CALCULATION OF RADIATION FROM DIELECTRIC WAVEGUIDE STEP DISCONTINUITIES USING A PEC APPROXIMATION

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I. Introduction

Discontinuity problems in dielectric waveguides are important in designing components such as gratings or corrugations, dielectric rod antennas, etc., in millimeter-wave, submillimeter-wave and optical integrated circuits. There are two classes of waveguides: the closed-type and the open-type. Discontinuities in the closed-type waveguide have been studied for many years [1]–[3]. However, those in the open-type waveguide were only solved by making various approximations, e.g., in [4], due to the complexity of the continuous spectrum of its radiation modes. Until recently, they are treated more rigorously [5]–[7].

It is customary to discretize the continuum in the open-type waveguides by introducing the Laguerre transform [5]. Rozzi [6] employed the Laguerre transform in a Ritz-Galerkin variational solution, while Chung [7] did it in his partial variational principles (PVP). These rigorous approaches can give more complete information about the discontinuities of the planar dielectric waveguides, e.g., transmitted (reflected) guided power, radiated power, radiation patterns, etc. But, owing to the continuum of radiation modes, their approaches need complicated formulations and calculations.

Almost all the approximate methods for the open-type waveguides neglect some terms in the formulation, e.g., the reflected radiated fields. Brooke and Kharadly [8] proposed a different approach. They used a bounded system to approximate the original unbounded problem by placing two perfect conductors distant from the dielectric slab as shown in Fig. 1. After placing the perfect conductors, the open-type system becomes a closed-type one and the continuous spectrum is replaced by a discrete set of modes. For the new closed-type system, we can use mode-matching method straight to solve the discontinuity problems. It is often to take the perfect conductors as perfect electric conductors (PEC's). We thus call this approach the PEC approximation.

Until now, all the approximate methods have been used to calculate powers only. In this paper we use the PEC approximation to calculate the radiation pattern of the step discontinuity in a planar dielectric waveguide. In the following section we give the formulation. Then, we present the radiation patterns for some transverse-electric (TE) cases in Section III and compare our results with those obtained from the rigorous approaches given by Rozzi and Chung. Finally, some conclusions in this study are made in Section IV.

II. The PEC Approximation

In the open-type dielectric waveguides, there are three classes of modes: guided, propagating radiation and evanescent radiation modes. They correspond respectively to slow, fast and evanescent modes in the closed-type waveguides (see Table 1 in [8]). In the PEC approximation, we use a closed-type system to approximate the open-type one. But for the sake of convenience, we will use the terms in the open-type waveguides to indicate the modes in the new closed-type system after taking the PEC approximation.

It is natural and necessary not to affect the guided modes when we adopt the PEC approximation. But the continuous spectrum of the radiation modes is now replaced

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by a discrete set of modes. The effect of the PEC's on the radiation modes is that of sampling. The sampled (radiation) modes are those with zero tangential E-field (E_y and E_z in Fig. 1) at the locations of the PEC's. If we change the locations of the PEC's, different set of modes are sampled. These sampled modes must be renormalized in the closed-type system.

The standard approach for the discontinuity problems is to use local normal mode expansion and match the boundary conditions at the interface. Because the mode spectrum now is discrete, we can solve it directly. We use $e_n^{(i)}$ and $h_n^{(i)}$ to represent the transverse electric and magnetic field components of the nth mode in the ith section of the waveguide system. Suppose that the incident wave is the fundamental mode in section 1 with $e_1^{(1)}$, the continuity of these transverse fields at the interface is expressed as

$$E_t = e_1^{(1)} + \sum_{n=1}^{\infty} a_n e_n^{(1)} = \sum_{n=1}^{\infty} b_n e_n^{(2)} \tag{1}$$

$$H_t = h_1^{(1)} - \sum_{n=1}^{\infty} a_n h_n^{(1)} = \sum_{n=1}^{\infty} b_n h_n^{(2)} \tag{2}$$

Using the orthogonality condition, we obtain the following equations for the coefficients a_n and b_n:

$$[R][b] = (1, 0, 0, \cdots)^T \tag{3}$$

$$[S][b] = [a] \tag{4}$$

where $[a] = (a_1, a_2, \cdots)^T$, $[b] = (b_1, b_2, \cdots)^T$, and the elements of $[R]$ and $[S]$ are

$$R_{ij} = \frac{\beta_i^{(1)^*}}{4\beta_i^{(1)}} \int_{-\infty}^{\infty} (e_j^{(2)} \times h_i^{(1)^*} - e_i^{(1)^*} \times h_j^{(2)}) \cdot \hat{z} dx$$

$$S_{ij} = \frac{\beta_i^{(1)^*}}{4\beta_i^{(1)}} \int_{-\infty}^{\infty} (e_j^{(2)} \times h_i^{(1)^*} + e_i^{(1)^*} \times h_j^{(2)}) \cdot \hat{z} dx$$

with $\beta_i^{(1)}$ being the propagation constant corresponding to the ith mode in section 1. We solve a_n and b_n from (3) and (4) and the radiation properties of the discontinuity can be obtained as described in the following section.

III. The Radiation Patterns

The radiation patterns are important in many applications such as in semiconductor laser devices and dielectric rod antennas. So far as we know, the only way to obtain the radiation patterns is to deal with the continuum of radiation modes rigorously like in [6] and [7].

In the rigorous form, the continuity conditions at the interface are

$$e_1^{(1)} + \sum_{n=1}^{N_{g1}} a_n e_n^{(1)} + \int_0^{\infty} c(\rho) e^{(1)}(\rho) d\rho = \sum_{n=1}^{N_{g2}} b_n e_n^{(2)} + \int_0^{\infty} d(\rho) e^{(2)}(\rho) d\rho \tag{5}$$

$$h_1^{(1)} - \sum_{n=1}^{N_{g1}} a_n h_n^{(1)} - \int_0^{\infty} c(\rho) h^{(1)}(\rho) d\rho = \sum_{n=1}^{N_{g2}} b_n h_n^{(2)} + \int_0^{\infty} d(\rho) h^{(2)}(\rho) d\rho \tag{6}$$

where N_{g1} and N_{g2} are the numbers of guided modes in sections 1 and 2, respectively, and ρ is the transverse wavenumber. The total transmitted and reflected radiated powers are $\int_0^{k_0} | d(\rho) |^2 d\rho$ and $\int_0^{k_0} | c(\rho) |^2 d\rho$, respectively, where k_0 is the wavenumber in free space. Because $\rho = k_0 \sin\theta$ (θ is shown in Fig. 1), $d\rho = k_0 \cos\theta d\theta$. We can use θ as the integration variable in the expression of the radiated power, and the integrand, $P(\theta)$, is the radiation pattern. That is,

$$P(\theta) = \begin{cases} |d(\rho)|^2 k_0 \cos\theta & \text{for} \quad 0^\circ < \theta < 90^\circ \\ |c(\rho)|^2 k_0 \cos\theta & \text{for} \quad 90^\circ < \theta < 180^\circ. \end{cases} \tag{7}$$

The coefficients a_n ($n > N_{g1}$) and b_n ($n > N_{g2}$) in (1) and (2) play similar roles as do $c(\rho)$ and $d(\rho)$ in (5) and (6), respectively. Because the PEC approximation has the sampling effect, we expect $a_n = Cc(\rho_n)$ and $b_m = Cd(\rho_m)$ if their corresponding propagation constants are the same (i.e., $\beta_n^{(1)} = \beta^{(1)}(\rho_n)$ and $\beta_m^{(2)} = \beta^{(2)}(\rho_m)$), where C is a constant. Equation (7) tells that we can calculate the radiation patterns from the coefficients of the propagating radiation modes (with $0 < \rho_n < k_0$) in the the PEC approximation. Certainly, we can only obtain the radiation pattern at the sampled angles, θ_n, this is,

$$P(\theta_n) = \begin{cases} |b_n|^2 k_0 \cos\theta_n & \text{for} \quad 0^\circ < \theta_n < 90^\circ \\ |a_n|^2 k_0 \cos\theta_n & \text{for} \quad 90^\circ < \theta_n < 180^\circ. \end{cases}$$

Fig. 2(a) shows a step discontinuty in a planar dielectric waveguide. We take the value of $k_0 d$ to be 1 and the refractive index of the slab, n, to be $\sqrt{5}$. The radiation patterns calculated by Rozzi based on a rigorous variational approach [6] and us are given in Fig. 2(b). Excellent agreement has been obtained. Fig. 3(a) shows an abruptly ended dielectric waveguide. The abruptly ended dielectric waveguides can serve as dielectric rod antennas. The values of $k_0 d$ and n are taken to be 0.8 and 2.236, respectively. The length of the second section, ℓ, is chosen as 0 and $1.7d$ in the calculation. Fig. 3(b) shows our results and those of [7] based on a rigorous partial variational formulation. Again, the agreement is good. The comparisons given in Figs. 2 and 3 demonstrate the usefulness of the PEC approximation in calculating the radiation patterns.

IV. Conclusions

In conclusion, we have demonstrated that the PEC approximation can be used to calculate the radiation patterns of the step discontinuities in planar dielectric waveguides. Although the results are given at discrete angles, interpolation can be used to obtain the entire pattern. Moreover, the larger the distance between the two PEC's we place, the more modes we sample and the more points of the radiation pattern we obtain. The PEC approximation enables us to use a simple model to solve the discontinuity problems in the open-type waveguides. It does not require complicated computations, but can provide accurate information about the dielectric waveguide discontinuities.

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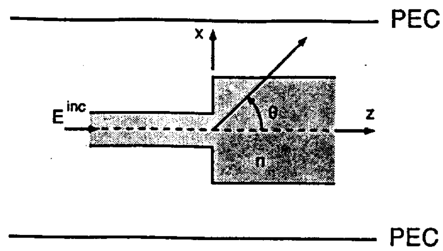

Fig. 1 The system of the step discontinuity in a planar dielectric waveguide under the PEC approximation.

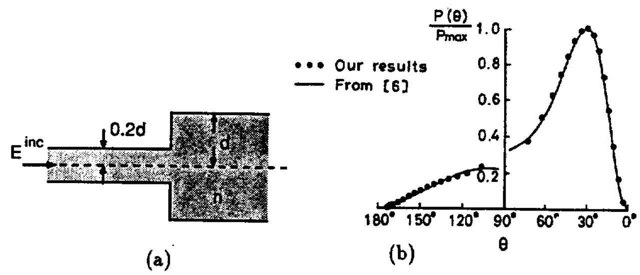

Fig. 2 Radiation from a step discontinuity in a planar dielectric waveguide: (a) sketch of the structure, (b) the calculated radiation pattern.

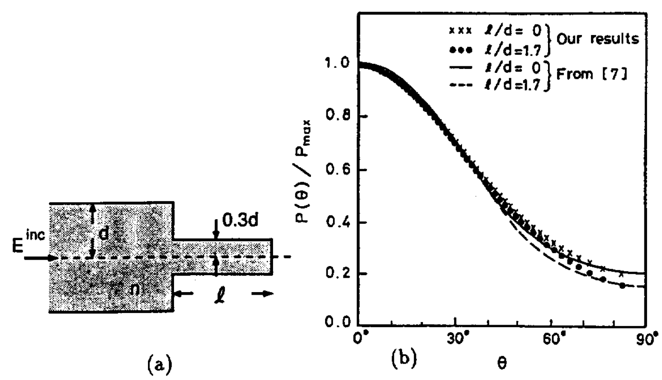

Fig. 3 Radiation form an abruptly ended dielectric waveguide: (a) sketch of the structure, (b) the calculated radiation pattern.