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Adaptive Sliding Mode Controller Design of a Maglev Guiding System for Application in Precision Positioning

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Abstract

In this paper, we have analyzed the dynamics of a single-axis maglev system and derived its analytical model with full DOFs (degrees-of-freedom). Then, an adaptive sliding mode controller which deals with unknown parameters is proposed here to accomplish both guidance and positioning in this system. Finally, stability is proved and simulation results are provided. From the simulation results, good performance of regulation for guiding-axes and of tracking for positioning-axis is achieved.

Keywords: Maglev guiding, Hybrid magnet, Adaptive control, Precision positioning.

1. Introduction

Recently, magnetic levitation is considered as one of the most suitable ways to achieve the high precision transportation. By Hollis *et al.*[1][2], it creates a stable state without any mechanical contact when the gravitational force is solely counterbalanced by magnetic forces. Of course, such contact-free levitation has to be realized over all possible degrees of freedom (DOFs) of the rigid body.

Previous work on maglev systems spans many fields. Some well known fields include maglev transportation [3][4], wind tunnel levitation[5] and anti-vibration tables[6]. Here, however, we will only investigate the maglev techniques for the field of short-range travel with precision positioning and then design and implement a prototype maglev system to verify its high performance.

In our foregoing research [8], we have analyzed the dynamics of a maglev guiding system and derived its analytical model with full DOFs. In this paper, we made more progress. The organization of this paper is as follows. Section 2 describes the design aspects of the proposed prototype system. Section 3 provides a detailed mathematical model. In section 4, an adaptive controller for the maglev system is developed which can achieve the regulating objective for guiding and the tracking objective for positioning. Section 5 presents simulation results to demonstrate the effectiveness of the system design including the adaptive controller. Finally, conclusions are drawn in section 6.

2.System Description and Modeling

In this section, the mechanical structure of a maglev guiding system will be described.

2-1.Maglev Guiding System

Without the presence of electrical field, the expression of the Lorentz force and torque exerted on a magnetic material can be simplified as

$$\mathbf{F}=(\mathbf{m}\cdot\nabla)\mathbf{B}, \tag{2-1}$$

$$\mathbf{T}=\mathbf{m}\times\mathbf{B}, \tag{2-2}$$

where **m** is the dipole moment of the magnetic material and B is the magnetic flux density. This means if we build flux somewhere in the space, then the flux can interact with a dipole moment (or a magnet).

Figure 2-1 shows the directions of force and torque acting on a single dipole moment due to the existing magnet flux density built by a current-carrying straight wire.

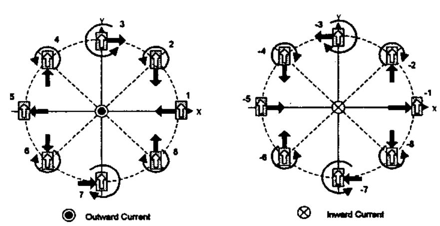

Fig.2-1: Forces and torques on a single dipole moment due to a current-carrying straight wire.

From the concepts above, with proper hardware design composed of the permanent magnets and coils, which provides both levitation force and stabilizing force. By applying control currents into coils, guidance as well as positioning task can be achieved.

2-2.Modified Voice Coil Motor

To provide the propulsion force in the Y-direction, we adopted a modified voice coil motor (VCM) here as shown in Fig.2-2.

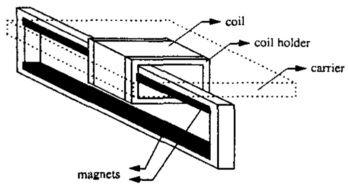

Fig.2-2 : Perspective of VCM and track

The propulsion force of VCM is resulting from the interaction between magnetic field generated by the permanent magnet and the coil. The working principle of VCM is

$$F = i\vec{L} \times \vec{B}, \tag{2-3}$$

which in our compliance with our design becomes

$$F = iLB, \tag{2-4}$$

2-3.System Overview

Given by the previous design considerations, we propose a four tracks arrangement as shown in Fig.2-3.

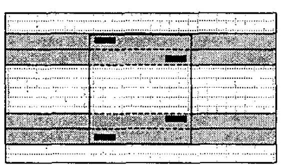

Fig.2-3 Top view of the maglev system

The four-tack arrangement is somewhat like the four wheels of an automobile as shown in Fig.2-4. By applying appropriate control current into each levitation and stabilization coil, all attitudes in free space can be regulated.

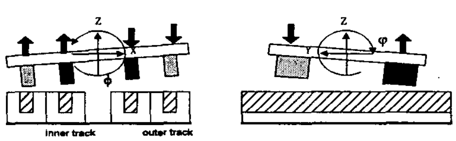

Fig.2-4 Four-track concept.

Though the platform is levitated by magnets and coils, the repulsive force will also generate undesirable torque on the carrier. The larger the levitating force implies the larger destabilizing force in lateral direction. Therefore, we design the stabilizers to provide control force in lateral direction. By adopting the stabililzers on, regulation of both lateral translation and rotation can be easily accomplished.

After all, we propose design of the prototype system as shown below.

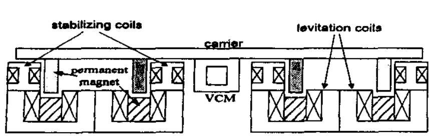

Fig.2-5 The front view of maglev system

3. Modeling of Maglev System

A complete analytical model which includes two lateral degrees of freedom (DOFs), a propulsion DOF and three vertical DOFs are imperative for designing a good controller. The six DOFs of the carrier X, Y, Z, φ, ϕ, and θ are defined as shown in Figure3-1.

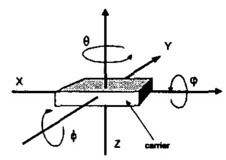

Fig.3-1 Definition of coordinate of the carrier.

The complete model of the maglev system can be derived by using Newton's law of motion and Euler's motion equations as follows [8]:

$$m\ddot{X} = 4K_{FXL}X + 2K_{IXS}(I_1 + I_2)$$

$$I_Z\ddot{\theta} = 4a^2K_{FXL}\theta + 2aK_{IXS}(I_2 - I_1)$$

$$m\ddot{Z} = 4K_{FZL}Z + 2K_{FZS}(-b_1I_1 + b_2I_2)\phi + K_{IZL}(I_A + I_B + I_C + I_D)$$

$$\begin{aligned} I_Y\ddot{\phi}_u = {} & 2K_{TYL}I_Y\left(\frac{1}{I_{a0}} + \frac{1}{I_{c0}}\right)X - 2aK_{TYL}I_Y\left(\frac{1}{I_{a0}} - \frac{1}{I_{c0}}\right)\theta \\ & + 2\left[K_{FZS1}(-b_1I_1 + b_2I_2) + K_{TYS}I_Y\left(\frac{I_1}{I_{a0}} + \frac{I_2}{I_{c0}}\right)\right]Z \\ & + 2a\left[K_{FZS1}(-b_1I_1 - b_2I_2) + K_{TYS}I_Y\left(\frac{I_1}{I_{a0}} - \frac{I_2}{I_{c0}}\right)\right]\psi \\ & + 2(b_1^2 - b_2^2)K_{FZL}\phi + K_{IZL1}\left[b_1(I_A - I_B) - b_2(I_C - I_D)\right] \end{aligned}$$

$$I_X\ddot{\psi} = -2aK_{FZS}(b_1I_1 + b_2I_2)\phi + 4a^2K_{FZL}\psi + K_{IZL}(I_A + I_B - I_C - I_D)$$

A model of the modified voice coil motor is adopted as given below, which describe the dynamics in the positioning DOF:

$$m\ddot{Y} = K_{vc}I_{vc} - f(Y) \tag{3-1}$$

where $f(Y)$ is a damping force resulting from the magnetic field of the all system. Such damping function is best modeled by a parabolic function because its value is the smallest in the middle of the track and gets larger and larger in the direction towards either head or end of the track.

To get a compact model form for convenience of analysis, all the coefficients are all abbreviated as simple notations. As a result, the concise model would appear as:

$$b_{11}\ddot{X} = a_{11}X + u_1 + v_1 \tag{3-2}$$

$$b_{22}\ddot{\theta} = a_{22}\theta - u_2 + v_2 \tag{3-3}$$

$$b_{33}\ddot{Z} = a_{33}Z + \mathbf{g}_3 + u_3 + v_3 \tag{3-4}$$

$$b_{44}\ddot{\phi} = a_{44}\phi + \mathbf{g}_4 + u_4 + v_4 \qquad (3\text{-}5)$$

$$b_{55}\ddot{\psi} = a_{55}\psi + \mathbf{g}_5 + u_5 + v_5 \qquad (3\text{-}6)$$

$$m\ddot{Y} = u_6 - f(Y) \qquad (3\text{-}7)$$

where a_{ii} and b_{ii} are constants, and $u_i, \forall i = 1 \sim 5$ are the control inputs. Additionally, $v_i, \forall i = 1 \sim 5$, represent the disturbances, and m stands for the mass of the carrier. Furthermore, $\mathbf{g}_3$, $\mathbf{g}_4$ and $\mathbf{g}_5$ are coupled terms [8].

4.Controller Design

An adaptive sliding mode controller is proposed here for this maglev system.

4-1-1.Guidance Control

We rewrite Eqs.(3-2)~(3-6) into the state space form as:

$$D_B\ddot{E} = -D_A E + U + G + V \qquad (4\text{-}1)$$

Apparently, E captures all guiding DOFs, and hence minimize E is equivalent to providing precise guiding. First, we define that:

$$S = G_D\dot{E} + G_P E \qquad (4\text{-}2)$$

Our goal now is to drive S to zero so that $\dot{E}$ and E converge to zero simutaneously, since the sliding mode S=0 characterizes stable dynamics. Choose a Lyapunov function candidate V as:

$$V = \frac{1}{2}S^T S \qquad (4\text{-}3)$$

If we can let $\dot{S} = -KS$, then, the derivative of V can be rewritten as:

$$\dot{V} = -KS^T S \le 0 \qquad (4\text{-}4)$$

which implies that S converges to zero exponentially in time t by use of Lyapunov theory. In turn, $\dot{E} \to 0$ and $E \to 0$ as has been mentioned earlier. Now, the necessary input condition that keeps $\dot{S} = -KS$ has been changed as follows:

$$U = D_B G_D^{-1}(-KG_D\dot{E} - KG_P E - G_P\dot{E}) + D_A E - G - V \qquad (4\text{-}5)$$

This is exactly the necessary control input to achieve sliding mode control provided all the coefficients are priory known. However, if this ideal case fails to hold, then we have to modify the control in Eq.(4-5) as follows:

$$U = \hat{D}_B G_D^{-1}(-KG_D\dot{E} - KG_P E - G_P\dot{E}) + \hat{D}_A E - \hat{G} - \hat{V} \qquad (4\text{-}6)$$

where $\hat{D}_B$, $\hat{D}_A$, $\hat{G}$ and $\hat{V}$ are estimates of D_B, D_A, G and V, respectively.

4-1-2.Adaptive Law for Guiding Control

Let

$$\dot{\hat{D}}_A = -\Gamma_3(G_D^{-1}S)E^T \qquad (4\text{-}7)$$

$$\dot{\hat{D}}_B = \Gamma_4(G_D^{-1}S)(-KG_D^{-1}S - G_D^{-1}G_P\dot{E})^T \qquad (4\text{-}8)$$

$$\dot{\hat{G}} = \Gamma_5(G_D^{-1}S) \qquad (4\text{-}9)$$

$$\dot{\hat{V}} = \Gamma_6(G_D^{-1}S) \qquad (4\text{-}10)$$

where $\Gamma_i, i = 1 \sim 4$ are all diagonal matrixes which are properly chosen. Integrate those equations above, we can get $\hat{D}_B$, $\hat{D}_A$, $\hat{G}$ and $\hat{V}$. Finally, by means of Eq.(4-6), we can derive the input command and the guidance task is achieved.

4-2-1.Positioning in the Y-direction

Also, an adaptive controller is proposed here for positioning. From Eq.(3-7), we derive

$$m\ddot{Y} = u_6 - f(Y) \qquad (4\text{-}11)$$

where

$$f(Y) = \begin{cases} F_0\left(Y - \dfrac{YL+YU}{2}\right)^2 & for \quad Y \ge \dfrac{YL+YU}{2} \\ -F_0\left(Y - \dfrac{YL+YU}{2}\right)^2 & for \quad Y < \dfrac{YL+YU}{2} \end{cases} \qquad (4\text{-}12)$$

$$u_6 = K_{vc} I_{vc} \qquad (4\text{-}13)$$

Fig.4-1: Motion along y-axis

where ***YL*** and ***YU*** are boundaries of travel range. Eq.(4-13) is the propulsion force generated by the voice coil motor, and it's relevant to current gain K_{vc} and the input current, I_{vc}.

Furthermore, Y_d is the desired position and v is the velocity of the platform in the Y-direction. We define:

$$v_e = v \qquad (4\text{-}14)$$

$$Y_e = Y_d - Y \qquad (4\text{-}15)$$

Now, we are going to regulate $Y \to Y_d$ and $v \to 0$, i.e., $Y_e \to 0$ and $v_e \to 0$. First, we define that:

$$S_6 = v_e + \lambda Y_e$$

where λ is constant and properly chosen. Choose a Lyapunov function candidate V_6 as:

$$V_6 = \frac{1}{2} S_6^T S_6 \tag{4-16}$$

If we can let $\dot{S}_6 = -K_v S_6$,where $K_v > 0$, then the derivative of V_6 can be rewritten as:

$$\dot{V}_6 = -K_v S_6^T S_6 \le 0 \tag{4-17}$$

which implies that S_6 converges to zero exponentially in t by use of Lyapunov theory. In turn, $Y_e \to 0$ and $v_e \to 0$. Just like the previous procedure, we derived the necessary input condition as follows:

$$u_6 = f(Y) - m\lambda\dot{Y} - mK_v\dot{Y} - mK_v[\dot{Y} + \lambda(Y_d - Y)] \tag{4-18}$$

Similarly, the condition of acquiring all the coefficients fails to hold. Thus, we modify the control input as:

$$u_6 = \hat{f}(Y) - \hat{m}\lambda\dot{Y} - \hat{m}K_v\dot{Y} - \hat{m}K_v[\dot{Y} + \lambda(Y_d - Y)] \tag{4-19}$$

where $\hat{f}(Y)$ and $\hat{m}$ are estimates of $f(Y)$ and m, respectively. After derivations, the estimates of these two terms be got as in the following:

4-2-2. Adaptive Law for Positioning Control

$$\dot{\hat{m}} = \gamma_1 S_6[-K_6\dot{Y} - K_6(Y_d - Y) - \lambda\dot{Y}] \tag{4-20}$$

$$\dot{\hat{f}}(Y) = -\gamma_2 S_6 \tag{4-21}$$

where $S_6 = v_e + \lambda Y_e = \dot{Y} + \lambda(Y_d - Y)$, and $\gamma_i > 0, \forall i = 1 \sim 2$, which are properly chosen. Integrate those equations above, we can get these two estimates and derive the input command. And the guidance task is accomplished as consequence.

4-3. Stability Analysis

We will prove the asymptotic convergence of the state and its time derivative, i.e., they approach to 0 as t tends to infinity by Lyapunov stability theory.

From the following two equations :

$$\dot{V} = -KS^T S \le 0 \tag{4-22}$$

$$\dot{V}_6 = -K_v S_6^T S_6 \le 0 \tag{4-23}$$

It shows that V is a suitable Lyapunov function, and, by Lyapunov stability criteria, we conclude that S, $\tilde{D}_A$, $\tilde{D}_B$ and $\tilde{V}$ along with S_6, $\tilde{m}$, and $\tilde{f}(Y)$ parameters are all bounded, $S, S_6 \in L_2$ and in turn $\dot{S}, \dot{S}_6 \in L_\infty$.

Thus, by using Barbalat's Lemma, we finally have that S and S_6 asymptotically stable. From the definition, we can further conclude every entry in $E = [X \;\; \theta \;\; Z \;\; \phi \;\; \psi]^T$, v_e and Y_e and their time derivative asymptotically stable. Finally, they approach to zero in finite time. This means the guidance and positioning tasks are all achieved successfully.

5.Simulation Results

In this section, we will give some simulation results in accordance with the model and controller we have derived so far. Table 5-1. shows the specification of the free body:

Mass		0.3 Kg
Moment of Inertia	I_x	0.025 $Kg \cdot m^2$
	I_y	0.015 $Kg \cdot m^2$
	I_z	0.015 $Kg \cdot m^2$
Gap	lateral	1.5 mm
	vertical	8 mm
Levitation PM (NdFeB) Dipole moment		0.6 $A \cdot m^2$

Table 5-1 Specification of the free body(carrier)

The six states are Y, θ, X, Z, ϕ and ψ. X and Z denote the translation displacement between the current position and the equilibrium point in the X-direction and the Z-direction. θ, ϕ and ψ stand for angular displacement of yaw angle, roll angle and pitch angle, respectively.

First, we will show the simulation result with initial conditions of the largest transitional and rotational error in the first simulation. This situation is encountered before the carrier is levitated from rest. After turning on the controller, the carrier is forced to the equilibrium point in 0.4 second. Fig.5-1 also shows the transient response of the largest transitional and rotational displacement error in the first place.

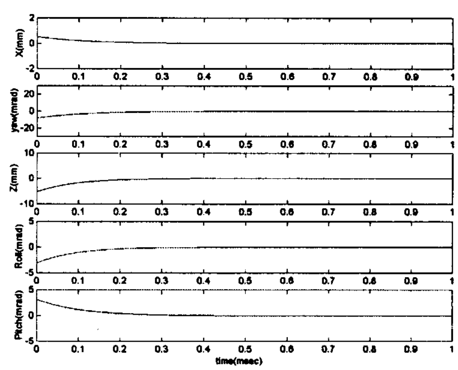

Fig.5-1 : The six DOF of carrier with maximum lateral translation and rotational initial condition.

The second simulation tests the capability of disturbance rejection. The disturbance is applied on the carrier after the carrier coming to steady-state first time. Fig.5-2 shows that it's robust to this kind of disturbance and go back to the equilibrium point in 0.4 second.

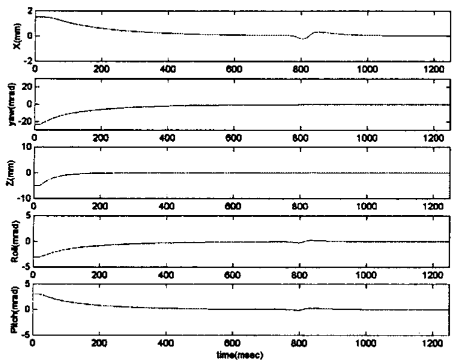

Fig.5-2 : The six DOF of upper track carrier with maximum lateral translation initial condition

Referring to these two simulation results above, the adaptive controller indeed ensure the system performance.

When it comes to position the carrier to the desired position, an input current is applied into the modified voice coil motor as control command, but introduce disturbances in the same time. In the third simulation, we command the carrier to travel from 3cm to 11cm on Y-axis, the longest permitting traveling range referring to our mechanical design. The result is shown as Fig.5-3. The flatform reaches the desired position in 2 seconds.

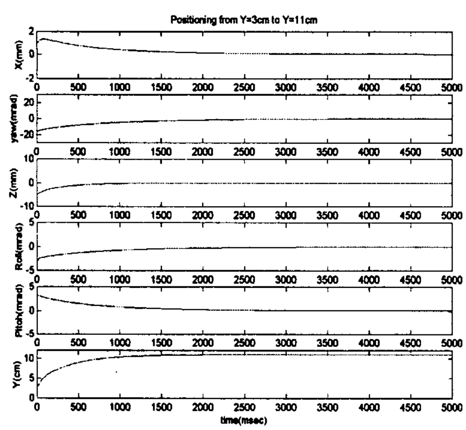

Fig.5-3 : Positioning the carrier from 3cm to 11cm

Referring to these simulations above, not only positioning task is achieved, but the errors of states in other five degrees of freedom are also guaranteed. The adaptive controller accomplish the goal of guidance and positioning.

6.Conclusions

Throughout the research presented in this thesis, many research results relevant to magnetically levitated systems have been surveyed. A prototype maglev guiding and positioning system is constructed, and the system's stabilizing controller is well developed. Finally, extensive simulations were conducted to demonstrate the feasibility and effectiveness of the overall system.

7.Reference

[1]. R. L. Hollis, and S. E. Salcudean, "On the Design and Control of Magnetically Levitated Robot Wrists, Robotics and Manufacturing, recent trends in research, education and application". *Proc. 3Rd Int. Symp. Robotis and Manufact*, vol. 3. ASME Press, New York, 1990

[2]. R. L. Hollis, and S. E. Salcudeam, "Six Degree of Freedom Magnetically Levitated Variable Compliance Fine Motion Wrist", *4th Int. Symp. On Robotics Research. Santa Cruz Ca. Aug. 1987, in Robotis Research*, vol. 4, MIT Press.

[3]. B. V. Jayawant, P. K. Sinha, and D. G. Aylwin, "Feedback Control System for D.C. Electromagnets in Passenger-Carrying Vehicles", *Int. J. Control*, vol. 24, No. 5, pp. 627-639, 1976

[4]. M. Proise, et al.:"System Concept Definition of the Grumman Superconducting Electromagnetic Suspension (EMS) Maglev Design", *Maglev '93 Conf. Argonne National Laboratory*, May 19-21,1993

[5]. E. E. Covert, M. Vlajinac, T. Stephens, and M. Finston,"Magnetic Balence and Suspension Systems for Use with Wind Tunnels", *Progress in Aerospace Science*, vol. 14., ed. D, Kuchemann, pp.27-107, Pergamon Press, 1973.

[6]. N. Kosuke, and I. Masashi, "A Noncontact Permanent Magnet Levitation Table with Electromagnetic Control and Its Vibration Isolation Method Using Direct Disturbance Cancellation Combining Optimal Regulators", *IEEE Tran. on Magnetics* ,vol. 31, no. 1, Jan 1995.

[7] I. Y. Wang, *A Magnetic Levitation Silicon Wafer Transport System*. Ph.D. Thesis, The University of Texas at Austin, 1993.

[8]. M. Y. Chen, M. J. Wang and L.C. Fu, "Modeling and Controller Design of a Maglev Guiding System for Application in Precision Positioning", *American Control Conference*,1999