ and $\boldsymbol{g}_i(\boldsymbol{x}) = [g_{i1}(\boldsymbol{x}), \cdots, g_{im}(\boldsymbol{x})]^T$ are unknown functions.

Let $\dot{\boldsymbol{y}}_{di} = [y_{di}, \dot{y}_{di}, \cdots, y_{di}^{(r_i-1)}]^T$ represents the known desired trajectory for $y_{di}(t)$, $i = 1, \cdots, r_i$. Let the tracking error as $\underline{\boldsymbol{e}} = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_m]^T$ with

$$\boldsymbol{e}_i = [y_{di} - y_i, \dot{y}_{di} - \dot{y}_i, \cdots, y_{di}^{(r_i-1)} - y_i^{(r_i-1)}]^T \tag{2}$$

and $\underline{\Lambda} = [\Lambda_1, \cdots, \Lambda_i]$, with $\Lambda_i = [\alpha_{i1}, \cdots, \alpha_{ir_i}]^T \in R^{r_i}$ be such that all roots of the polynomial

$$h_i(s) = s^{(r_i)} + \alpha_{i1}s^{(r_i-1)} + \cdots + \alpha_{i(r_i-1)}\dot{s} + \alpha_{ir_i} \quad (3)$$

are in the open left-half plane, $i = 1,\cdots,m$. The control aim is to determine a controller for the composite nonlinear system described by (1) so that the tracking error $\underline{e} = [e_1,\cdots,e_m]^T$ will be attenuated to an arbitrarily small residual tracking error set.

Conceptually, the control strategy consists of two design goals: the first is to force the system toward a desired dynamics and the second is to maintain the system on that differential geometry. The equivalent control $\hat{\boldsymbol{u}}_{eq}$ is estimated by using an adaptive mechanism that forces the system state to slide on the sliding surface. Another is the hitting control $\hat{\boldsymbol{u}}_h$ that drives the states toward the sliding surface. Thus the control law can be represented as

$$\boldsymbol{u} = \hat{\boldsymbol{u}}_{eq} + \hat{\boldsymbol{u}}_h \quad (4)$$

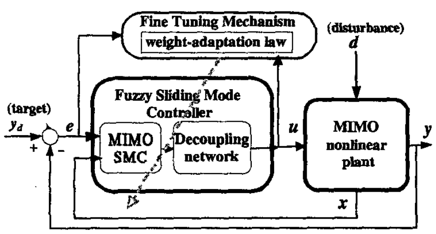

Fig. 1:The configuration of the adaptive fuzzy sliding mode control system

3 Design of the fuzzy sliding mode controller

The proposed robust fuzzy sliding mode controller is composed of the following three parts: a MIMO SMC, a fine-tuning mechanism on the consequent membership functions of multi-layer fuzzy system, and a decoupling network. Fig. 1 shows the configuration of the MIMO sliding mode controller and the interconnections compensating network of the adaptive fuzzy control system. The multi-layer fuzzy system and the decoupling network are nominal designs based on on-line approximation of the unknown nonlinear functions of the plant. The fine-tuning mechanism is designed to encounter the equivalent uncertainty resulted by the plant uncertainty, the function approximation error, or the external disturbances.

For the systems given in (1), the *i*th sliding surface is s_i. Hence, this MIMO SMC also has m switching surfaces to form a switching manifold so that the system exhibits desirable behavior when its trajectories are confined in the sliding surface. Define a generalized error vector as follows:

$$\boldsymbol{s} = [s_1(e_1,t), s_2(e_2,t),\cdots,s_m(e_m,t)]^T = \begin{bmatrix} e_1^{(r_1-1)} + \alpha_{11}e_1^{(r_1-2)} + \cdots + \alpha_{1,(r_1-1)}e_1 \\ e_2^{(r_2-1)} + \alpha_{21}e_2^{(r_2-2)} + \cdots + \alpha_{2,(r_2-1)}e_2 \\ \vdots \\ e_m^{(r_m-1)} + \alpha_{m1}e_m^{(r_m-2)} + \cdots + \alpha_{m,(r_m-1)}e_m \end{bmatrix} \quad (5)$$

Since (5) satisfies Hurwitz stability criterion from (3), maintaining system states on surface $\boldsymbol{s}(t)$ for all $t > 0$ is equivalent to the tracking problem $\boldsymbol{y} = \boldsymbol{y}_d$. The tracking control problem can be formulated by keeping the error vector $\underline{\boldsymbol{e}}$ on the sliding surface defined as follows:

$$\dot{\boldsymbol{s}} = \boldsymbol{y}_d^{(r)} - \boldsymbol{y}^{(r)} + \begin{bmatrix} \sum_{i=1}^{r_1-1}\alpha_{1i}e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1}\alpha_{2i}e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1}\alpha_{mii}e_m^{(r_m-i)} \end{bmatrix} \quad (6)$$

In the design of sliding mode controller, an equivalent control is given first such that each state Lyapunov-like condition holds for system stability [2]:

$$\tfrac{1}{2}\tfrac{d}{dt}(s_i^2) \le -\eta_i|s_i|,\ \eta_i > 0,\ i = 1,\cdots,m \quad (7)$$

or in sum:

$$\frac{1}{2}\frac{d}{dt}(s^2 s) \le -\sum_{i=1}^{m}\eta_i|s_i|,\ \eta_i > 0, i = 1,\cdots,m \quad (8)$$

Therefore the control problem is to obtain the optimal control input $\boldsymbol{u}^*$ that guarantees the sliding condition (8).

If the control of nonlinear MIMO systems (1) uses SMC directly but does not take the interconnections among subsystems into consideration, then the control law $\boldsymbol{u}^0$ can be chosen as follows

$$\boldsymbol{u}^0 = \hat{D}^{-1}\left(\begin{bmatrix} \sum_{i=1}^{r_1-1}\alpha_{1i}e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1}\alpha_{2i}e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1}\alpha_{mi}e_m^{(r_m-i)} \end{bmatrix} - \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{y}_d^{(r)} - \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})\right) \quad (9)$$

where $\boldsymbol{K}$ is $m \times m$ positive definite diagonal gain matrix, $\hat{\boldsymbol{D}} = Block\ diag[g_{ii}]$ and

$$\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s}) = [K_1\,\mathrm{sgn}(s_1),\cdots,K_m\,\mathrm{sgn}(s_m)]^T \quad (10)$$

where $K_i > 0$ and $\mathrm{sgn}(s_i)$ is defined as

$$\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases} \quad (11)$$

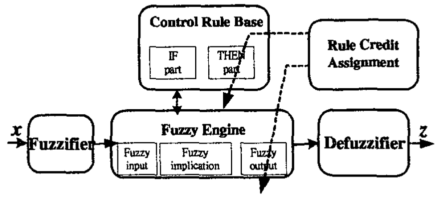

Fig. 2: Diagrammatic representation of fuzzy system with adjustable rule credit assignment.

Since exact decoupling requires exact mathematical model of the controlled plant, the interconnections compensating network is needed. The proposed fuzzy sliding mode controller has a neural part to release the interaction among the subsystems and a fuzzy part to asymptotically cancels the non-linearity in the system. The output of the controller is combined with $\boldsymbol{u}^0$ and its modification by decoupling network

$$\boldsymbol{u}(t) = \boldsymbol{u}^0(t) + \boldsymbol{M}\boldsymbol{u}^0(t) \tag{12}$$

To derive a stable weight adaptation in control matrix, the matrix $\boldsymbol{M}$ be chosen as

$$\boldsymbol{M} = -(\boldsymbol{I}_m + \hat{\boldsymbol{C}}^{-1}\hat{\boldsymbol{D}})^{-1} \tag{13}$$

where $\boldsymbol{I}_m$ denotes a $m \times m$ identity matrix and

$$\hat{\boldsymbol{C}} = \begin{bmatrix} 0 & g_{12} & \cdots & g_{1m} \\ g_{21} & 0 & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & 0 \end{bmatrix} \tag{14}$$

Using (9), (12), (13) and the matrix inversion $(A+BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}$ [9], the formulation of SMC resolves into

$$\boldsymbol{u} = G^{-1}\left(\begin{bmatrix} \sum_{i=1}^{r_1-1} \alpha_{1i} e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1} \alpha_{2i} e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1} \alpha_{mi} e_m^{(r_m-i)} \end{bmatrix} - \boldsymbol{f}(\boldsymbol{x}) + y_d^{(r)} - \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})\right) \tag{15}$$

where $\boldsymbol{G} = \hat{\boldsymbol{C}} + \hat{\boldsymbol{D}}$. By plugging $\boldsymbol{u}$ into (6), we will have $\dot{s} = -\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})$. Thus, the sliding condition (7) can be easily verified.

However, $\boldsymbol{f}$ and $\boldsymbol{G}$ are unknown, only their estimations $\hat{\boldsymbol{G}}$ and $\hat{f}$ can be used to construct $\boldsymbol{u}$. To solve this problem, we propose the adaptive scheme using the fuzzy logic system.

4 Description of the adaptive fuzzy system

The fuzzy system can uniformly approximate nonlinear continuous functions to arbitrary accuracy [10, 11]. Thus we will introduce fuzzy systems, which are expressed as a series expansion of fuzzy basis functions, to model the uncertainties $\boldsymbol{G}(\boldsymbol{x})$ and $\boldsymbol{f}(\boldsymbol{x})$ by tuning the parameters of the corresponding fuzzy systems. The basic configuration of the fuzzy logic system comprises four principal components: fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier [10]. The configuration of the fuzzy system with adjustable rule credit assignment is shown in Fig. 2 [12].

The fuzzy logic system performs a mapping from $\boldsymbol{U} \subset R^n$ to $\boldsymbol{V} \subset R^m$. Let $\boldsymbol{U} = U_1 \times \cdots \times U_n$ and $\boldsymbol{V} = V_1 \times \cdots \times V_m$ where $U_k \subset R$, $k = 1,2,\cdots,n$ and $V_i \subset R$, $i = 1,2,\cdots,m$. A multivariable system can be controlled by the following N linguistic rules

$R^{(l)}$: IF x_1 is A_1^l and $\cdots$ and x_n is A_n^l

THEN z_1 is B_1^l and $\cdots$ and z_m is B_m^l (16)

where $l = 1,\cdots,N$, $x_k, k = 1,2,\cdots,n$, are the input variables to the fuzzy system, $z_i, i = 1,2,\cdots,m$, are the output variables of fuzzy systems, and the antecedent fuzzy sets A_k^l in U_k and the consequent fuzzy sets B_i^l in V_i are linguistic terms characterized by the fuzzy membership functions $\mu_{A_k^l}(x_k)$ and $\mu_{B_i^l}(z_i)$. The fuzzy logic systems with center-average defuzzifier, product inference and singleton fuzzifier is defined as [10]

$$z_i(\boldsymbol{x}) = \frac{\sum_{l=1}^{N} \mu^l(\boldsymbol{x}) \cdot q_i^l}{\sum_{l=1}^{N} \mu^l(\boldsymbol{x})} \tag{17}$$

where $\mu^l(\boldsymbol{x}) = \Pi_{k=1}^{n} \mu_{A_k^l}(x_k)$ is the matching degree of the lth, and q_i^l is the center of the consequent membership function of the lth rule. If q_i^l is chosen as the design parameter, the adaptive fuzzy system can be viewed as the type of neural network.

Therefore, (17) can be rewritten as

$$z_i(\boldsymbol{x}) = \phi^T \psi(\boldsymbol{x}) \tag{18}$$

where $\phi = [q_i^1,\cdots,q_i^N]^T$ is a parameter vector, and $\psi = (\xi_1,\cdots,\xi_N)^T$ is a regressor, and where the fuzzy basis function is defined as [10]

$$\xi_l = \frac{\Pi_{k=1}^{n} \mu_{A_k^l}(x_k)}{\sum_{l=1}^{N}\left(\Pi_{k=1}^{n} \mu_{A_k^l}(x_k)\right)} \tag{19}$$

5 Learning algorithm and performance analysis

In this section, we show how to derive an adaptive law to adjust the regulating factor such that the estimated equivalent control $\hat{u}_{eq}$ can be optimally approximated to the equivalent control of the SMC under the situations of unknown function $\boldsymbol{f}$ and $\boldsymbol{G}$. Then, we construct the hitting control to guarantee system's stability.

We now define the control $\boldsymbol{u} = \hat{\boldsymbol{u}}_{eq} + \hat{\boldsymbol{u}}_h$ where auxiliary input as

$$\hat{\boldsymbol{u}}_{eq} = G^{-1}\left(\begin{bmatrix} \sum_{i=1}^{r_1-1} \alpha_{1i} e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1} \alpha_{2i} e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1} \alpha_{mi} e_m^{(r_m-i)} \end{bmatrix} - \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{y}_d^{(r_x)} + \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{x})\eta_\Delta\right)$$

and the hitting control $\hat{\boldsymbol{u}}_h = G^{-1}\boldsymbol{u}_h^{\cdot}$. Define the parameters θ_i^* and $\boldsymbol{w}_{ij}^*$ of the best function approximation as

$$\theta_i^* \equiv \arg \min_{\theta_i \in \Omega_{\theta_i}} [\sup_{x\in\Omega_x} | f_i(\boldsymbol{x}) - \hat{f}_i(\boldsymbol{x} | \theta_i) |] \qquad (20)$$

$$\boldsymbol{w}_{ij}^* \equiv \arg \min_{w_{ij}\in\Omega_{w_{ij}}} [\sup_{x\in\Omega_x} | g_{ij}(\boldsymbol{x}) - \hat{g}_{ij}(\boldsymbol{x} | \boldsymbol{w}_{ij}) |] \qquad (21)$$

where Ω_{θ_i} and $\Omega_{w_{ij}}$ are constraint sets for θ_i and $\boldsymbol{w}_{ij}$, defined as $\Omega_{\theta_i} = \{\theta_i : |\theta_i| \le M_{\theta_{i,Max}}\}$ and $\Omega_{w_{ij}} = \{w_{ij} : |w_{ij}| \le M_{w_{ij,Max}}\}$ where $M_{\theta_{i,Max}}, M_{w_{ij,Max}}$ are specified by the designer. The fuzzy logic systems $\hat{f}_i(\boldsymbol{x}|\theta_i)$ and $\hat{g}_{ij}(\boldsymbol{x}|\boldsymbol{w}_{ij})$ are

$$\hat{f}_i(\boldsymbol{x}|\theta_i) = \theta_i^T \xi_f\,(\boldsymbol{x}) = \xi_f{}^T(\boldsymbol{x})\theta_i \qquad (22)$$

$$\hat{g}_{ij}(\boldsymbol{x}|\boldsymbol{w}_{ij}) = \boldsymbol{w}_{ij}^T \xi_g\,(\boldsymbol{x}) = \xi_g{}^T(\boldsymbol{x})\boldsymbol{w}_i \qquad (23)$$

where $\xi_f(x)$ and $\xi_g(x)$ are vectors of fuzzy bases, θ_i and $\boldsymbol{w}_{ij}$ are the corresponding parameters of fuzzy logic systems. Thus equation (6) can be rewritten as

$$\begin{aligned} \dot{s} &= [\hat{\boldsymbol{f}}(s|\underline{\theta}) - \boldsymbol{f}(\boldsymbol{x})] + [\hat{G}(\boldsymbol{x}) - G(\boldsymbol{x})]\hat{\boldsymbol{u}}_{eq} - \boldsymbol{u}_h - \boldsymbol{K}\cdot\mathrm{sgn}(s)\cdot\eta_\Delta \\ &= [\hat{\boldsymbol{f}}(\boldsymbol{x}|\underline{\theta}) - \hat{\boldsymbol{f}}(\boldsymbol{x}|\underline{\theta}^*)] + [\hat{G}(\boldsymbol{x}|\underline{w}) - \hat{G}(\boldsymbol{x}|\underline{w}^*)]\hat{\boldsymbol{u}}_{eq} \\ &\quad + \zeta_f + \zeta_G \hat{\boldsymbol{u}}_{eq} - \boldsymbol{u}_h - \boldsymbol{K}\cdot\mathrm{sgn}(s)\cdot\eta_\Delta \\ &= \begin{bmatrix} \tilde{\theta}_1^T \xi_f \\ \tilde{\theta}_2^T \xi_f \\ \vdots \\ \tilde{\theta}_m^T \xi_f \end{bmatrix} + \begin{bmatrix} \tilde{w}_{11}^T\xi_g & \tilde{w}_{12}^T\xi_g & \cdots & \tilde{w}_{1m}^T\xi_g \\ \tilde{w}_{21}^T\xi_g & \tilde{w}_{22}^T\xi_g & \cdots & \tilde{w}_{2m}^T\xi_g \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{m1}^T\xi_g & \tilde{w}_{m2}^T\xi_g & \cdots & \tilde{w}_{mm}^T\xi_g \end{bmatrix} \begin{bmatrix} \hat{u}_{eq1} \\ \hat{u}_{eq2} \\ \vdots \\ \hat{u}_{eqm} \end{bmatrix} \\ &\quad - \begin{bmatrix} K_1\cdot\mathrm{sgn}(s_1)\cdot\eta_{\Delta 1} \\ K_2\cdot\mathrm{sgn}(s_2)\cdot\eta_{\Delta 2} \\ \vdots \\ K_m\cdot\mathrm{sgn}(s_m)\cdot\eta_{\Delta m} \end{bmatrix} - \boldsymbol{u}_h + \zeta_f + \zeta_G \hat{\boldsymbol{u}}_{eq} \end{aligned}$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$, $\tilde{w}_{ij} = w_{ij} - w_{ij}^*$, denotes the parameter estimation errors, the minimum approximation errors as $\zeta_f = f(\boldsymbol{x}) - \hat{f}(\boldsymbol{x}|\theta^*)$, $\zeta_G = G(\boldsymbol{x}) - \hat{G}(\boldsymbol{x}|w^*)$. Our design objective involves specifying the control and adaptive laws for θ_i and $\boldsymbol{w}_{ij}$ such that the sliding condition (8) is guaranteed.

Theorem 1: Consider nonlinear plant (1) with controller (4), the tracking error allows us to use the following adaptive law

$$\dot{\theta}_i = -s_i \gamma_i \xi_f \qquad (24)$$

$$\dot{\boldsymbol{w}}_{ij} = -s_i \beta_{ij} \xi_g \hat{u}_{fj} \qquad (25)$$

$$\begin{aligned} u_{hi} = \mathrm{sgn}(s_i)[| f_i |_{\max} + \sum_{j=1}^{m} | g_{ij} |_{\max} \cdot | \hat{u}_{eqj} | \\ + | y_{di}^{r} | + | \sum_{j=1}^{r_j-1} \alpha_{1j} e_i^{(r_j-1)} |] \end{aligned} \qquad (26)$$

Proof: Now consider the Lyapunov candidate

$$V = V_1 + V_2 + \cdots + V_m \qquad (27)$$

where $V_i = \frac{1}{2}(s_i^T s_i + \frac{1}{r_i}\tilde{\theta}_i^T\tilde{\theta}_i + \sum_{j=1}^{m} \frac{1}{\beta_{ij}} \tilde{w}_{ij}^T \tilde{w}_{ij})$.

By the fact $\dot{\tilde{\theta}}_i = \dot{\theta}_i$, $\dot{\tilde{\boldsymbol{w}}}_{ij} = \dot{\boldsymbol{w}}_{ij}$, we obtain the derivative of V_i as

$$\dot{V} = \dot{V}_1 + \cdots + \dot{V}_m, \qquad (28)$$

$$\begin{aligned} \dot{V}_i &= s_i^T \dot{s}_i + \tfrac{1}{r_i}\tilde{\theta}_i^T \dot{\tilde{\theta}}_i + \sum_{j=1}^{m} \tfrac{1}{\beta_{ij}} \tilde{w}_{ij}^T \dot{\tilde{w}}_{ij} \\ &= s_i[\tilde{\theta}_i^T \xi_f + \zeta_{fi} + \sum_{j=1}^{m} (\tilde{w}_{ij}^T \xi_g + \zeta_{Gij})\hat{u}_{eqj} \\ &\quad - K_i \cdot \mathrm{sgn}(s_i)\cdot\eta_{\Delta i} - u_{hi}] + \tfrac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i + \sum_{j=1}^{m} \tfrac{1}{\beta_{ij}} \tilde{w}_{ij}^T \dot{\tilde{w}}_{ij} \\ &= \tfrac{1}{\gamma_i}\tilde{\theta}_i^T [s_i \gamma_i \xi_f + \dot{\tilde{\theta}}_i] + \sum_{j=1}^{m} \tfrac{1}{\beta_{ij}} \tilde{w}_{ij}^T [s_i \beta_{ij} \xi_g \hat{u}_{eqj} + \dot{\tilde{w}}_{ij}] \\ &\quad + s_i\zeta_{fi} - s_i K_i \cdot \mathrm{sgn}(s_i)\cdot \eta_{\Delta i} + s_i \sum_{j=1}^{m} \zeta_{Gij}\hat{u}_{eqj} - s_i u_{hi} \end{aligned}$$

If we choose the adaptive law as $\dot{\tilde{\theta}}_i = -s_i r_i \xi_f$ and $\dot{\tilde{\boldsymbol{w}}}_{ij} = -s_i \beta_{ij} \xi_g \hat{u}_{eq}$, then

$$\dot{V}_i \le s_i \zeta_{fi} + s_i \sum_{j=1}^{m} \zeta_{Gij} \hat{u}_{fj} - s_i u_{hi}$$

where we use the fact that u_{hi} has the same sign with s_i.

In order to complete the FSMC design, it is necessary to show that the hitting control is enough to force the state trajectory toward the sliding surface as well as to establish asymptotic convergence of the tracking error. Consider the Lyapunov function candidate:

$$V_i = \tfrac{1}{2} s_i^2 \qquad (29)$$

Taking the derivative of (29), one has

$$\dot{V}_i = s_i(-f_i - \sum_{j=1}^{m} g_{ij}\hat{u}_{eqj} + \sum_{j=1}^{r_j-1}\alpha_{1j}e_i^{(r_j-1)}$$
$$+ y_{di}^{r}) - s_i u_{hi} \quad (30)$$

To ensure (30) is less than zero, the hitting control should be selected as

$$u_{hi} = \mathrm{sgn}(s_i)[| f_i |_{\max} + \sum_{j=1}^{m} | g_{ij} |_{\max} \cdot | \hat{u}_{eqj} |$$
$$+ | y_{di}^{r} | + | \sum_{j=1}^{r_j-1}\alpha_{1j}e_i^{(r_j-1)} |]$$

This means that the inequality $\dot{V}_i = s_i\dot{s}_i < 0$ is obtained and the hitting control actually achieves a stable FSMC system.

From the above discussion, we use a FSMC to estimate the equivalent control of SMC system. Conceptually, the equivalent control is desired when the state trajectory is near s_i=0, while the hitting control is determined in the case of $s_i \neq 0$ [3]. A fuzzy rule base is of the form

If s_i is *ZO* Then u_i is $u_i = \hat{u}_{eqi}$ (31)

If s_i is *NZ* Then u_i is $u_i = \hat{u}_{eqi} + \hat{u}_{hi}$ (32)

where *ZO* and *NZ* denote zero and nonzero fuzzy sets, respectively, and input variable s_i is given in (5). The control law of the fuzzy controller is

$$u_i = \frac{\mu_{ZO}(s)\hat{u}_{eqi} + \mu_{NZ}(s)[\hat{u}_{eq} + \hat{u}_{hi}]}{\mu_{ZO}(s) + \mu_{NZ}(s)} \quad (33)$$

where $\mu_{ZO}(s)$ and $\mu_{NZ}(s)$ is the membership functions of fuzzy sets *ZO* and *NZ*, respectively. The membership functions of fuzzy sets *ZO* and *NZ* are selected to overlap and be symmetric to satisfy $\mu_{ZO}(s) + \mu_{NZ}(s) = 1$.

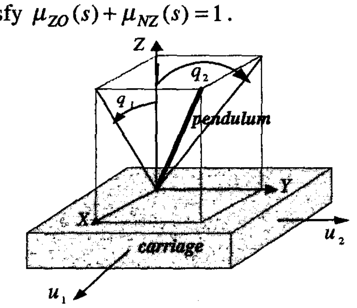

Figure 3:An inverted 2D pendulum system.

6 Simulation

In this section we demonstrate the geometric configuration of an inverted pendulum manipulator with 2 degrees of freedom in the rotational angles described by Euler angles q_1 and q_2 [13], as shown in Fig. 3. On the assumption that the rotational angles of the two planar pendulums are small, motions of the two planar pendulums can be considered independent of each other. The external forces, u_1 and u_2, are applied to keep the pendulum at the upright position in X and Y direction, m_0 and m_1 are the mass of the carriage and pendulum., respectively.

The dynamic equations describing the motion of an inverted pendulum system are derived by the Lagrange scale function $L(\boldsymbol{q},\dot{\boldsymbol{q}})$ [14]. After some manipulation, the dynamic equations are of the following form

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ C_2(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} + \boldsymbol{Z}(\boldsymbol{q}) = N\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where

$$H_{11} = m_1 l^2(1/3 + \cos^2 q_2 + \sin^2 q_1 \sin^2 q_2)$$
$$-(m_1^2 l^2 / m_0+m_1)(\cos^2 q_1 \cos^2 q_2 + \sin^2 q_1 \sin^2 q_2)$$
$$H_{12} = H_{21} = [(2m_1^2 l^2 / m_0+m_1) - m_1 l^2]\cos q_1 \sin q_1 \cos q_2 \sin q_2$$
$$H_{22} = m_1 l^2(1/3 + \sin^2 q_2 + \cos^2 q_1 \cos^2 q_2)$$
$$-(m_1^2 l^2 / m_0+m_1)(\sin^2 q_1 \sin^2 q_2 + \cos^2 q_1 \cos^2 q_2)$$
$$C_1 = (m_1^2 l^2 / m_0+m_1)(\dot{q}_1^2 + \dot{q}_2^2)\cos q_1 \sin q_1(\cos^2 q_2 - \sin^2 q_2)$$
$$+(2m_1^2 l^2 / m_0+m_1)\dot{q}_1\dot{q}_2(\cos^2 q_1 - \sin^2 q_1)\cos q_2 \sin q_2$$
$$+m_1 l^2(\dot{q}_1^2 + \dot{q}_2^2)\cos q_1 \sin q_1 \sin^2 q_2 - 2m_1 l^2 \dot{q}_1\dot{q}_2 \cos^2 q_1 \cos q_2 \sin q_2$$
$$C_2 = (m_1^2 l^2 / m_0+m_1)(\dot{q}_1^2 + \dot{q}_2^2)(\cos^2 q_1 - \sin^2 q_1)\cos q_2 \sin q_2$$
$$+(2m_1^2 l^2 / m_0+m_1)\dot{q}_1\dot{q}_2 \cos q_1 \sin q_1(\cos^2 q_2 - \sin^2 q_2)$$
$$+m_1 l^2(\dot{q}_1^2 + \dot{q}_2^2)\sin^2 q_1 \cos q_2 \sin q_2 - 2m_1 l^2 \dot{q}_1\dot{q}_2 \cos q_1 \sin q_1 \cos^2 q_2$$
$$\boldsymbol{Z} = -m_1 g l[\sin q_1 \cos q_2, \cos q_1 \sin q_2]^{\mathrm{T}}$$
$$N = m_1 l / m_0+m_1 \begin{bmatrix} \cos q_1 \cos q_2 & -\sin q_1 \sin q_2 \\ -\sin q_1 \sin q_2 & \cos q_1 \cos q_2 \end{bmatrix}$$

The kinematics and inertial parameters of the pendulum system are chosen as $m_0 = 1kg$, $m_1 = 0.5kg$, $l = 0.5m$, and initial states $q_1(0) = q_2(0) = 0.2\,rad$, $\dot{q}_1(0) = \dot{q}_2(0) = 0\,rad/s$. The trajectories to be followed are described by two decoupled linear systems, the desired coefficients are specified to be $\alpha_{i1} = 2$, $\alpha_{i2} = 1$, $i = 1,2$. The pendulum is given the following target joint rotations:

$$q_{d1} = (\pi/15)\sin t, q_{d2} = (\pi/15)\cos t + (\pi/30)\cos t$$

The membership functions of states q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ (represented by generic variable x_i) for the qualitative statements are defined as $NB: \exp(-0.5(x_i + 0.4)^2)$, $NS: \exp(-0.5(x_i + 0.2)^2)$, $PB: \exp(-0.5(x_i - 0.4)^2)$, $PS: \exp(-0.5(x_i - 0.2)^2)$, $ZE: \exp(-0.5x_i^2)$. Consider the design parameters are given by $\gamma_i = 0.1$, $\beta_{ij} = 0.01$, $| f_i |_{\max} = 16$, $| g_{ij} |_{\max} = 1.9$, $K_i = 1$, $\eta_{\Delta i} = 0.02$, $i, j = 1,2$.

The curves and phase plane of $q_1(t)$ and $q_2(t)$ are given in Fig. 4-7, respectively. From these simulation results, the track error has been attenuated efficiently. Thus we see that our adaptive fuzzy sliding mode controller can control the inverted pendulum to follow the desired trajectory without using any linguistic information.

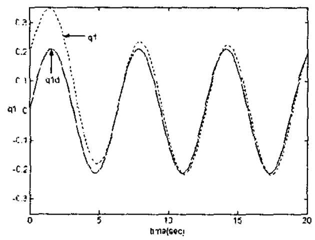

Fig. 4:The tracking curves of the q_1 and q_{d1}

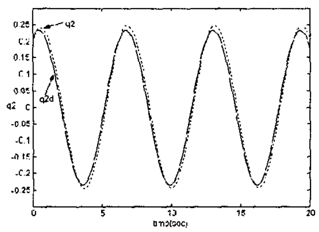

Fig. 5:The tracking curves of the q_2 and q_{d2}

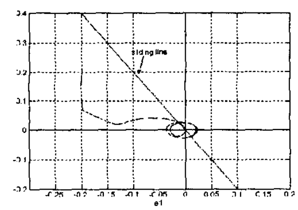

Fig. 6:State Trajectory of $e_1 - \dot{e}_1$

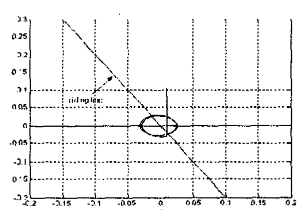
Fig. 7:State Trajectory of $e_2 - \dot{e}_2$

7 Conclusions

In this paper, a fuzzy controller based sliding mode is proposed for the trajectory tracking of MIMO system with unknown nonlinear dynamics. When matching with the model occurs, the overall control system is equivalent to a stable dynamic system. The bounds of the fuzzy modeling error are estimated adaptively using an learning algorithm and the global asymptotic stability of the algorithm is established via Lyapunov function. The overall robust adaptive scheme is shown to guarantee that the output tracking error can converge to a residual set ultimately.

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