Performance analysis of ASK optical receivers in the presence of optical channel noise

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Abstract: The performance of an optical ASK receiver is analysed with the consideration of optical channel noise. Three noises are considered: the LO-channel noise, the signal-channel noise, and the shot noise. Their statistical properties and spectral distributions are investigated. Using correlation functions and a conditional probability scheme, we prove that the noises are independent Gaussian distributions. Through spectral analysis, the effect of optical filter bandwidth on the receiver performance is examined. The study reveals that the receiver performance degrades if a wide bandwidth optical filter is employed. In addition, there may exist an optimum amplifier gain to obtain a maximum signal-to-noise ratio when a high gain receiver preamplifier and a wide bandwidth optical filter are employed. The bit error probability is also evaluated. The results show that systems with little laser phase noise are subject to degradation by optical channel noise than those with larger phase noise.

1 Introduction

The progress of semiconductor laser and single-mode fibre technologies directs the optical communications research from intensity modulation systems toward sophisticated coherent systems [1]. The introduction of coherent schemes not only improves the receiver sensitivity but also enhances frequency selection capability which makes a densely multiplexed optical network possible [2]. Many theoretical efforts were devoted to the analysis of coherent receivers to find appropriate ways to combat various noises [3–6]. Usually the noises under consideration are the phase noises of the signal and local oscillator (LO), circuit noise, and photodetector (PD) shot noise, whereas the optical channel is assumed to be noise free.

As communication distance extends, optical amplifiers such as fibre Raman amplifiers [7, 8] or semiconductor laser amplifiers [9, 10] can be used as in-line amplifiers to optically amplify the signal. When optical amplifiers are used, spontaneous emission noise is introduced which comes along with the signal and reaches the receiver. On the other hand, the optical amplifier can be employed as a receiver preamplifier to improve performance in intensity modulated systems [11, 12]. For an ideal heterodyne detection coherent system there is little benefit in using optical preamplifiers such as a semiconductor laser amplifier under the condition that the LO power is high. However, the special feature of long range amplification of a backward fibre Raman amplifier can be adopted in preamplifier application which is expected to further extend the system transmission distance. In this case the spontaneous emission noise is also received by the coherent receiver. Under both conditions the received signal consists of not only phase noise but also optical channel noise. Because of the nonlinear optical-to-electrical conversion process, the optical channel noise may have a complicated effect on the receiver performance. The performance of an amplitude shift keying (ASK) coherent system employing an in-line semiconductor laser amplifier was analysed by Olsson [13], where a basic formulation similar to a direct detection system was presented for coherent systems. Here we analyse the performance of a heterodyne ASK optical receiver in the presence of optical channel noise from a rather different aspect. We concentrate on the statistics and the spectral behaviours of the various noises generated in the photodetection process and evaluate their impacts on system bit error probability.

2 Analysis

The system block diagram of the optical ASK receiver in a noisy optical channel is shown in Fig. 1. For simplicity and because the spontaneous noises are randomly generated with spectral distributions usually much wider than a coherent signal, we assume that the optical channel noise is white Gaussian. The optical filter at the input of the receiver is used to limit the noise spectrum. After passing through the optical filter, the signal and channel noise are incident on the photodetector (PD) to produce photocurrents. A bandpass filter is used to eliminate outside-of-band noise and an ASK demodulator is employed to recover the baseband signal.

2.1 Time domain analysis

A heterodyne detection optical receiver is polarisation-sensitive. Here we assume the signal and LO polarisations are well controlled to be the same and consider the part of channel noises along the same polarisation as the signal and LO. Let the optical filter be centred at the signal frequency with bandwidth BW_0 which is assumed to be sufficiently wide to pass the signal undistorted. The output of the optical filter can be written as

$$s(t) = b_k\sqrt{(2P_s)}\cos(w_s t + \phi_s(t)) + n_c \quad (1)$$

where P_s, w_s, and $\phi_s(t)$ are the power, angular frequency, and phase noise of the signal, n_c is the part of optically

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filtered channel noise with the same polarisation as the signal, and b_k takes the values of '1' or '0'. The spectrum of n_c is assumed to be uniformly distributed in the filter

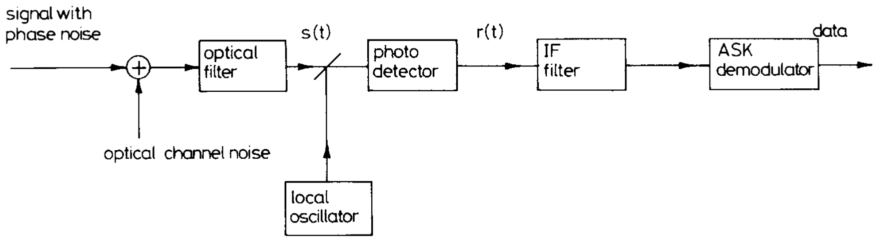

Fig. 1 *System block diagram of an optical ASK receiver in the presence of optical channel noise*

passband with two-sided spectral density $N_w/2$. We can expand n_c at the carrier frequency as [14]

$$n_c = n_i \cos w_s t + n_q \sin w_s t \tag{2}$$

where n_i and n_q are independent Gaussian processes with uniform spectral density N_w within baseband bandwidth BW_0. The LO output is expressed as

$$L(t) = \sqrt{(2P_L)} \cos (w_L t + \phi_L(t)) \tag{3}$$

where P_L, w_L, and ϕ_L are the power, angular frequency and phase noise of LO. To simplify the notation, the time dependence of phase noises is dropped if it is irrelevant. The PD output current can be written as

$$r(t) = R[s(t) + L(t)]^2 + n_{sh} \tag{4a}$$

where R is the responsivity of the PD and n_{sh} is the shot noise. By neglecting the optical frequency terms, which are irresponsive to the PD and the low frequency terms, $r(t)$ can be expressed as an intermediate frequency (IF) signal plus four noises, given by

$$r(t) = 2b_k R\sqrt{(P_s P_L)} \cos (w_{IF} t + \phi) + R\frac{n_i^2 + n_q^2}{2} + n_s + n_L + n_{sh} \tag{4b}$$

where

$$n_s = Rb_k\sqrt{(2P_s)}(n_i \cos \phi_s - n_q \sin \phi_s) \tag{5}$$

$$n_L = R\sqrt{(2P_L)}[n_i \cos (w_{IF} t + \phi_L) - n_q \sin (w_{IF} t + \phi_L)] \tag{6}$$

Here $\phi = \phi_s - \phi_L$ is the phase difference between the signal and LO and $w_{IF} = w_s - w_L$ is the intermediate angular frequency. For simplicity the circuit noise is neglected. The first term of eqn. 4b denotes the IF signal, whereas the second term is the self-product of channel noise which is a result of the incident channel noise power on the PD. The third results from the cross-product of the signal and channel noise, denoted as signal-channel noise n_s; the fourth term is the cross-product of the LO and channel noise, denoted as LO-channel noise n_L; and the last term is the PD shot noise n_{sh} which is a consequence of the quantum nature of the photodetection process. For a system with appropriate bit error probability, P_L and P_s should be much larger than n_i^2 and n_q^2. Thus we neglect the term $(n_i^2 + n_q^2)/2$ in the following discussion.

At first we investigate the statistical properties of n_L and n_s. The autocorrelation functions of n_L and n_s, denoted as $R_{nL}(\tau)$ and $R_{ns}(\tau)$, are given by (see Section 7)

$$R_{nL}(\tau) = 4R^2 P_L R_{ni}(\tau) R_\alpha(\tau) \tag{7}$$

$$R_{ns}(\tau) = 4R^2 P_s R_{bk}(\tau) R_{ni}(\tau) R_\beta(\tau) \tag{8}$$

where $R_{ni}(\tau)$, $R_\alpha(\tau)$, $R_{bk}(\tau)$, $R_\beta(\tau)$, respectively, represent the autocorrelation functions of n_i, $\cos (w_{IF} t + \phi_L)$, b_k, and $\cos \phi_s$. On the other hand the crosscorrelation function of n_L and n_s is formulated as [see Section 7]

$$R_{Ls}(\tau) = 0 \tag{9}$$

Hence n_L and n_s are uncorrelated.

Let $P(n_L)$ denote the probability density function of n_L. By definition we have

$$P(n_L) = \int_{-\infty}^{\infty} P(n_L \mid \phi_L) P(\phi_L)\, d\phi_L \tag{10}$$

where $P(n_L \mid \phi_L)$ denotes the conditional probability of n_L given ϕ_L. For a fixed ϕ_L, we can express n_L by an envelope function as

$$n_L = u \cos (w_{IF} t + \phi_L + \theta) \tag{11}$$

Because n_i and n_q are independent Gaussian random variables, thus u is the well known Rayleigh distribution with variance σ_L^2, given by

$$\sigma_L^2 = 2R^2 P_L \sigma_c^2 \tag{12}$$

where σ_c^2 is the variance of n_i and n_q which is equal to $N_w BW_o$. σ_c^2 is also the power of spontaneous noises which polarise along the signal polarisation. And θ is uniformly distributed within $[-\pi, \pi]$ [15]. After some mathematical manipulations, we can prove that the conditional probability is Gaussian distributed and, most importantly, independent of ϕ_L as [15]

$$P(n_L \mid \phi_L) = \frac{1}{\sqrt{(2\pi)}\sigma_L} \exp\left(-\frac{n_L^2}{2\sigma_L^2}\right) = P(n_L) \tag{13}$$

Thus we conclude that n_L is a zero mean Gaussian variable with variance σ_L^2. Similarly, based on a conditional probability scheme we can again prove that n_s is a zero mean Gaussian variable with variance σ_s^2 as

$$\sigma_s^2 = 2b_k R^2 P_s \sigma_c^2 \tag{14}$$

Since n_L and n_s are Gaussian distributed and uncorrelated, they are also mutually independent [14]. Because the PD shot noise n_{sh} depends on the incident optical power only, it is independent of ϕ_L and ϕ_s. Practically the channel noise power is much smaller than the signal and LO, n_{sh} is therefore independent of n_i and n_q. Thus n_{sh} is independent of both n_L and n_s. We thus treat the three noises n_L, n_s, and n_{sh} as mutually independent Gaussian noises.

2.2 Spectral analysis

We proceed to examine the power spectral densities (PSD) of the noises and use this information to further investigate their respective impact on the receiver performance. The PSDs of n_i and n_q, which are identical, are baseband distributed with bandwidth BW_0 and density

N_w, given by

$$S_{nq}(f) = S_{ni}(f) = N_w \quad -\frac{BW_0}{2} \leqslant f \leqslant \frac{BW_0}{2}$$
$$= 0 \quad \text{elsewhere} \tag{15}$$

The PSDs of $\cos(w_{IF}t + \phi_L)$ and $\sin(w_{IF}t + \phi_L)$ are given by [3]

$$S_\alpha(f) = \frac{1}{2\pi B_L}\left[\frac{B_L^2}{B_L^2 + 4(f+f_{IF})^2} + \frac{B_L^2}{B_L^2 + 4(f-f_{IF})^2}\right] \tag{16}$$

where B_L denotes the LO laser linewidth in terms of Hz and f_{IF} is the intermediate frequency. Eqn. 16 indicates that $S_\alpha(f)$ is composed of two Lorentzian distributions centered at $\pm f_{IF}$. From eqn. 7 we can obtain the PSD of n_L as

$$S_{nL}(f) = 4R^2P_L S_{ni}(f) * S_\alpha(f) \tag{17}$$

where an asterisk denotes convolution. Mathematically we expect that $S_{nL}(f)$ has its maximum at $\pm f_{IF}$ and spreads over BW_0, if $BW_0 \gg B_L$. From eqns. 15–17, $S_{nL}(f)$ is readily calculated as

$$S_{nL}(f) = \frac{R^2P_LN_w}{\pi}\left[\tan^{-1}\left(\frac{f+f_{IF}+BW_0/2}{B_L/2}\right) - \tan^{-1}\left(\frac{f+f_{IF}-BW_0/2}{B_L/2}\right) + \tan^{-1}\left(\frac{f-f_{IF}+BW_0/2}{B_L/2}\right) - \tan^{-1}\left(\frac{f-f_{IF}-BW_0/2}{B_L/2}\right)\right] \tag{18}$$

On the other hand, the PSDs of $\cos\phi_s$ and $\sin\phi_s$ are again the same, given by [3]

$$S_\beta(f) = \frac{1}{\pi B_s}\left[\frac{B_s^2}{B_s^2 + 4f^2}\right] \tag{19}$$

where B_s is the signal laser linewidth in terms of Hz. It is easy to see that $S_\beta(f)$ is Lorentzian distributed at baseband. Referring to eqn. 8 the PSD of n_s is readily obtained as

$$S_{ns}(f) = 4R^2P_sS_{bk}(f) * S_{ni}(f) * S_\beta(f) = \frac{2R^2P_sN_w}{\pi}S_{bk}(f) * \left[\tan^{-1}\left(\frac{f+BW_0/2}{B_s/2}\right) - \tan^{-1}\left(\frac{f-BW_0/2}{B_s/2}\right)\right] \tag{20}$$

where $S_{bk}(f)$ is the PSD of b_k, given as

$$S_{bk}(f) = \frac{1}{4}\delta(f) + \frac{T\sin^2(\pi fT)}{(2\pi fT)^2} \tag{21}$$

where $\delta(f)$ is the Delta function and T is the bit duration.

We assume that the PD shot noise is a white Gaussian process with PSD as

$$S_{nsh}(f) = eR(P_s + P_L) \quad -\infty < f < \infty \tag{22}$$

where e is the electron charge, equal to 1.6×10^{-19} Coulomb. Since P_L and P_s are assumed to be much larger than the channel noise power so that the shot noise owing to channel noise is neglected in eqn. 22. An illustrative example of the PSD of the three noises is shown in Fig. 2. In this and the following examples we consider an optical ASK system with bit rate $(Br) = 140$ Mbit/s, $P_L = 1$ mW, $B_L = B_s = 10$ MHz, and $R = 1$ A/W. The Figure shows that n_s is a baseband noise which depends

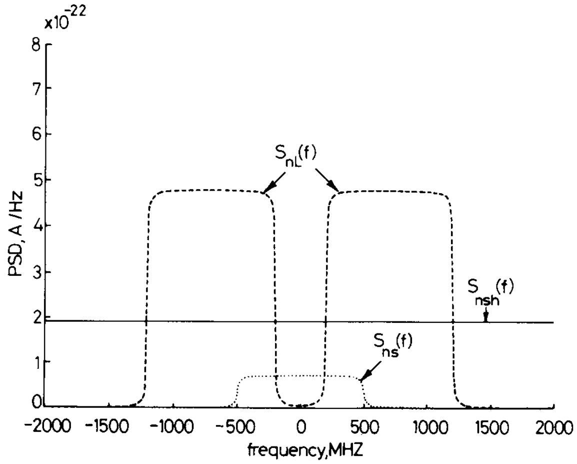

Fig. 2 *Illustrative example of PSD for the three noises*
$P_s = -7$ dBm; $BW_0 = 1$ GHz; $f_{IF} = 700$ MHz; $N_w = 3e$ W/Hz

on the channel noise and signal phase noise while n_L is a bandpass noise centered at $\pm f_{IF}$ and depends on the channel noise and LO phase noise. Since the signal locates at $\pm f_{IF}$, n_L is expected to be a critical noise to the system. However, depending on the bandwidth of the optical filter and the received signal power, n_s may also degrade the system performance.

We place a linear bandpass filter (BPF) after the PD to eliminate outside-of-band noise. It is well known that a linear transformation of Gaussian random variable yields Gaussian random variables [14]. Since the three noises are independent Gaussian random variables, they are expected to be independently Gaussian distributed at the BPF output. Thus we can calculate their variance at the BPF output through spectral domain. For simplicity we take the BPF as an ideal rectangular filter centered at $\pm f_{IF}$ with bandwidth BW_e and transfer function

$$H(f) = 1 \quad \pm f_{IF} - \frac{BW_e}{2} \leqslant f \leqslant \pm f_{IF} + \frac{BW_e}{2}$$
$$= 0 \quad \text{elsewhere} \tag{23}$$

After passing through the BPF, the noise powers of n_L, n_s, and n_{sh}, denoted as N_L, N_s, and N_{sh}, are obtained by integrating their PSD within the passband of BPF, given, respectively, as

$$N_L = \int_{f_{IF}-BW_e/2}^{f_{IF}+BW_e/2} S_{nL}(f)\,df + \int_{-f_{IF}-BW_e/2}^{-f_{IF}+BW_e/2} S_{nL}(f)\,df \tag{24}$$

$$N_s = \int_{f_{IF}-BW_e/2}^{f_{IF}+BW_e/2} S_{ns}(f)\,df + \int_{-f_{IF}-BW_e/2}^{-f_{IF}+BW_e/2} S_{ns}(f)\,df \tag{25}$$

$$N_{sh} = 2eR(P_s + P_L)BW_e \tag{26}$$

N_L, N_s, and N_{sh} not only denote the filtered noise power but also represent the variance of the three Gaussian noises which are critical factors to evaluate the bit error probability. Figs. 3 and 4 illustrate the dependence of N_L and N_s on the optical filter bandwidth. The rapid increase of N_L as BW_0 increases toward $4f_{IF}$ is a result of spectrum folding of $S_{nL}(f)$. This results in a factor of 2 increase in N_L. The same phenomenon occurs for N_s. However, the increase of N_s is caused by the extension of the baseband spectrum $S_{ns}(f)$ toward the BPF passband as BW_0 increases toward $2f_{IF}$. Also note that N_s is essentially zero for small BW_0 since there is no noise power within the BPF passband.

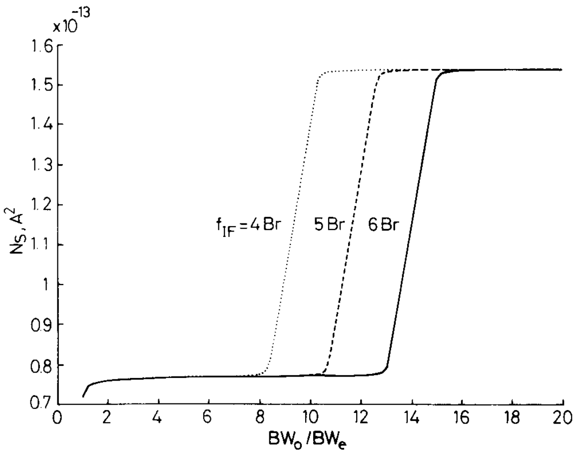

Fig. 3 *N_L as a function of BW_0/BW_e*
$BW_e = 240$ MHz; $N_w = e$ W/Hz

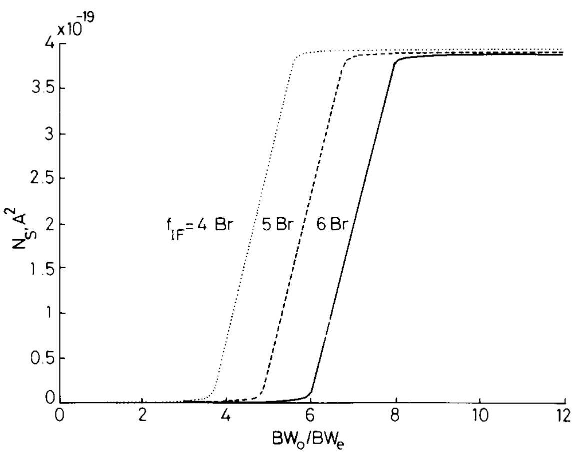

Fig. 4 *N_s as a function of BW_0/BW_e*
$BW_e = 240$ MHz; $N_w = e$ W/Hz; $P_s = -50$ dBm

3 Discussion

We assume that BW_e is wide enough to pass the signal undistorted in the presence of phase noise. Thus at the BPF output the problem can be formulated as a binary AM signal plus three independent Gaussian noises. With channel noise, the IF signal to noise ratio is written as

$$SNR = \frac{2R^2 P_s P_L}{N_L + N_s + N_{sh}} \tag{27}$$

Here we consider two cases. First, let the optical channel noise be induced by repeater optical amplifiers so that the received signal power P_s is much smaller than P_L. In this case it is clear that $N_L \gg N_s$. For $N_w = 0.1\ e$ (note that the unit of N_w is W/Hz, is different from that of e), because N_{sh} dominates the noise terms so that SNR is nearly independent of BW_0 as shown in Fig. 5. Thus the optical filter has little benefit to the system which can be omitted. For $N_w = 10\ e$, N_L dominates and SNR suffers about 3 dB decrease as BW_0 increases toward 4 IF because of band folding of $S_{nL}(f)$. In this case a narrow band optical filter is helpful to increase SNR.

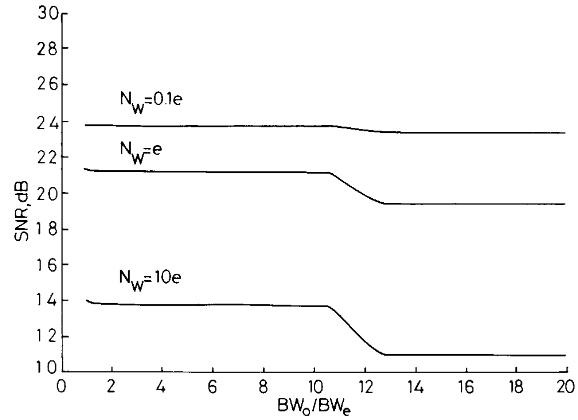

Fig. 5 *Relation between SNR and optical filter bandwidth for several values of N_w*
$BW_e = 240$ MHz; $f_{IF} = 700$ MHz; $P_s = -50$ dBm

Secondly, we are concerned with the case where the optical channel noise is introduced by an optical preamplifer. For example, a fibre Raman amplifier (FRA) can be used to extend transmission distance. Since the spontaneous emission noise induced by an FRA is a wideband noise and proportional to the amplifier gain in the linear gain region [16], we can model the output noise PSD of an FRA as an equivalent input noise PSD amplified by the FRA. Thus the output noise PSD and signal power are written as

$$N_w = GN_0 \tag{28}$$

$$P_s = GP_r \tag{29}$$

where N_0, P_r, and G express the equivalent input noise PSD, the input signal power, and the gain of the FRA, respectively.

The relation between SNR and BW_0 in the presence of an optical preamplifier is shown in Fig. 6. We observe

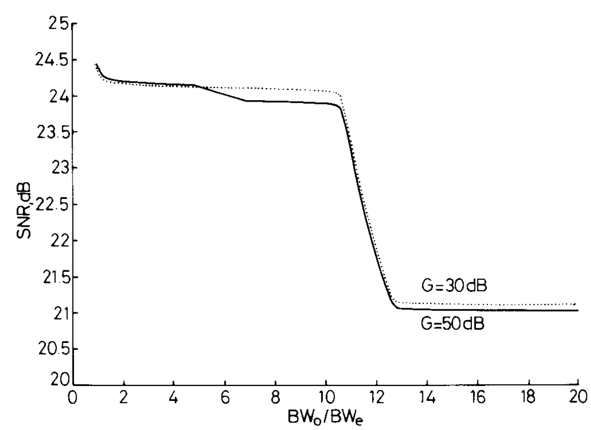

Fig. 6 *Relation between SNR and optical filter bandwidth for two preamplifier gains*
$BW_e = 240$ MHz; $N_0 = 0.1\ e$ W/Hz; $f_{IF} = 700$ MHz; $P_r = -60$ dBm

that a narrow band optical filter can eliminate N_s and prevent band folding of $S_{nL}(f)$ so as to increase SNR. The dependence of SNR on G is depicted in Fig. 7. For a given P_r and N_0. It is easy to see that the IF signal power and N_L are proportional to G because they are, respectively, proportional to P_s and N_w. On the other hand, since $S_{ns}(f)$ is proportional to both P_s and N_w, N_s is proportional to G^2 while N_{sh} is proportional to $P_L + GP_r$. If the optical filter bandwidth is narrow enough, for example a narrow band tunable optical filter is employed, n_s can be completely filtered by the BPF. In this case only N_L and N_{sh} are significant. When G is small, N_{sh} dominates because $e \gg N_w$ and $P_L \gg GP_r$ so that the noise power is nearly independent of G. As a result, SNR increases with G. When G is large enough, N_L dominates so that SNR increases little with G. On the other hand, if a wide bandwidth optical filter is used, the contribution of n_s should be taken into account. When G is small, SNR increases with G. Because of the G^2 dependence of N_s we can expect that N_s will dominate the noise contribution if G is large enough and SNR decreases as G increases. Consequently there exists an optimum amplifier gain to achieve maximum SNR as shown in Fig. 7. Because of the G^2 dependence of N_s, a narrow bandwidth optical filter should be used to eliminate this noise when the preamplifier gain is large.

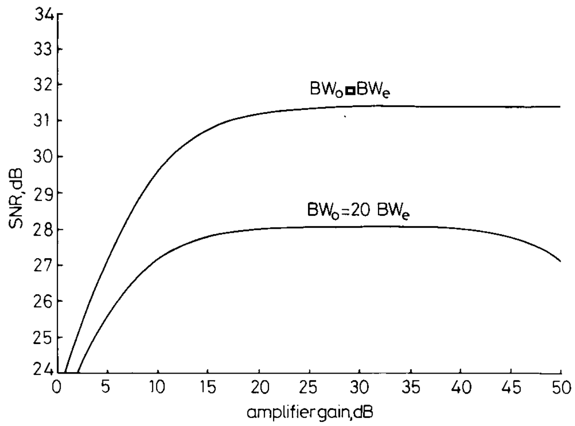

Fig. 7 *SNR as a function of preamplifier gain*
$BW_0 = 240$ MHz; $N_0 = 0.2\,e$ W/Hz; $f_{IF} = 700$ MHz; $P_r = -50$ dBm

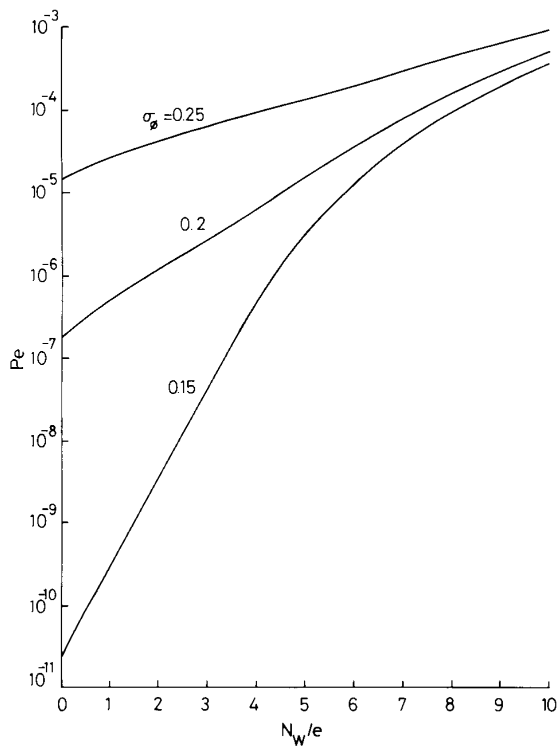

Fig. 8 *Degradation of P_e against channel noise PSD*
$BW_e = 240$ MHz; $P_s = -50$ dBm; $f_{IF} = 700$ MHz

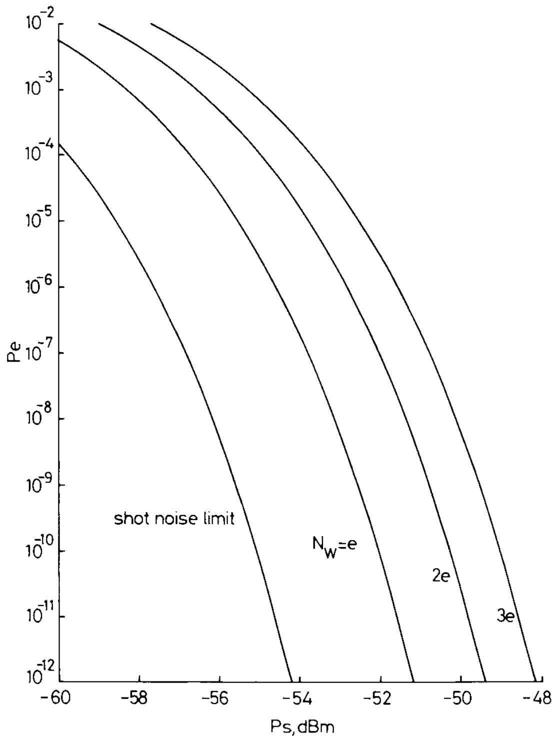

Fig. 9 *P_e as a function of P_s for $\sigma_\phi = 0$*
$BW_e = 240$ MHz; $P_s = -50$ dBm; $f_{IF} = 700$ MHz

We use a synchronous ASK demodulator after the IF filter to recover the baseband data. Here we consider an ideal case where the IF carrier can be tracked from the signal and used to reproduce the baseband waveform. The bit error probability P_e of such a synchronous detector had been circumstantially derived in Reference 5 which can be expressed as a function of IF SNR. For simplicity we set the threshold at the middle of '0' and '1', thus P_e can be written as

$$P_e = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{g(x)} e^{-x^2}e^{-y^2}\,dy\,dx + \frac{1}{2\sqrt{(\pi)}}\int_{0.5\sqrt{(SNR)}}^{\infty} e^{-x^2}\,dx \tag{30}$$

where

$$g(x) = \sqrt{(SNR)}[0.5 - \cos(\sqrt{(2)}\sigma_\phi x)] \tag{31}$$

and σ_ϕ^2 is the variance of ϕ. Fig. 8 illustrates the bit error probability with respect to N_w. It is seen that the bit error rate degrades seriously when N_w is large, particularly for small σ_ϕ. We can use a narrower bandwidth optical filter to reduce the impact of the channel noise but cannot completely eliminate it since the LO-channel noise possesses the same spectrum as the IF signal.

The degradation of P_e from shot noise limited curve owing to channel noise is presented in Fig. 9. We see that there is about 3 dB degradation from the shot noise limit for $N_w = e$. Thus the parameter e is a good parameter to estimate the degradation of P_e in the presence of optical channel noise.

4 Conclusion

We have theoretically analysed the performance of an optical ASK receiver in the presence of optical channel noise. The channel noise can come from repeater optical amplifiers or receiver preamplifiers. Two new noises, characterised as signal-channel noise and LO-channel noise, which result from the multiplication of channel noise with the signal and local oscillator lasers in the optical-to-electrical conversion process, are introduced. The statistical studies show that these noises are independent and Gaussian distributed, and are independent of the shot noise. Through spectral analysis we have made clear the effect of optical filter bandwidth on the system performance. The results show that a narrow band optical filter is necessary to suppress channel noise if strong channel noise is present, while it is less useful if the channel noise is weak since shot noise dominates. We also find that there exists an optimum preamplifier gain which results in a maximum signal-to-noise ratio if wide bandwidth optical filter is used. The examples reveal that strong channel noise can seriously degrade the system performance. However, it can be partly relieved by using a narrow bandwidth optical filter.

5 Acknowledgment

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7 Appendix

Let $R_{nL}(\tau)$ denote the autocorrelation function of n_L. By definition we have

$$\begin{aligned} R_{nL}(\tau) &= E[n_L(t)n_L(t+\tau)] \\ &= 2R^2 P_L\, E[n_i(t)n_i(t+\tau)\cos(w_{IF}t + \phi_L(t)) \\ &\quad \times \cos(w_{IF}(t+\tau) + \phi_L(t+\tau)) \\ &\quad + n_q(t)n_q(t+\tau)\sin(w_{IF}t + \phi_L(t)) \\ &\quad \times \sin(w_{IF}(t+\tau) + \phi_L(t+\tau)) \\ &\quad - n_i(t)n_q(t+\tau)\cos(w_{IF}t + \phi_L(t)) \\ &\quad \times \sin(w_{IF}(t+\tau) + \phi_L(t+\tau)) \\ &\quad - n_q(t)n_i(t+\tau)\sin(w_{IF}t + \phi_L(t)) \\ &\quad \times \cos(w_{IF}(t+\tau) + \phi_L(t+\tau))] \end{aligned} \tag{32}$$

where 'E' denotes mathematical expectation. Because n_i, n_q, and ϕ_L are independent and zero mean random variables, the last two terms of eqn. 32 are diminished. Thus

$$\begin{aligned} R_{nL}(\tau) &= 2R^2 P_L \\ &\quad \times \{E[n_i(t)n_i(t+\tau)]E[\cos(w_{IF}t + \phi_L(t)) \\ &\quad \times \cos(w_{IF}(t+\tau) + \phi_L(t+\tau))] \\ &\quad + E[n_q(t)n_q(t+\tau)]E[\sin(w_{IF}t + \phi_L(t)) \\ &\quad \times \sin(w_{IF}(t+\tau) + \phi_L(t+\tau))]\} \\ &= 2R^2 P_L[R_{ni}(\tau)R_{cos}(\tau) + R_{nq}(\tau)R_{sin}(\tau)] \end{aligned} \tag{32}$$

where $R_{ni}(\tau)$, $R_{nq}(\tau)$, $R_{cos}(\tau)$, and $R_{sin}(\tau)$, respectively, represent the auto-correlation functions of n_i, n_q, $\cos(w_{IF}t + \phi_L)$, and $\sin(w_{IF}t + \phi_L)$. It is easy to prove that $R_{ni}(\tau) = R_{nq}(\tau)$ [14] and $R_{cos}(\tau) = R_{sin}(\tau)$ [3], thus we can further simplify eqn. 32 as

$$R_{nL}(\tau) = 4R^2 P_L R_{ni}(\tau)R_\alpha(\tau) \tag{34}$$

where for simplicity we use $R_\alpha(\tau)$ to represent $R_{cos}(\tau)$ and $R_{sin}(\tau)$. In a similar manner we can obtain the autocorrelation function of n_s, $R_{ns}(\tau)$, as

$$R_{ns}(\tau) = 4R^2 P_s R_{bk}(\tau)R_{ni}(\tau)R_\beta(\tau) \tag{35}$$

where $R_{bk}(\tau)$ is the auto-correlation function of b_k where $R_\beta(\tau)$ is that the cos ϕ_s and sin ϕ_s.

We next examine the relation between n_L and n_s. The crosscorrelation function of n_L and n_s is formulated as

$$\begin{aligned} R_{Ls}(\tau) &= E[n_L(t), n_s(t+\tau)] \\ &= 2R^2 b_k \sqrt{(P_s P_L)} \\ &\quad \times E[n_i(t) n_i(t+\tau) \cos(w_{IF} t + \phi_L(t)) \\ &\quad \times \cos \phi_s(t+\tau) + n_q(t) n_q(t+\tau) \\ &\quad \times \sin(w_{IF} t + \phi_L(t)) \sin \phi_s(t+\tau) \\ &\quad - n_i(t) n_q(t+\tau) \cos(w_{IF} t + \phi_L(t)) \\ &\quad \times \sin \phi_s(t+\tau) - n_q(t) n_i(t+\tau) \\ &\qquad \times \sin(w_{IF} t + \phi_L(t)) \cos \phi_s(t+\tau)] \end{aligned} \quad (36)$$

In a heterodyne coherent system, n_i, n_q, ϕ_L and ϕ_s are mutually independent. Using the zero mean property of n_i and n_q, the last two terms diminished and eqn. 36 can be rewritten as

$$\begin{aligned} R_{Ls}(\tau) &= 2R^2 b_k \sqrt{(P_s P_L)} \\ &\quad \times \{E[n_i(t) n_i(t+\tau)] E[\cos(w_{IF} t + \phi_L(t))] \\ &\quad \times E[\cos \phi_s(t+\tau)] + E[n_q(t) n_q(t+\tau)] \\ &\quad \times E[\sin(w_{IF} t + \phi_L(t))] E[\sin \phi_s(t+\tau)]\} \end{aligned} \quad (37)$$

Since ϕ_L and ϕ_s are random Brownian motions [3], the expectation values of cos $\phi_s(t+\tau)$ and sin $\phi_s(t+\tau)$ are zero. Thus we have

$$R_{Ls}(\tau) = 0 \quad (38)$$