The Effect of Channel Errors on a Contention–based Reservation TDMA Protocol

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Abstract

A contention–based TDMA protocol was previously proposed for data transmissions. It was proved to have satisfactory performance for an error free channel. To maintain operational in erroneous channel a new modified protocol is proposed and analyzed. Results derived include the average frame length, the average number of wasted slots, the average number of residual packets, and the average delay. Numerical examples shown also include the effect of channel errors and the margin regained by proper error control codes.

I.Introduction

Numerous researches on satellite communication protocol are focused on normal operations in channel without errors. Contention–based time division multiple access protocols have good performance in application such as very small aperture terminal (VSAT) [1], [3] from medium to heavy traffic when the channel is error free. For the protocol to operate in a channel which may contain errors, a modified protocol is proposed as follows. Let M be the number of users, F a positive integer such that (F–1) slots < round trip propogation delay< F slots, and $N=\lceil M/F \rceil$ the smallest integer greater than or equal to M/F. The frame structure shown in Fig. 1 consists of three parts:

a)Reservation Subframe: It is used to reserve the number of slots in the variable subframe. Each user has one m–bit vector to reserve the number of slots to be used in the variable subframe

b)Fixed Subframe: Users of the same group share one fixed slot. If more than one user in the same group transmit in the shared fixed slot , a collision results. The length of the fixed subframe should not be shorter than the round–trip signal propogation delay, this is to guarantee that the slot assignment for the variable subframe can be done properly.

c)Variable Subframe: If a user transmits his data packet in the fixed subframe and encounters a collision or his transmission in the fixed subframe succeeds but he still has more data packets waiting for transmission, he will have to use the variable subframe. The number of slots occupied by this user of course is determined by the content of his request issued in his reservation vector. Also shown in Fig. 1 is an example to demonstrate the observations of the protocol. Assume specifically M=23, F=3, N=8, m=3 and in the enlarged reservation subframe j ($1 \leq j \leq 3$) is used to denote group j. Suppose at the beginning of the current frame, $U_{1,1}$, $U_{4,1}$ and $U_{2,3}$ has one, three, and two packets waiting for transmission respectively. From Fig. 1, we observe $U_{1,1}$, $U_{4,1}$, and $U_{2,3}$ in their reservation vector set (001), (011), and (010) respectively. Also we see that $U_{1,1}$ and $U_{4,1}$ collide in the first slot while $U_{2,3}$ has a successful transmission in the third slot of the fixed subframe and the second slot is empty because there are no packets from group 2. In the variable subframe $U_{1,1}$ and $U_{4,1}$ consume one and three slots due to their collision in the fixed subframe. However, $U_{2,3}$ needs only one more slot in the variable subframe.

In principle, slot assignment should be determined using both the information carried in the reservation subframe and the condition (empty, success, or collision) detected from the corresponding slot in the fixed subframe. If channel is noiseless the condition in the fixed subframe can be determined unambiguously from the contents in the reservation subframe. However, slot assignment is definitely disrupted by channel errors. First, the condition in the fixed subframe can no longer be deduced alone from reservation subframe unambiguously. Second, the number of slots assigned to any particular user may not be the same as it originally requested. How does channel error affect the performance is the subject to be studied in this paper. But when channel error occurs, we have to first modify the protocol so that slot assignment can still be done jointly using the information carried in the reservation subframe and the condition in the fixed subframe. The frame structure of the modified protocol is sketched in Fig. 2. Compare with Fig. 1 we observe that the four arrowed reservation vectors are in error. Since the status (collision, success, or empty) in the first slot of the fixed subframe is not yet available when the fixed subframe ends, we have to first invest a time slot, denoted by V_1 in Fig. 2, in the variable subframe. In other words, the variable subframe now has at least one slot. In order to make better use of the channel resource, V_1 is used as follows. Let F_i denote the ith slot in the fixed subframe, we always let $U_{1,1}$ use V_1 and the other users in group 1 use F_1 (Notice that in Fig. 2, $U_{1,1}$ transmits in V_1 and $U_{4,1}$ transmits in F_1.) Therefore if $U_{1,1}$ indeed transmits in V_1 its success is guaranteed. Additional slots in the variable subframe assigned to $U_{1,1}$ are determined from the observed $R_{1,1,k}$, the reservation

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vector of $U_{1,1}$. In Fig. 2, $U_{1,1}$ only uses V_1 in the variable subframe since the observed $R_{1,1,k}$ is one. The status in F_1 belongs to one of the following.

1.1) F_1 is observed to be empty. In this case, it can be concluded that each of $U_{2,1},\dots, U_{N,1}$ has no packets to transmit and requests no slots in the variable subframe no matter $R_{i,1,k}$ is in fact correct or not.

1.2) F_1 is observed to be successful. In this case we assume that the identity of the user who actually transmits, say $U_{i,1}$, can be determined from the observed contents of the packet. Then only the observed $R_{i,1,k}$ is used for slot assignment. In Fig. 2 F_1 is used by $U_{4,1}$ although the observed $R_{4,1,k}=0$.

1.3) F_1 is observed to contain a collision. In this case the observed $R_{i,1,k}$'s are used for slot assignment in the variable subframe.

As for group $j\geq 2$ if the status of F_j is available when all the preceeding groups have finished their use of the variable subframe then no extra slot has to be appended and slot assignment for group j can be explained similarly.

In Fig. 2, V_2 is an appended slot but wasted since $U_{1,2}$ is inactive and the status of F_2 is empty therefore the reservation vectors $R_{1,2,k}$ and $R_{3,2,k}$ are ignored. The success status of F_3 implies that the only active user in group 3 is $U_{2,3}$ and of course $R_{4,3,k}$ is again ignored.

II. Analysis

Assumptions

1) The only effect of the channel error is on the content of the reservation vector. The data slots are not so critical as the reservation vector and can also be corrected by the upper layer protocols [2].

2) The condition of a slot in the fixed subframe is empty, success, or collision and is always correctly received.

3) All the network users observe identical channel conditions.

4) The input to each terminal is assumed to be a Bernoulli process. The input process to different terminals are assumed to be independent.

5) Each user has an infinite buffer.

II.1 Transient–State Analysis

$Q^b_{i,j,k}$: queue length of $U_{i,j}$ at the beginning of frame k

$Q^e_{i,j,k}$: queue length of $U_{i,j}$ at the end of frame k

$A_{i,j,k}$: number of packets arrive at $U_{i,j}$ in frame k

$RP_{i,j,k}$: number of residual packets of $U_{i,j}$ in frame k

$L_{i,j,k}$: number of slots used by $U_{i,j}$

m: number of bits in a reservation vector

$d=2^m-1$: maximum slots reserved per user

slotb: number of bits per slot

L_k: length of frame k

$$=\text{length of reservation subframe}+\sum_{j=1}^{F}\sum_{i=1}^{N_j} L_{i,j,k}+F$$

$$=m*M/\text{slotb}+\sum_{j=1}^{F}\sum_{i=1}^{N_j} L_{i,j,k}+F$$

$$\approx \sum_{j=1}^{F}\sum_{i=1}^{N_j} L_{i,j,k}+F$$

$$c_d(x)=\begin{cases} d, \text{if } x\geq d \\ x, \text{if } -d<x<d \\ -d, \text{if } x\leq -d \end{cases}$$

p_e: the probability that a bit is in error and all bit errors are independent

$e(x\to y)$: the probability of changing x to y under bit error probability p_e, where $-d\leq x,y\leq d$.

Physically $e(x\to y)$ represents the probability of changing an issued request to the observed request, e.g. $e(x\to y)=p_e^{d(x,y)}(1-p_e)^{m-d(x,y)}$, in particular $e(x\to x)=(1-p_e)^m$.

$L_{i,j,k}$

Due to our modification in Fig. 2, we must divide users into three categories and treat them differently. They are $\{U_{1,1}\}$, $\{U_{1,j}, j\geq 2\}$, and $\{U_{i,j}, i\geq 2\}$. For $U_{1,1}$ we construct the following recursive relations.

$$(1)P(L_{1,1,k}=\ell)=\begin{cases} 0, \ell=0 \\ \sum_{q=0}^{\infty} P(Q^b_{i,j,k}=q) \\ [e(c_d(q)\to 0)+e(c_d(q)\to 1)], \ell=1 \\ \sum_{q=0}^{\infty} P(Q^b_{1,1,k}=q)e(c_d(q)\to\ell), d\geq\ell\geq 2 \end{cases}$$

Now consider $L_{1,j,k}$ for $j\geq 2$. Let A_j be the event that an extra slot has to be appended for group j and will be used by $U_{1,j}$. Fig. 3 sketches the possible events which lead to $L_{1,j,k}=1$ under A_j. Traverse from the root to any of the leaves represents one of such events. The corresponding probability can be obtained for $P(L_{1,j,k}=1\mid A_j)$. Through similar procedures we can obtain the probability for $P(L_{1,j,k}=1\mid \overline{A}_j)$.

Finally we have

$$(2)P(L_{1,j,k}=\ell)=P(A_j)P(L_{1,j,k}=\ell\mid A_j)+P(\overline{A}_j)P(L_{1,j,k}=\ell\mid\overline{A}_j)$$

where $P(A_j)$ and $P(\overline{A}_j)$ will be obtained later. The detailed derivations of $P(L_{1,j,k}=\ell\mid A_j)$ and $P(L_{1,i,k}=\ell\mid\overline{A}_j)$ will be shown in [4].

As for $P(L_{i,j,k}=\ell)$ with $i\neq 1$ we proceed as follows. Define $U_j=\{U_{1,j},\ldots,U_{N_j,j}\}$, the set of users in group j. Then

$$(3)\ P(L_{i,j,k}=0)=P(N_a(U_j-\{U_{i,j}\})>1)P(Q^b_{i,j,k}=0)e(0\to 0)+P(N_a(U_j)=0)+P(N_a(U_j-\{U_{i,j}\})=1)P(Q^b_{i,j,k}=0)+P(N_a(U_j-\{U_{i,j}\})>0)*\sum_{q=1}^{\infty}P(Q^b_{i,j,k}=q)e(c_d(q)\to 0)$$

$$(4)\ P(L_{i,j,k}=\ell)=P(N_a(U_j-\{U_{i,j}\})\geq 2)P(Q^b_{i,j,k}=0)e(0\to\ell)+\sum_{q=1}^{\infty}P(Q^b_{i,j,k}=q)e(c_d(q)\to\ell),\ \ell\geq 1$$

where

$$P(N_a(U_j-\{U_{i,j}\})>0)=1-\prod_{\substack{s=1\\ s\neq i}}^{N_j}P(Q^b_{s,j,k}=0)$$

$$P(N_a(\{U_j-\{U_{i,j}\})>1)=1-\prod_{\substack{s=2\\ s\neq i}}^{N_j}P(Q^b_{s,j,k}=0)-\sum_{\substack{s=2\\ s\neq i}}^{N_j}P(Q^b_{s,j,k}>0)$$

In numerical calculations we start from k=1 and assume $Q^b_{i,j,1}=0$ for all i and j. From this set of initial conditions we can proceed to compute the distributions of $L_{i,j,1}$ and $Q^b_{i,j,2}$ and then $L_{i,j,2}$ and $Q^b_{i,j,3}$, so on and so forth until they converge.

<u>$WF_{j,k}$</u>

We use $WF_{j,k}$ to denote the number of slots wasted by group j in the fixed subframe. The waste is due to either collision or no one uses it. We have

$$(5)\ P(WF_{j,k}=1)=\begin{cases}P(N_a(U_1-\{U_{1,1}\})=0)+P(N_a(U_1-\{U_{1,1}\})\geq 2), & j=1\\ P(N_a(U_j)=0)+P(N_a(U_j)\geq 2), & j>1\end{cases}$$

Now $P(A_j)$ can be obtained as follows.

$$(6)\ P(A_j)=P\Big(\sum_{s=1}^{j-1}\sum_{r=1}^{N_s}L_{r,s,k}-(j-1-\sum_{s=1}^{j-1}WF_{s,k})=j-1\Big)$$

$$(7)\ P(\overline{A}_j)=1-P(A_j)$$

<u>$WV_{i,j,k}$</u>

$WV_{i,j,k}$ denotes the number of slots wasted by $U_{i,j}$ in the variable subframe. The waste of course is due to overreservation of slots in the variable subframe. We have

for i=1, j=1

$$(8)\ P(WV_{1,1,k}=w)=\begin{cases}\sum_{q=1}^{d-1}P(Q^b_{1,1,k}=q)e(q\to q+1)+P(Q^b_{1,1,k}=0)[e(0\to 0)+e(0\to 1)], & w=1\\ \sum_{q=1}^{d-w}P(Q^b_{1,1,k}=q)e(q\to q+w)+P(Q^b_{1,1,k}=0)e(0\to w), & w>1\end{cases}$$

Now for i=1, j>1, through similar reasonings we have

$$(9)\ P(WV_{1,j,k}=w)=P(A_j)P(WV_{1,j,k}=w\mid A_j)+P(\overline{A}_j)P(WV_{1,j,k}=w\mid\overline{A}_j)$$

In general for $i\neq 1$, j>1 we have

$$(10)\ P(WV_{i,j,k}=w)=\sum_{q=1}^{d-w}P(Q^b_{i,j,k}=q)e(q\to q+w)+P(N_a(U_j-\{U_{i,j}\})\geq 2)P(Q^b_{i,j,k}=0)*e(0\to w),\ w\geq 1$$

<u>L_k</u>

Let L_k be the length of frame k expressed in number of slots. Ignoring the length of reservation subframe we have

$$(11)\ L_k(z)=\Big[\sum_{j=1}^{F}\sum_{i=1}^{N_j}L_{i,j,k}(z)\Big]\Big[\sum_{j=1}^{F}WF_{j,k}(z)\Big]$$

<u>$A_{i,j,k}$</u>

We use $A_{i,j,k}$ to denote the number of packets arriving at the queue of $U_{i,j}$ in frame k. Then

$$(12)\ A_{i,j,k}(z)=L_k(1-\sigma_{i,j}+\sigma_{i,j}z)$$

<u>$RP_{i,j,k}$</u>

Let $RP_{i,j,k}$ denote the number of packets among $Q^b_{i,j,k}$ which have not yet been transmitted by the end of frame k, i.e. the number of $U_{i,j}$'s residual packets in frame k. Clearly,

$$(13)\ RP_{i,j,k}=(Q^b_{i,j,k}-L_{i,j,k})^+$$

Where $(x)^+=x$ if $x>0$ and 0 otherwise.

<u>$Q^b_{i,j,k}$</u>

$Q^b_{i,j,k}$ has been used to denote the queue length of $U_{i,j}$ at the beginning of frame k. Clearly

$$(14)\ Q^b_{i,j,k}(z)=RP_{i,j,k-1}(z)A_{i,j,k-1}(z)$$

II.2 Steady–State

In numerical calculations, steady–state values can be obtained from the given initial conditions, say $L_1=F+F$ and $Q^b_{i,j,1}=0$ for all i,j, by iteratively applying the recursive relations constructed in (1)–(14) and the values should converge when k becomes big enough.

Let $D_{i,j}$ be the delay of a packet generated by $U_{i,j}$. $D_{i,j}$ can be decomposed into waiting time $W_{i,j}$ and service time $S_{i,j}$, i.e.

(15) $D_{i,j}=W_{i,j}+S_{i,j}$

In the sequel we shall use $X_{i,j}$ to denote the steady–state value of the random variable $X_{i,j,k}$, i.e. $X_{i,j}=\lim_{k\to\infty} X_{i,j,k}$.

Intuitively, $E(W_{i,j})$ can be written as

(16) $E(W_{i,j})=E(L)/2+E(RP_{i,j})/\sigma_{i,j}$

To derive $E(S_{i,j})$ we define several additional notations as follows.

(17) $\overline{V}_j$ = the average number of data slots in the variable subframe occupied by group j

$$=\sum_{i=1}^{N_j} E(L_{i,j})-[1-WF_j]$$

(18) $\overline{G}_j$= the average number of data slots in the variable subframe occupied by the users of group 1 to group j–1 plus the length of the fixed subframe

$$=\sum_{j'=1}^{j-1} \overline{V}_{j'}+F$$

(19) $\overline{H}_{i,j}$= the average number of data slots occupied by $U_{1,j}$ to $U_{i-1,j}$

$$=\sum_{i'=1}^{i-1} E(L_{i',j})$$

(20) $X_{i,j}$=an indicator random variable for which $X_{i,j}=1$ if $U_{i,j}$ collides with other users, otherwise 0.

(21)
$$C_{i,j}=P[X_{i,j}=1] = \begin{cases} 0, i=j=1 \\ P(N_a(U_j-\{U_{1,1},U_{i,1}\})\geq 1), i\neq 1, j=1 \\ P(N_a(U_j-\{U_{i,j}\})\geq 1), j\neq 1 \end{cases}$$

(22) $a_{i,j}(q,\ell)=\lim_{k\to\infty} P(Q^b_{i,j,k}=q,L_{i,j,k}=\ell)$

(23)
$$u(x)=\begin{cases} 1 \text{ if } x>0 \\ 0 \text{ if } x\leq 0 \end{cases}$$

(24)
$$E(S_{11})=\frac{1}{1-P(Q^b_{1,1}=0)}\sum_{q=1}^{\infty}\sum_{\ell=1}^{\infty} a_{1,1}(q,\ell)[u(\ell+1-q)(F+(q+1)/2)+u(q-\ell)(F+(\ell+1)/2)]$$

(25)
$$E(S_{1,j})=\frac{1}{1-P(Q^b_{1,j}=0)}\{P(A_j)\{\sum_{q=1}^{\infty}\sum_{\ell=1}^{\infty} a_{1,j}(q,\ell)[(u(\ell+1-q)(j+c_{1,j}F+(q-1)(F+j)+q(q-1)/2)/q +u(q-\ell)(j+c_{1,j}F+(\ell-1)(F+j)+\ell(\ell-1)/2)/\ell]\} +(1-P(A_j))\{\sum_{q=1}^{\infty}\sum_{\ell=0}^{\infty} a_{1,j}(q,\ell)[(u(\ell+1-q)(j+(q-1)\overline{G}_j +q(q-1)/2+c_{1,j}(\overline{G}_j+q-j))/q+u(q-\ell)(j+(\ell-1)\overline{G}_j+\ell(\ell-1)/2+c_{1,j}(\overline{G}_j+\ell-j))/\ell]\}\}, j>1$$

(26)
$$E(S_{i,j})=\frac{1}{1-P(Q^b_{i,j}=0)}\{\sum_{q=1}^{\infty}\sum_{\ell=0}^{\infty} a_{i,j}(q,\ell)[u(\ell++1-q)(j+(q-1)(\overline{G}_j+\overline{H}_{i,j})+(q(q-1))/2+c_{i,j}(\overline{G}_j+\overline{H}_{i,j}+q-j))/ q+u(q-\ell)(j+(\ell-1)(\overline{G}_j+\overline{H}_{i,j})+\ell(\ell-1)/2+c_{i,j}(\overline{G}_j+\overline{H}_{i,j}+\ell j))/\ell]\} \; i\neq 1$$

III. Numerical Examples

Presented here are some representative examples. In the following examples we consider a uniform system with M=20 ,m=3 and F=4, i.e. $\sigma_{1,1}=...=\sigma_{5,1}=\sigma_{1,2}=...=\sigma_{5,4}=\sigma/M$.

Fig. 4 shows the variation of the average frame length $\overline{L}_k$ versus k with $Q^b_{i,j,0}=0$ for all i,j with $p_e=0.1$ $\sigma=0.3$ and 0.9. The purpose of this figure is to demonstrate how $\overline{L}_k$ converges to its steady–state value. Fig. 5 shows the steady–state expectation of the overall average of queue lengths at the beginning of a frame, i.e. $\overline{Q}=(\sum_{i,j}\overline{Q}_{i,j})/M$ in which $\overline{Q}_{i,j}$ denotes the steady–state expectation of $Q_{i,j}$. In Fig. 5 $\overline{Q}$ is plotted against σ. each curve in Fig. 5 is labeled by a number which is the corresponding value of p_e. Fig. 6 shows the effect of channel errors on the delay performance of our protocol, i.e. $\overline{D}=(\sum_{i,j}\overline{D}_{i,j})/M$ versus σ. Fig. 7 is used to show the average of the total number of wasted slots per frame. The improvement achieved via error control codes is illustrated in Figs 8 and 9. For simplicity we only plot $\overline{D}$, the most representative performance measure, versus σ. The results in Fig. 8 and 9 are obtained solely from computer simulations. Results of $\overline{Q}$, $\overline{TW}$, which show consistent trends have also been obtained. In Fig. 8 we use the simple repetition code while in Fig. 9 the famous BCH(n,k,t) code is used. The protection against channel noise via coding is done only for the reservation subframe. When repetition code is used, each reservation vector is repeated r times. The value of r in Figs. 8 is 7. When BCH code is used the coding is done at once for the entire reservation subframe. The values of (n,k,t) in Figs. 9–10 are (127,64,10) and (511,67,87). The value of k in each of those two sets is good enough to accommodate a system whose M=20 and m=3 since Mm=60. We witness that

BCH codes offer better improvement over repetition codes at the expense of more overheads. In particular channel errors are almost all corrected when BCH(511,67,87) code is used.

IV. Conclusions

Based on the protocol considered in [3] we have proposed a modified protocol which can work properly for a noisy channel. Another major contribution of this paper is to derive the results, although in recursive form, of the key performance measure. The validity of these results has not only been verified by computer simulations; but also supported by those intuitively reasonable trends demonstrated in our numerical calculations. In addition, the trends demonstrated in our numerical calculations are intuitively reasonable which provides a further justification of our results.

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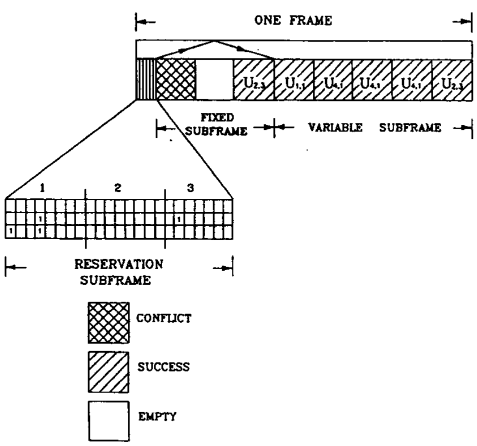

Fig. 1 Frame structure of the protocol considered in [3]

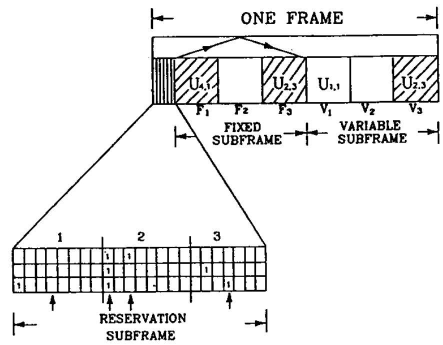

Fig.2 Frame structure of the modified protocol

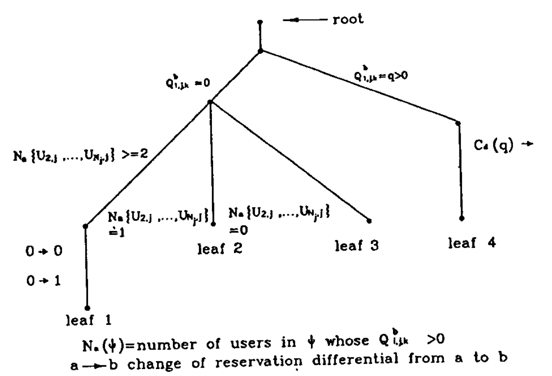

Fig.3 The events which lead to $L_{1,j,k} = 1$ under A_j

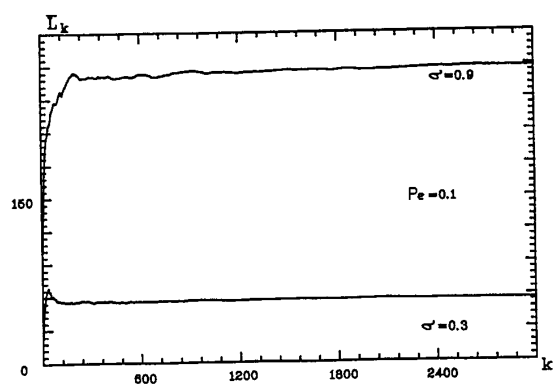

Fig. 4 L_k vs k p_e=0.1

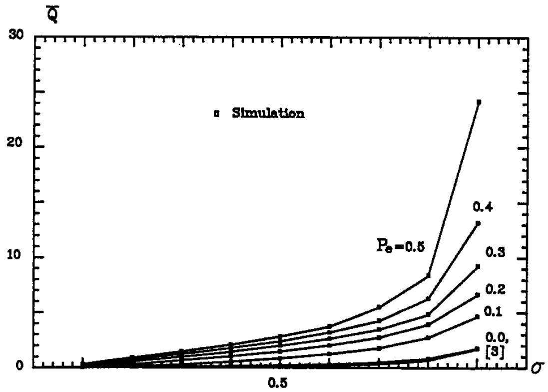

Fig. 5 $\overline{Q}$ vs σ

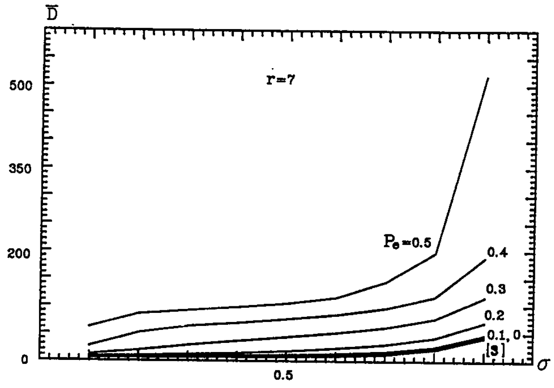

Fig. 8 $\overline{D}$ vs σ under repetition code r=7

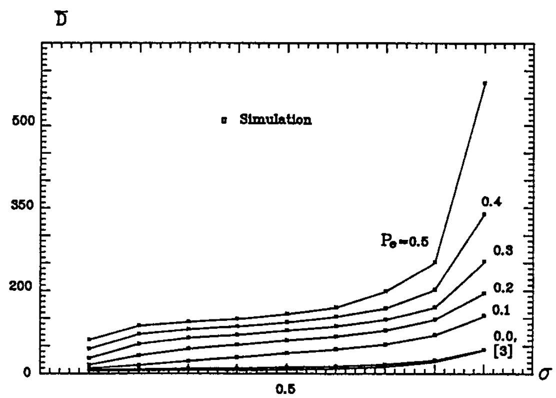

Fig. 6 $\overline{D}$ vs σ

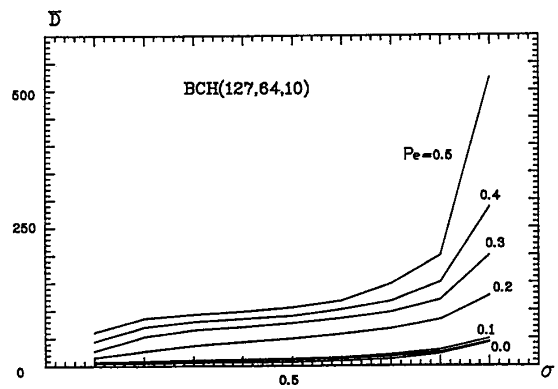

Fig. 9 $\overline{D}$ vs σ under BCH code BCH (127,64,10)

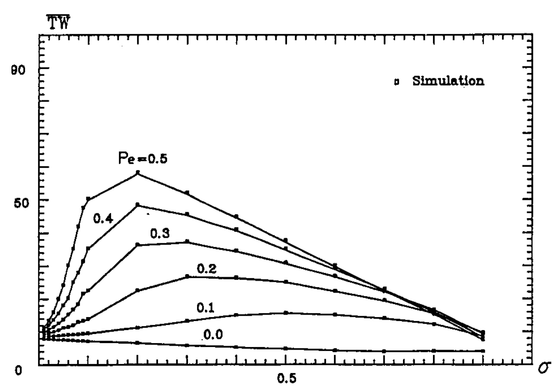

Fig. 7 $\overline{TW}$ vs σ

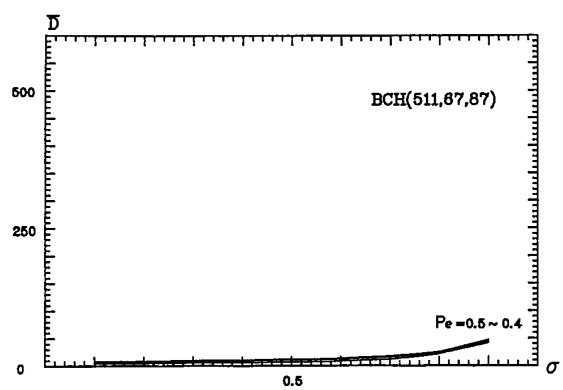

Fig. 10 $\overline{D}$ vs σ under BCH code BCH (511,67,87)