The thru-line-symmetry (TLS) calibration method for on-wafer scattering matrix measurement of four-port networks

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***Abstract* — The multimode TRL calibration method is known as an approach for the measurement of multi-conductor transmission line devices. The propagation constants of different modes propagating along the multi-conductor transmission line must be different using this method. However, in general multi-port networks, the propagating constants at each port may be equal. We here then present the thru-line-symmetry (TLS) calibration method for the calibration of the equal propagation constant case. The calibration equations with the associated calibrators for on-wafer scattering matrix measurement of four-port network are developed. The measured scattering matrix of a branch line coupler using proposed calibration method shows good agreement with simulation. This method can also be extended for the calibration of multi-port networks.**

***Index Terms* —scattering parameters measurement, multi-port network, calibration.**

I. INTRODUCTION

The TRL calibration method [1] has been widely used in the calibration of two-port networks. The multimode TRL calibration method proposed in [2] then gives a new formulation to deal with circuits with multiple conductors. In general, N signal lines with ground plane can support N modes propagating simultaneously on a multi-conductor transmission line. This method gives a concise formulation and multi-port calibrators to reduce the number of calibrators required as compared with [3]. However, in the formulation, propagation constants are assumed to be different among different modes. In a general multi-port network, one can combine certain physical transmission lines together and consider them as a single multi-conductor transmission line mathematically. The signal propagating in a transmission line can be considered as a mode. Therefore, for a multi-port network having the same transmission lines, the propagating constants are identical. The formulation of multimode TRL then has to be modified to deal with the case of equal propagating constant. In this study, we find that as more propagation constants become equal, more unknowns are required in the calibration process. Therefore, we developed a "symmetry" calibrator to provide more information than the "R" calibrator in the multimode TRL calibration method. In other words, it gives more equations for solving the additional unknowns.

In the following, the formulation of proposed TLS calibration method is presented. The reasons for considering new symmetry calibrator are also addressed. The experiment arrangement, proposed calibrators and results are given in Sec. III. Finally, the conclusion is given in Sec. IV.

II. FORMULATION

In this section, the formulation for four-port scattering matrix calibration is described, because this is the maximum number of available ports of the network analyzer used in our laboratory. The formulation can however be generalized for networks with more than four ports without much difficulty.

A. Generalized transfer and scattering matrices

The device-under-test (DUT) shown in Fig.1 has four physical ports. Only one propagating mode exists in the transmission line where each physical port is defined. The signals propagating in these four transmission lines have identical propagating constants. One can combine physical ports 1 and 3 as the multimode port 1 and physical ports 2 and 4 as the multimode port 2. The selection of multimode port is arbitrary. The reference planes of physical ports and multimode ports are defined at the same location. The incident and reflected wave amplitude and the wave vector A_i and B_i of multimode ports are given as

$$A_1 = \begin{bmatrix} a_1 \\ a_3 \end{bmatrix},\ A_2 = \begin{bmatrix} a_2 \\ a_4 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} b_1 \\ b_3 \end{bmatrix},\ B_2 = \begin{bmatrix} b_2 \\ b_4 \end{bmatrix}. \quad (1)$$

In the following equations, the capital letters are referred to matrices, while scalar quantities are denoted by lowercase letters. The same representation of general transfer matrix and scattering matrix in [2] are used here. According to the basic measurement arrangement shown in Fig.2, the measured transfer matrix M is related to the transfer matrices of error box A, DUT and error box B as

$$M = ATB^{-1}. \tag{2}$$

B. Thru-line-symmetry (TLS) calibration method

In the following derivation, the ports numbers are referred to the multimode ports for convenience. Additional notice will be given when the physical ports are referred. From (2), the transfer matrix N_x of DUT will lead to the following measured transfer matrix M_x as

$$M_x = AN_xB^{-1} \text{ and } N_x = A^{-1}M_xB. \tag{3}$$

One can start with the "thru (or T)" and "line (or L)" calibrators. The transfer matrices of "T" calibrator N_1 and "L" calibrator N_2 are given by

$$N_1 = \begin{bmatrix} I & \bar{0} \\ \bar{0} & I \end{bmatrix} \text{ and}$$

$$N_2 = diag(e^{-\gamma l}, e^{-\gamma l}, e^{\gamma l}, e^{\gamma l}), \tag{4}$$

where I denotes an identity matrix and $\bar{0}$ is the null matrix. γ is the propagation constant and l is the line length. As in [2], P and Q are defined as $P = N_2N_1^{-1}$ and $Q = M_2M_1^{-1}$. Since P and Q are similar matrices, they have the same eigenvalues. Λ is a diagonal matrix with elements from eigenvalues of P or Q. Since Q is completely known from measurement, the unknown line length of "L" calibrator can be solved using these solved eigenvalues.

The relation between P, Q and Λ is given as

$$\Lambda = X^{-1}PX = Y^{-1}QY, \tag{5}$$

where the columns of X (respectively, Y) are composed of the eigenvectors of P (respectively, Q). Since the eigenvectors are not unique, they are known except for a constant when the eigenvalues are distinct. As there are equal eigenvalues, they can be expressed as a linear combination of a set of solved eigenvectors. Since P and Q are fully known, one can find a set of their eigenvectors X_0 and Y_0. In addition, since $\lambda_1 = \lambda_2$, the first eigenvector (i.e., the first column of X) in X can be written as a linear combination of the first and second eigenvectors in X_0. The second eigenvector in X can also be expressed as a linear combination of the first and second eigenvectors in X_0. However, the first and second eigenvectors in X should be linearly independent.

Similar combinations can be applied for the third and the fourth eigenvectors in X. Therefore, X can be expressed as

$$X = X_0\beta \text{ and } \beta = \begin{bmatrix} \Phi_1 & \bar{0} \\ \bar{0} & \Phi_2 \end{bmatrix} \tag{6}$$

where Φ_1 and Φ_2 are 2x2 matrices. The similar expression can be obtained for Y as

$$Y = Y_0\delta \text{ and } \delta = \begin{bmatrix} \Delta_1 & \bar{0} \\ \bar{0} & \Delta_2 \end{bmatrix}, \tag{7}$$

where Δ_1 and Δ_2 are 2x2 matrices. The partially determined A can then be expressed as

$$A = YX^{-1} = Y_0KX_0 = A_0K \text{, where } K = \begin{bmatrix} K_1 & \bar{0} \\ \bar{0} & K_2 \end{bmatrix},$$

$$K_1 = \Delta_1\Phi_1^{-1} \text{ and } K_2 = \Delta_2\Phi_2^{-1}, \tag{8}$$

where A_0 is fully known while K remains unknown so far. The measurement of the "T" calibrator can also be used to partly derive B as $B = M_1^{-1}AN_1 = M_1^{-1}A_0K = B_0K$.

In the multimode TRL calibration method, the K matrix is a diagonal matrix with only four elements. However, in the equal propagation constant case, there are at most 8 non-zero elements in K matrix. Since there are more unknowns in K matrix, one needs a calibrator that can provide more information than the "R" calibrator in the multimode TRL.

In this study, we propose to use a "symmetry (or S)" calibrator with the following properties:

1. $S_{11} = S_{22}$ and $S_{11} = S_{11}^t$. This is the same characteristics as the "R" calibrator.
2. $S_{12} = S_{34}$ and $S_{14} = S_{23}$. The subscripts here are the port number of physical ports.

As the "S" calibrator is connected, its transfer matrix N_3 is related to the measured transfer matrix M_3 by

$$N_3 = A^{-1}M_3B = K^{-1}A_0^{-1}M_3B_0K = K^{-1}PK, \tag{9}$$

where $P = A_0^{-1}M_3B_0$. Equation (9) can be divided into four 2x2 sub-matrices as

$$N_3 = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} = \begin{bmatrix} K_1^{-1}P_{11}K_1 & K_1^{-1}P_{12}K_2 \\ K_2^{-1}P_{21}K_1 & K_2^{-1}P_{22}K_2 \end{bmatrix}. \tag{10}$$

By using the property 1 of "S" calibrator, one can find an equation relating K_1 and K_2 as

$$K_2K_1^{-1}P_{12}P_{21}^{-1}K_2K_1^{-1} = LP_{12}P_{21}^{-1}L = -P_{22}^{-1}P_{21}, \tag{11}$$

where $L = K_2K_1^{-1}$. Now there are four equations for solving four unknown elements in L. Since they are not linear equations, the solution might not be unique. We will discuss how to resolve the ambiguity problem later.

Let's assume a solution of L is found. As the solution is L_0, one can use it to solve K_1. In the multimode TRL, K_1 is a 2x2 diagonal matrix. There are only two unknowns to be solved. Since a constant term can be factored out, only one unknown remains in the four-port multimode TRL calibration method. $\Gamma = \Gamma^t$ provides one additional equation that can be used to solve the only one unknown in K_1. In the equal propagation constant case, there are three unknowns instead of one has to be solved. Therefore, one needs at least two more equations. The "S" calibrator can provide two additional equations.

Since $K_2 = \pm L_0 K_1$ from (11), the left hand side of (10) can be written as a function of K_1 only. From the property 2 of "S" calibrator, one can find that $N_{22,11} = N_{22,22}$ and $N_{22,12} = N_{22,21}$. This then gives two additional equations for solving K_1. The solutions for K_1 might not be unique. For each solved L_0 and the corresponding K_1, one can calculate the scattering matrix of "S" calibrator. In our computer simulation, we found that one can determine the correct solution set of L_0 and K_1 from the a priori knowledge of the signs of scattering parameter phase terms of the "S" calibrator. As K is solved, the transfer matrix N_x of DUT can be calculated from the measured transfer matrix M_x by using $N_x = K^{-1} A_0^{-1} M_x B_0 K$.

III. Experiment arrangement and results

To measure a four-port network, one can directly use a multi-port vector network analyser such as Agilent E5071B or a two-port vector network analyser as proposed in [4] or the port reduction methods proposed in [5-6].

The test structure and proposed calibrators for on-wafer measurement are shown is Fig. 3. The circuits are fabricated on 8 mil thick Rogers RO4003C substrate. Since the four probes on a probe station are located at north, east, south and west directions, a test structure shown in Fig. 3(a) incorporating 45° bends are used for the connection to probes. As the two reference planes are connected together, it becomes a "thru" calibrator. The "line" calibrator is shown in Fig.3(b). The "symmetry" calibrator is shown in Fig.3(c) by adding a transmission line at the midpoints of two transmission lines in a "line" calibrator. The length and impedance of this additional transmission line is not critical based on the requirement of "symmetry" calibrator. A branch-line coupler with the test structure as shown in Fig. 3(d) is used as a DUT to test the proposed calibration method. We use the Agilent E5071B four-port network analyzer to measure the four-port scattering matrices of these networks. The network analyzer is first calibrated to coaxial ports using the Agilent N4431A-010 ECal. The proposed TLS calibration method is then used to de-embed the effects of test structure and probes. The de-embedded and simulated magnitude and phase results of the coupler are shown in Fig.4. They are in a good agreement.

IV. Conclusion

A novel TLS calibration method is developed for the equal propagation constant case in multimode networks measurement. A set of calibrators that is suitable for on-wafer four-port network measurement is described. The measured scattering matrix of a branch-line coupler shows good agreement with simulation. Although the formulation given is for four-port network measurement, this method can be extended to multi-port networks without much difficulty.

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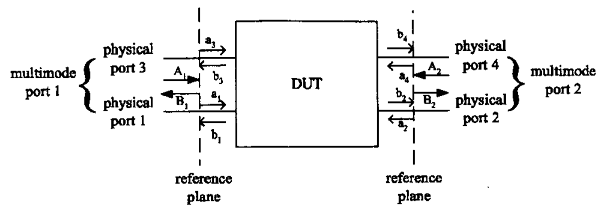

Fig.1 Representation of a four physical-port or a two multimode-port DUT.

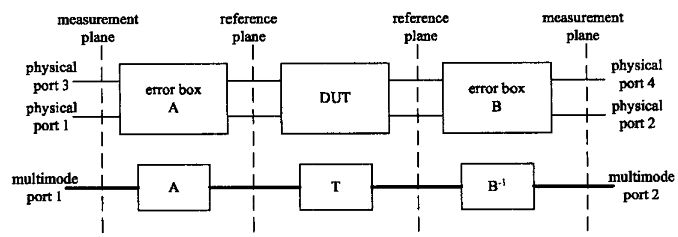

Fig.2 The basic arrangement for four port DUT measurement.

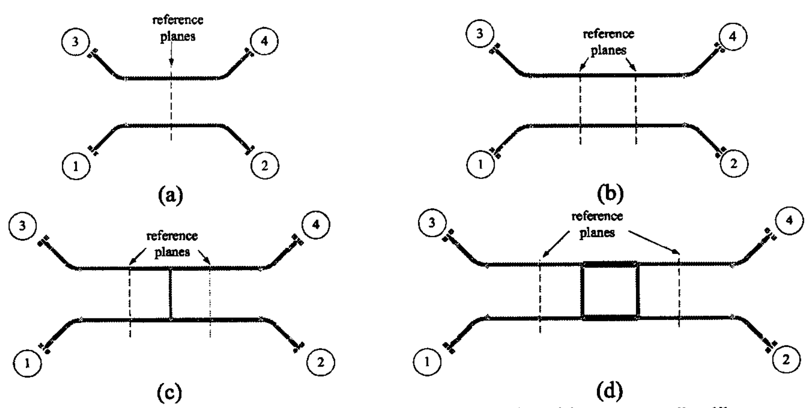

Fig.3 On-wafer test structures (a) in "thru" calibrator, (b) with "line" calibrator, (c) with "symmetry" calibrator, and (d) with a branch-line coupler as DUT. Note only the physical ports are labelled.

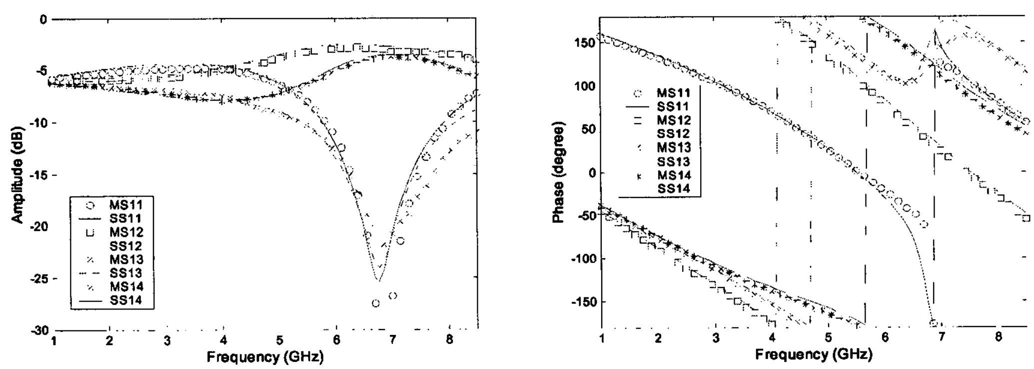

Fig.4 Results of measured (MS_{ij}) and simulated (SS_{ij}) scattering parameters of the branch-line coupler.