Near Optimum Low Complexity MMSE Multi-user Detector

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Abstract-**In this paper we propose a novel multiuser detector for the synchronous CDMA system. Its BER performance approaches to the optimum MMSE multiuser detector while the complexity only polynomially increases to the number of active users. We also demonstrate its complexity and performance over optimum MMSE multiuser detector and decorrelator. In addition to synchronous AWGN channel case, we also provide the mechanism for the case of asynchronous case, Rayleigh fading channel case, and frequency selective channel case.**

I. Introduction

Direct-sequence code division multiple access (DS-CDMA) has been adopted in various mobile communication systems. In CDMA systems, multiple access interference (MAI) dominates the performance of a receiver. Multi-user detection (MUD) eliminate the MAI to improve the performance of the receiver. The MUD usually collects the information, which passes through the matched filters of all users from the received signal, into a vector. Then the MUD takes the vector processing to reduce the MAI and makes decisions. The optimum multi-user detector was first proposed by S. Verdu [1]. Its complexity increases exponentially with the number of users. As a result, many suboptimal receivers have been developed [2] and these include the linear multiuser detectors and decision-driven-based detectors. The later can be further classified as successive cancellation, multistage detection, and decision feedback detection.

The optimum MMSE multiuser detector has good capability to combat the MAI, but the computational complexity increases exponentially with the number of users and is not practical. It is hard to achieve both low complexity and good BER (Bit Error Rate) performance. We propose a new MUD structure to resolve this dilemma. The proposed MUD uses one simple detector to reduce the candidate decision set. This reduces the complexity of MMSE MUD significantly but the bit-error-rate performance maintains as good as the optimum case.

The organization of this paper is as follows. Section II shows the system model with synchronous channel. In Section III, we describe the structure of the proposed MUD and the complexity analysis is given. In section IV, the mechanism for the practical channel characteristics are presented. Finally, the conclusion can be found in section V.

II. System Model

We consider K users in a synchronous CDMA. The received signal is

$$v(t) = \sum_{k=1}^{K} A_k \sum_i b_k(i) c_k(t - iT_o) + n(t)$$

where A_k is the transmitted amplitude of user k, $b_k(i)$ is the i^{th} information bit of user k, $c_k(t)$ is the signature waveform of user k, T_o is the symbol duration, and $n(t)$ is the AWGN.

The output vector of the bank of K matched filters is

$$\bar{y} = \tilde{R}\tilde{A}\bar{b} + \bar{n}$$

where $\tilde{A} = diag\{A_1, \ldots A_K\}$, $\bar{b} = [b_1, \ldots, b_K]^t$, $\bar{n}$ is a zero mean Gaussian vector with covariance matrix $\tilde{R}$, and $\tilde{R}$ is the cross-correlation matrix whose ij coefficients are $R_{ij} = \int_0^{T_o} c_i(t) c_j(t) dt$.

III. Near Optimum Low Complexity MMSE MUD

The form of optimum MMSE MUD is

$$\bar{b}_{MMSE} = \mathrm{sgn}\{\sum_i \bar{b}_i * p(\bar{b}_i \mid \bar{v})\} \qquad (3\text{-}1)$$

where $\bar{v}$ is the vector output of the matched filter bank, $\bar{b}_i$ is a candidate decision vector, $p(\bar{b}_i \mid \bar{v})$ is a conditional pdf, and sgn{} is the decision operator.

In equ. (3-1), the summation is over all possible vectors $\bar{b}_i$ introducing exponential order of complexity. A sub-optimum multi-user detection is thus useful.

Here after, we make the common assumption [5,6] that powers, phases and time delays of all users can be estimated by the receiver.

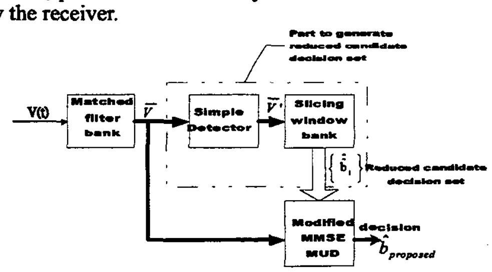

Fig. 1 The structure of Proposed MUD

The proposed structure is presented in Fig. 1. There are three parts of the proposed multi-user detector, one is the matched filter bank, one is for generating the reduced decision set (dashed box), and the other is modified MMSE MUD.

A. Matched Filter Bank

In synchronous channel, K users, we need K matched

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filters. Each matched filter matches each user signature waveform.

B. Generating of Reduced Decision Set

At Fig. 1, the part to generate reduced decision set contains one simple detector and a slicing window bank. The function of simple detector is to partition the total candidate decision set into several sets, Fig. 2, and there exist one property between every two sets, which we call that distance. The distance of each partition is similar to Euclidean distance in traditional single user detector. As like the concept of single user detection, the error probability is dominated by the minimum distance. Due to the sharpness property of AWGN, the error probability should have the form $P_e \approx N_o Q(\frac{d_{\min}}{\sigma})$, where $d_{\min}$ is the minimum distance between sets. If we know the error probability of optimum MMSE MUD, we can find out one partition distribution such that the $d_{\min}$ is large enough and the error rate of proposed MUD is close enough with the optimum MMSE MUD. The function of slicing window is to point out the set, which the distance between the set and received signal is the minimum.

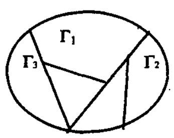

Fig. 2 Idea of slicing window bank

For easy to illustrate, we use the case of BPSK modulation and the simple detector is replaced by decorrelator.

The decorrelator outputs the result of matched filter bank multiplied by the inverse cross-correlation matrix $\tilde{R}^{-1}$. Each element of the output signal vector of the decorrelator goes through a window slicer, Fig. 3. The term of window decision is that the slicer first set a window range and decide whether the received signal falls inside or outside the window. To form a window, we set two threshold t_1 and t_2. The output o_k follows the rules described in Fig. 4.

After window slicing, we collect the outputs of all windows to a vector set. Assume o_k stands for the output set of the k^{th} window slicer. Conceptually, we can use the idea of binary tree. All nodes at level K can trace back to top level, and we can get the set of vectors $\hat{\hat{b}}_o \sim \hat{\hat{b}}_N$ where N is the total number of the output vector set.

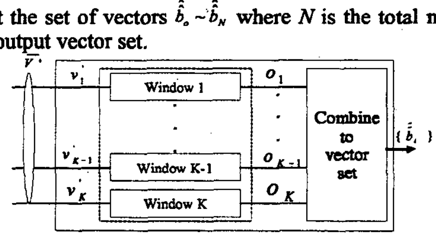

Fig. 3 Diagram of slicing window bank

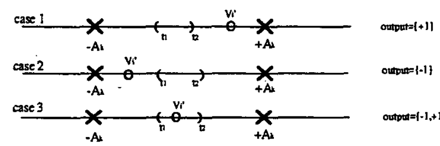

Fig. 4 Output of each window

C. The Modified MMSE MUD

Rewrite the MMSE MUD again,

$$\bar{b}_{MMSE} = \text{sgn}\{\sum_{b_k \in \Omega_o} \bar{b}_k * p(\bar{b}_k \mid \bar{v})\}$$

Ω_o is the total candidate decision vector set which contain 2^K vectors. We need to modify the optimum MMSE MUD to

$$\bar{b}_{proposed} = \text{sgn}\{\sum_{b_k \in \Omega_1} \bar{b}_k * p(\bar{b}_k \mid \bar{v})\}$$

We change the decision set from the total candidate decision set Ω_o to the reduced candidate decision set Ω_1.

D. Complexity Analysis

Here, we discuss the computation complexity of MMSE MUD, decorrelating MUD, and proposed MUD separately. For easy to illustrate, we also still discuss the simple case which appeared in previous section.

For the MMSE MUD case,

$$\bar{b}_{MMSE} = \text{sgn}\left\{\sum_{b_k \in \Omega_o} \bar{b}_k c_1 \exp[c_2(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)' \tilde{R}^{-1}(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)]\right\}.$$

Assume the computation complexity to get $\bar{b}_k c_1 \exp[c_2(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)' \tilde{R}^{-1}(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)]$ is CP_1. Then the total computation complexity of MMSE is $2^K * CP_1$.

Assume the total computation complexity of decorrelating MUD is CP_2.

For the case of proposed MUD,

$$\bar{b}_{proposed} = \text{sgn}\left\{\sum_{b_k \in \Omega_1} \bar{b}_k c_1 \exp[c_2(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)' \tilde{R}^{-1}(\bar{v} - \tilde{R}\tilde{A}\bar{b}_k)]\right\}.$$

The computation complexity in getting the set Ω_1 consist of CP_2, and that of window slicing, says CP_3.

Now, we need to calculate the mean number of vectors in the set Ω_1. Let $\bar{v}_1'$ be the output vector of the decorrelator, and each element of $\bar{v}_1'$ is v_k'. Assume the probability that v_k' falls in the region $(-t,t)$ is P_t. Then the mean number of vectors in Ω_1 is $\mu = \sum_{k=0}^{K} 2^k C_k^K (1-P_t)^{K-k} P_t^k$.

The total computation complexity of the proposed MUD is

$CP_3 + CP_2 + N * CP_1$

For most cases, P_t is very small, and we can neglect the high order terms of N, then,

$$N \approx 1 * (1-P_t)^K + 2 * C_2^K (1-P_t)^{K-2} P_t^2. \quad (3\text{-}2)$$

We will roughly compare the complexity of these three

MUDs by calculating the number of computing action. From Table 1, the complexity of proposed MUD only increases polynomially to the number of active users.

Table 1 Complexity comparison between MUD of decorrelating, proposed, and MMSE.

	Decorrelating MUD	Proposed MUD	MMSE MUD
Number of adders	K^2	$N(2K^2+2K+3K)+K^2$	$2^K(2K^2+2K)$
Number of multipliers	K^2	$N(2K^2+2K+2)+K^2$	$2^K(2K^2+2K+2)$
Number of comparators.	0	$3KN$	0
Number of memory storage.	0	$2KN$	0
Number of exp.	0	N	2^K

E. Simulation for the case of AWGN channel

We use the model that appeared in section II and the environments are AWGN, equal power of each user, and equal cross-correlation.

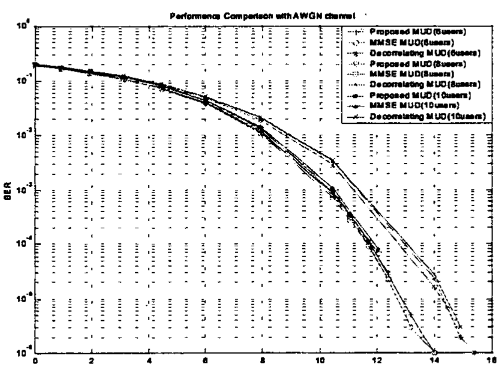

Fig. 5 Simulation Curves of the three MUD at AWGN channel, equal power, equal correlation=0.4.

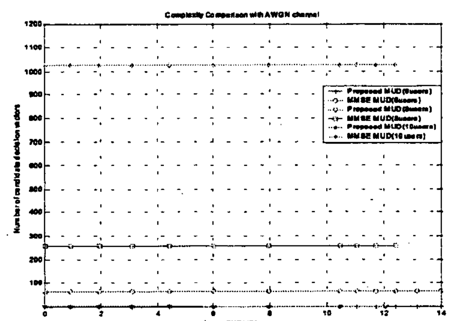

Fig. 6 Complexity comparison at AWGN.

We can see that as the number of users increases, the BER performance relation between three MUD almost the same. From equ (3-2), N is usually only a bit larger than one. From Fig. 6, as the number of users increases, the number of candidate decision vectors of the proposed MUD is stalled in a small number, 1~3. But the number of candidate decision vectors of the MMSE MUD increases exponentially with the number of active users.

IV. Mechanism for Practical Channel Characteristics

A. System Model with Asynchronous Channel

The system model with asynchronous channel ever appears in [2]. And we follow the same expression to describe the channel model. For K simultaneous users and $2M+1$ bits per frame, the received signal

$$v(t)=\sum_{i=-M}^{M}\sum_{k=1}^{K}A_k b_k[i]c_k(t-iT_o-\tau_k)+n(t)$$

where $b_k[i]$ is the i^{th} information bit of user k, $c_k(t)$ is the signature waveform of user k , T_o is the symbol duration, τ_k is the time delay of user k.

Without lost of generality, we assume $\tau_k>\tau_j$ for $k>j$. We need K matched filter bank to match each signature waveform of each user. If the total transmitted bits of a frame are $2M+1$, we need to see $2M+1$ bits each decision. The output of the matched filter bank is

$$\bar{v}_M=\tilde{R}_M\tilde{A}_M\bar{b}_M'+\bar{n}_M \qquad (4\text{-}1)$$

Followings are the detail descriptions of the notation in Equ.(4-1). $\bar{v}_M$ is a K(2M+1)*1 vector. $\tilde{R}_M$ is the correlation matrix, which is defined as

$$\tilde{R}_M=\begin{bmatrix}\tilde{R}[0] & \tilde{R}'[1] & \cdots & \cdots & \tilde{0} & \tilde{0}\\ \tilde{R}[1] & \tilde{R}[0] & \tilde{R}'[1] & \cdots & \tilde{0} & \tilde{0}\\ \tilde{0} & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \tilde{R}[1] & \tilde{R}[0] & \tilde{R}'[1]\\ \tilde{0} & \tilde{0} & \cdots & \tilde{0} & \tilde{R}[1] & \tilde{R}[0]\end{bmatrix}.$$

This matrix is composed of $(2M+1)*(2M+1)$ matrixes. $\tilde{0}$ is a $K*K$ zero matrix. $\tilde{R}[0]$ is a $K*K$ correlation matrix. The lk element of $\tilde{R}[0]$ is $\rho_{lk}=\int_0^T s_l(t-\tau_l)s_k(t-\tau_k)dt$. The diagonal elements of $\tilde{R}[0]$ are all 1. $\tilde{R}[1]$ is a K*K correlation matrix. $\tilde{R}[1]$ is a upper triangular matrix and the lk^{th} elements of $\rho_{lk}=\int_0^T s_l(t-\tau_l)s_k(t-\tau_k)dt$ for k>l. $\tilde{A}_M=diag\{\tilde{A}\ \cdots\ \tilde{A}\}$, where $\tilde{A}$ is the K*K amplitude matrix defined the same as synchronous case.

The desired signal vector $\bar{b}_M$ is a K(2M+1)*1 vector and defined as $\bar{b}_M=[\bar{b}'[-M]\ \ldots\ \bar{b}'[0]\ \ldots\ \bar{b}'[M]]^T$, where $\bar{b}[i]$ is a K*1 signal vector at symbol time i. The noise vector $\bar{n}_M$ is a (2M+1)*K vector.

The Equ. (4-1) is similar to synchronous case. The difference is the size of vector and that of matrix. We can follow the same method in the synchronous case.

In order to apply to the asynchronous channel, we must take some modification with the proposed MUD at section III. Following are the modifications:

1.In order to form a received vector $\bar{v}_M$, each matched filter needs to match the received signal (2M+1) times.

2.$\widetilde{R}$ changes to $\widetilde{R}_M$. $\widetilde{A}$ changes to $\widetilde{A}_M$. $\bar{b}$ changes to $\bar{b}_M$.

3.We need a (2M+1)*K window slicer.

B. System Model with Rayleigh Fading Channel

We consider the simple flat fading channel case and the received waveform is $v(t)=\sum_{k=1}^{K}A_k\beta_k\sum_{i}b_k(i)c_k(t-iT_o)+n(t)$ where β_k is the fading parameter of user k.

We can express the output of the bank of matched filter as a vector form, $\bar{y}=\widetilde{R}\widetilde{A}\widetilde{\beta}\bar{b}+\bar{n}$ .The fading coefficient matrix is a diagonal matrix with fading coefficient on diagonal.

We can see that the received signal vector almost the same as the non-fading case. The only difference between them is that the received vector of the fading case has additional fading matrix. The combination of the diagonal matrix $\widetilde{\beta}$ and $\widetilde{A}$ can be regarded as a fading amplitude diagonal matrix, and we can directly apply the proposed MUD.

Fig. 7 is the simulation curves of the three MUD for 6 users case at Rayleigh fading channel. The received power of each user is assumed to be equal. The cross-correlation between users are equal to 0.4. From the simulations, the performance gets some degradation for three cases due to fading. The performance of the proposed MUD is still near MMSE MUD.

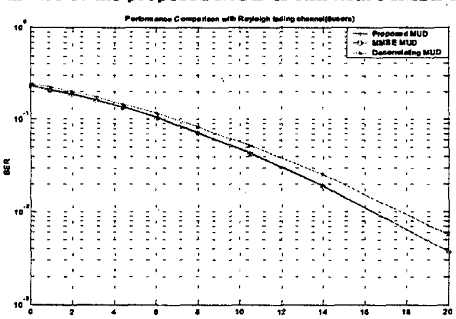

Fig. 7 Simulation Curves of the three MUD for 6 users case at Rayleigh fading channel

C. System Model with Multi-path Fading Channel

Consider the CDMA system with K simultaneous users, and each user transmits information bits of block length $2M$+1. Suppose that there are L resolvable paths for each user and the channel is slow frequency-selective fading. Therefore, we can write the received signal in the form,

$$v(t)=\sum_{i=-M}^{M}\sum_{l=0}^{L}\sum_{k=1}^{K}\beta_{lk}A_k b_k[i]c_k(t-nT_o-\tau_k-\tau_l)+n(t) \quad (4\text{-}2)$$

where β_{lk} is the fading coefficient of path l of user k, τ_k is the time delay of user k, τ_l is the time delay of path l.

We can treat each path as one user, so we regard Equ. (4-2) as a K*L users with asynchronous channel and we need K*L matched filters. These matched filters need to match the received signal from first symbol time to $2M$+1[th] symbol time per decision. Then we can follow the same model of asynchronous channel case. The output of the bank of matched filter at i^{th} symbol time is

$$\bar{v}[i]=\widetilde{R}[-1]\widetilde{P}\widetilde{A}\bar{b}[i-1]+\widetilde{R}[0]\widetilde{P}\widetilde{A}\bar{b}[i]+\widetilde{R}[1]\widetilde{P}\widetilde{A}\bar{b}[i+1] \quad (4\text{-}3)$$

The notations of Equ.(4-3) describe below. $\bar{b}[i]$ defined as $\bar{b}[i]=[b_1[i],b_2[i],...,b_K[i]]^t$. $\widetilde{A}$ is the amplitude matrix with diagonal element A_i. The matrix $\widetilde{P}$ is a mapping matrix, which maps K users signal vector to K*L users signal vector, $[\beta_{1,1}A_1b_1[i], \;...\; ,\beta_{1,L}A_1b_1[i], \;...\; ,\beta_{K,L}A_Kb_K[i]]^t$.

The matrix $\widetilde{P}$ defines as

$$\widetilde{P}=\begin{bmatrix} \beta_{1,1} & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & 0 \\ \beta_{1,L} & 0 & ... & ... & 0 \\ 0 & \beta_{2,1} & ... & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & \beta_{2,L} & 0 & ... & ... \\ ... & 0 & \beta_{3,1} & ... & ... \\ ... & 0 & ... & ... & 0 \\ ... & ... & ... & ... & 0 \\ ... & ... & ... & 0 & \beta_{K,1} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & \beta_{K,L} \end{bmatrix}.$$

The correlation matrix $\widetilde{R}[i]$ where the K*L matched filters match the part of i^{th} symbol time signal is defined as

$$\widetilde{R}[i]=\begin{bmatrix} \widetilde{R}_{1,1}[i] & \widetilde{R}_{1,2}[i] & ... & ... & \widetilde{R}_{1,K}[i] \\ \widetilde{R}_{2,1}[i] & ... & ... & ... & ... \\ ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... \\ \widetilde{R}_{K,1}[i] & ... & ... & ... & R_{K,K}[i] \end{bmatrix}.$$

The matrix $\widetilde{R}_{m,n}[i]$ indicates the correlation between user m and user n from path 1 to path L. Each element in the matrix $\widetilde{R}_{m,n}[i]$ is defined as

$$[\widetilde{R}_{m,n}[i]]_{u,v}=\int_{iT_o+\tau_m}^{(i+1)T_o+\tau_m}c_m(t-\tau_u-\tau_m)*c_n(t-\tau_v-\tau_n)dt$$

We need to match the received signal from first symbol time to $2M$+1[th] symbol time per decision. From Equ. (4-3) we can write the received vector

$$\bar{y}_F=\widetilde{R}_F\widetilde{P}_F\widetilde{A}_F\bar{b}_F+\bar{n}_F \quad (4\text{-}4)$$

The matrix $\widetilde{A}_F$ is a (K*(2M+1))*(K*(2M+1)) matrix and is defined as $\widetilde{A}_F=diag\{\widetilde{A} \;\cdots\; \widetilde{A}\}$. The matrix $\widetilde{R}_F$ is a ((2M+1)*K*L)*((2M+1)*K*L) matrix and is defined as

$$\widetilde{R}_F=\begin{bmatrix} \widetilde{R}[0] & \widetilde{R}[1] & 0 & ... & 0 \\ \widetilde{R}[-1] & \widetilde{R}[0] & \widetilde{R}[1] & ... & ... \\ 0 & \widetilde{R}[-1] & ... & ... & ... \\ ... & 0 & ... & ... & ... \\ ... & ... & ... & \widetilde{R}[0] & \widetilde{R}[1] \end{bmatrix}.$$

The matrix $\widetilde{P}_F$ is a (K*L*(2M+1))*(K*(2M+1)) matrix and is defined as

$$\tilde{P}_F = \begin{bmatrix} \tilde{P} & 0 & \cdots & 0 & 0 \\ 0 & \tilde{P} & \cdots & \cdots & 0 \\ \cdots & 0 & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \tilde{P} \end{bmatrix}.$$

The vector $\bar{b}_F$ and $\bar{n}_F$ is a K*(2M+1) vector and defines as $\bar{b}_F = [\bar{b}'[-M] \ \cdots \ \bar{b}'[0] \ \cdots \ \bar{b}'[M]]^T$ and $\bar{n}_F = [n_{-M} \ \ldots \ n_0. \ \ldots \ n_M]^T$.

At Equ. (4-4), the form is somewhat similar to the case for synchronous AWGN. We need to do some modifications, and then we can apply the method proposed in section III.

1. We need $K*L$ matched filters.
2. To form a received vector $\bar{y}_F$, each matched filter needs to match the received signal (2M+1) times.
3. Replace $\tilde{R}$ with $\tilde{R}_F$. Replace $\bar{b}$ with $\bar{b}_F$. Replace $\bar{A}$ with $\bar{P}_F\bar{A}_F$.
4. Due to L diversity, we need to do the maximum ratio combining [3] before window slicer, Fig. 8. we need (2M+1)*K window slicer.

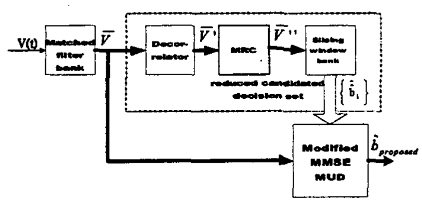

Fig. 8 Modification of proposed MUD for Multi-path channel

Fig. 9 is the simulation Curves of the three MUDs for 3 users case at frequency selective fading channel (2 ray). We adopt one of the two-rays channel model suggested by 3GPP, [4]. And we use the Gold code as our spreading sequence.

From the result of the simulation, Fig. 9, we can see that the BER performance of MMSE MUD is about 4dB better than that of the decorrelating MUD. The performance of the proposed MUD is near MMSE MUD.

V. Conclusions

In this paper, we have proposed a MMSE base multiuser detection. Base on the main idea of reduced decision set, the complexity of the proposed MUD significantly lower than the optimum MMSE MUD, but the performance of the proposed multiuser detection approaches that of optimum MMSE MUD. The complexity analysis indicates that the complexity of the proposed MUD is polynomially increased with the number of active users.

Another contribution of this paper is the extension of the proposed MUD to the case of asynchronous channel, Rayleigh fading channel, and frequency selective channel. Through some simple modifications, the proposed MUD can be applied in these cases of channel. And the simulations show that the performance also good in the cases of these channels.

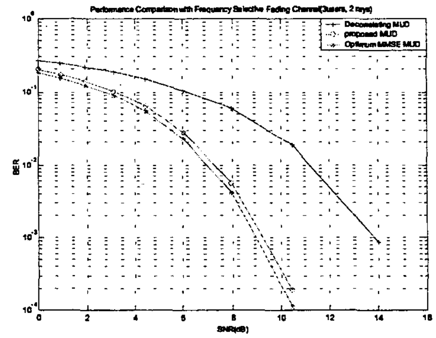

Fig. 9 Simulation Curves of the three MUD for 3 users case at frequency selective fading channel (2 ray).

VI. Acknowledgment

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