fading of the desired packet than to that of the interferers. For high values of m_o and m_u, the throughput approaches that of the ideal channel i.e. $G\exp(-G)$.

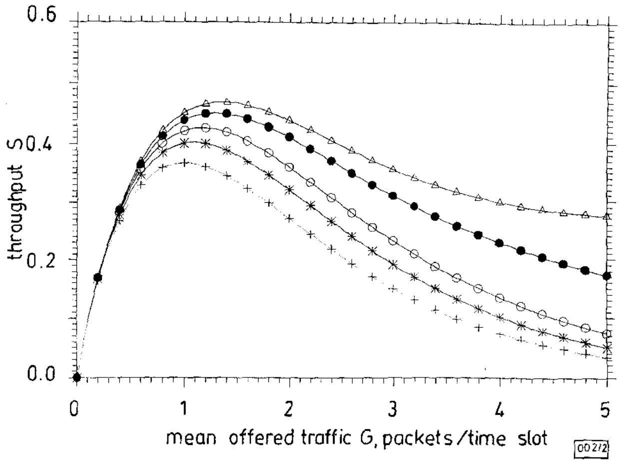

Fig. 2 *Effect of shadow spread on channel throughput with z_o = 6dB, m_o = m_u = 2.3, and $\rho_{o,u}$ = 0.4*

Effect of $\sigma_o = \sigma_u$
× 0dB
○ 3dB
● 6dB
△ 9dB
+ ideal channel

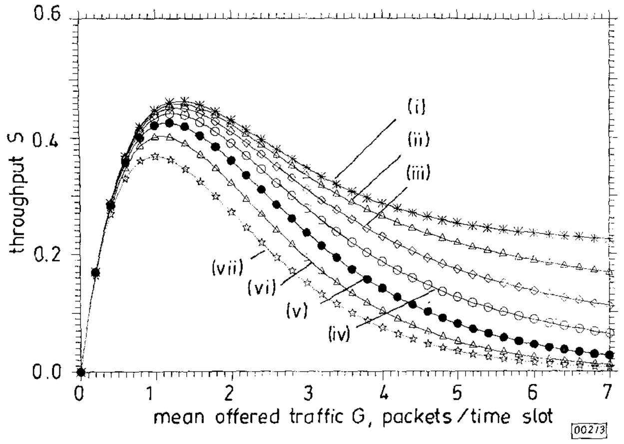

Fig. 3 *Effect of correlation coefficient $\rho_{o,u}$ on channel throughput with z_o = 6dB, m_o = m_u = 2.3, and $\sigma_o = \sigma_u$ = 6dB*

(i) 0.0
(ii) 0.2
(iii) 0.4
(iv) 0.6
(v) 0.8
(vi) 1.0
(vii) ideal channel

Figs. 2 and 3 show the effect on the channel throughput of the shadow spread and correlation between packets. The results show that the presence of shadowing increases the throughput significantly. The throughput was observed to be highly sensitive to the shadow spread and to the correlation coefficient. It becomes higher with increasing shadow spread and decreasing correlation coefficient.

Conclusions: This Letter presents an exact formulation of the throughput of slotted ALOHA protocol in a multiuser radio system with Nakagami fading and correlated shadowing. The fading parameter of the interfering packets was observed to affect the throughput more than that of the desired packet. Correlated shadowing was observed to have a significant effect on the channel throughput.

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Constructions of link-fault-tolerant *q*-ary *k*-cube networks

Jichinag Tsai and Sy-Yen Kuo

Indexing terms: Fault tolerant computing, Codes

A technique for constructing fault-tolerant q-ary k-cube networks for an arbitrary positive integer q is investigated. Extra sets of links are added to construct the fault-tolerant network such that in the presence of link faults, the remaining healthy portion is guaranteed to contain an original fault-free network. Linear codes over an integer ring are used for this approach.

Introduction: Hypercubes, tori, meshes and Omega networks are well-known interconnection networks for parallel computers. The structure of these networks can be described in a topology called the q-ary k-cube. This topology has a k-dimensional grid structure with q nodes in each dimension such that every node is connected to its two immediate neighbours, modulo q, in each dimension. Since links in a q-ary k-cube can be regularly grouped into sets, we can add extra sets of links in certain ways to enhance the fault-tolerance ability, so that even when link faults occur, regardless of their distribution, a network of original structure can still be found within the remaining healthy portion of the constructed fault-tolerant network.

Latifi and El-Amawy [1] added another set of links to the hypercube, called the folded hypercube, to tolerate a single fault. Later, Shih and Batcher [2] presented an *ad hoc* scheme, which can find appropriate redundant sets of links that can tolerate two or three faults for a hypercube. Using techniques from coding theory, Bruck *et al.* [3] constructed f-fault-tolerant q-ary k-cubes when q is prime and $f \leq q - k + 1$. In [4], they continued to developed a more general version capable of handling an arbitrary number of faults even when q is a prime number. In this Letter, we discuss how to construct fault-tolerant q-ary k-cubes for an arbitrary positive integer q. We also show how these constructions can be used to tolerate link faults in mesh networks.

Mathematical model: Our approach for handling faults is based on a graph model. In this model the network architecture can be viewed as a graph, where nodes represent processors and edges represent communication links between nodes. The graph of a q-ary k-cube can be described using a more general framework [3].

Definition: Let q and k be positive integers and the set $S \subseteq Z_q^k$, where $Z_q = \{0, 1, ..., q-1\}$. The graph $N(q, k, S)$ is a graph with q^k nodes indexed by distinct k-tuple vectors over Z_q, and with the edges specified by the set of vectors in S. Namely, two nodes, e.g. X and Y, are connected by an edge iff there exists a vector $V \in S$ such that $X + V = Y$ or $Y + V = X$ where addition is vector addition performed over Z_q. □

If the graph $N(q, k, S)$ is isomorphic to a q-ary k-cube, there is a path from all-0 node to every other arbitrary node since $N(q, k, S)$ is a connected graph. Each path corresponds to a linear combination of vectors in S. Hence, the vectors in S necessarily span the Z_q-module Z_q^k (when q is prime, Z_q^k is a vector space). The following theorem emerges as a result [3].

Theorem 1: The graph $N(q, k, S)$ is isomorphic to the q-ary k-cube iff S is a set of k linearly independent vectors over Z_q. □

From the foregoing discussion, we know that edges specified by extra vectors can be added to a q-ary k-cube to construct a fault-tolerant graph. The method for constructing f-fault-tolerant q-ary k-cubes is to find a set of n k-tuple vectors over the ring Z_q, with the property that any $n - f$ vectors in the set span the module Z_q^k, i.e. they contain a subset of k linearly independent vectors. This follows because, in the worst case, f faulty edges can affect at most f distinct vectors. We denote such a set by $S_q(k, f)$, where $n = |S_q(k, f)|$ is the number of vectors in the set. We then have the following corollary.

Corollary: Any $n - f$ vectors of $S_q(k, f)$ where $n = |S_q(k, f)|$ contain a subset of k linearly independent vectors over Z_q. □

Proposed method: The method for constructing an f-edge-fault-tolerant q-ary k-cube is based on coding theory. The main issue in the theory of codes is to construct a large set of vectors over a finite ring to form a code with the property that the Hamming distance between any two vectors is larger than a predefined parameter d (the minimum distance). An (n, k, d) q-ary linear code is a code of length n, dimension k, and minimum distance d over Z_q. This code can be described by a $k \times n$ matrix, called the generator matrix, which spans the code. We denote this matrix as $G_q(n, k, d)$. An n-tuple vector V is in the code iff there exists a k-tuple vector X over Z_q such that $X \cdot G_q = V$, where computations are performed over Z_q. That is, any linear combination of the k row vectors (of length n) in G_q forms a vector in the code.

The key in [4] is to prove the equivalence between the constructions of fault-tolerant q-ary k-cube graphs and the constructions of generator matrices for q-ary linear codes when q is a prime. Their result can be directly generalised to the arbitrary positive integer q as the following theorem.

Theorem 2: The matrix $S_q(k, f)$ and the generator matrix $G_q(n, k, f + 1)$ are equivalent. □

The idea behind Theorem 2 is that an $(n, k, f + 1)$ q-ary linear code is an f-erasure-correcting code. When f erasures (errors whose positions are known) occur in an n-tuple codeword, these f erroneous symbols can be corrected by other $n - f$ remaining healthy symbols through the parity check equations of this code. Columns of the generator matrix $G_q(n, k, f + 1)$ also have this property. The values of any f columus of $G_q(n, k, f + 1)$ can be derived from the values of the other $n - f$ columns through the parity check equations with n column vectors instead of n corresponding symbols. Because $G_q(n, k, f + 1)$ has k linearly independent columns, any f columns of these k linearly independent columns is recoverable from the other $n - f$ columns of $G_q(n, k, f + 1)$. Hence, these k linearly independent columns can be expressed with linear combinations of any $n - f$ columns. Therefore, any $n - f$ columns of $G_q(n, k, f + 1)$ contain a subset of k linearly independent columns. This satisfies the requirements of $S_q(k, f)$.

We now give an example to illustrate our results.

Example: In [5], Shankar derived BCH codes over an arbitrary finite integer ring. Using this technique, we obtain the generator matrix of an (8, 4, 4) linear code over Z_9 in the following:

$$G_9(8,4,4) = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 4 & 0 & 4 \\ 0 & 1 & 0 & 0 & 5 & 1 & 5 & 2 \\ 0 & 0 & 1 & 0 & 2 & 4 & 1 & 4 \\ 0 & 0 & 0 & 1 & 5 & 0 & 5 & 1 \end{pmatrix}$$

The previous matrix can be used as $S_9(4, 3)$ to construct a fault-tolerant 9-ary 4-cube, in which three link faults are fully tolerable. In this case, four extra sets of links are added.

Reconfiguration process: We need to find a subset of k linearly independent columns from the submatrix formed by the remaining columns of $S_q(k, f)$ after excluding the columns that have faulty edges to reconfigure the system. This problem can be solved using the Gaussian elimination to transform this submatrix into row echelon form. For our purposes, Gaussian elimination is not an expensive algorithm because it is applied to matrices of moderate size, and is more efficient than the exhaustive search used in [2].

Fault tolerance for meshes: We now discuss how our approach deals with link faults in meshes. If the k dimensions of a q-ary k-cube have only, at most, one faulty link each, after removing the faulty links, the remaining part still contains a mesh of the same width and dimensions. This is because it is possible to rotate each dimension independently, so as to place the single faulty link in that dimension (if it exists) in the position of a wrap-around link in the q-ary k-cube, which is not required in the mesh. We have that any $n - f$ vectors of $S_q(k, f)$ contain a subset of k linearly independent vectors that can form a q-ary k-cube. Since the k vectors can only have at most, one faulty edge each, in order to contain a k-dimensional mesh of the form $q \times q \times ... \times q$ as a subgraph, these $n - f$ vectors of $S_q(k, f)$ also have to have only at most one faulty edge each. Other remaining f vectors then necessarily have, at most, two faulty edges each. Thus, there can be at most $(2f + 1)$ faulty edges. Hence, the graph $N(q, k, S_q(k, f))$ is $(2f + 1)$-edge-fault-tolerant for such a mesh. Note that $(2f + 1)$ is the maximum number of edge faults which can be tolerated.

Summary: In this Letter, we have presented a scheme for adding extra sets of links to a q-ary k-cube to enhance its fault-tolerance ability for an arbitrary positive integer q based on coding theory. This means that the proposed method generalises the techniques of the previous work in [4]. Although this method focuses on link faults, we can establish the most robust connections with proper redundancy.

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Efficent operator pipelining in a bit serial genetic algorithm engine

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Indexing terms: Systolic arrays, Genetic algorithms

The authors propose a bit serial pipeline used to perform the genetic operators in a hardware genetic algorithm. The bit-serial nature of the dataflow allows the operators to be pipelined, resulting in an architecture which is area efficient, easily scaled and is independent of the lengths of the chromosomes. An FPGA implementation of the device achieves a throughput of >25 million genes per second.