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Adaptive Nonlinear Control for Manipulators with Joint Elasticity

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Abstract

The problem of designing adaptive control scheme for a flexible-joint manipulator is investigated and solved in this paper. A nonlinear model-based control scheme for the tracking of link variables is presented. Moreover, to circumvent the performance degradation due to parametric uncertainty, an adaptive control scheme is further proposed. In the developed control laws, only the position and velocity information of the actuators and links are utilized. Numerical simulations are also given to show the effectiveness of the proposed control laws.

1 Introduction

Traditionally, robots were modeled by chains of rigid links with rigid transmission units in order to simplify dynamics analysis and the subsequent controller design. In other words, the effects of flexibility of the mechanical structure of the manipulator were frequently observed in practice, but have been virtually ignored in the design of their controllers. Many of today's robots (typically with semi-rigid joints) are driven by actuators with high gear ratios, such as harmonic drives for high torque and/or low operation speed. Although the joint flexibility has demonstrated some potential merits, the difficulty with controlling such a flexible mechanical system with high performance made most robot designers prefer to manufacture mechanically rigid arms with stiff joints. Hence, in this paper, we will tackle the problem of controlling for flexible-joint robots and propose a complete solution to it.

Computed torque control (or static feedback linearization schemes) has been extensively applied to the control of rigidly jointed robots. Considerable numbers of researchers have also shown that static feedback linearization schemes can be applied to flexible-joint robots. ([1]-[2]) Some others [3], [4], have also successfully applied dynamic feedback linearization techniques to controlling robots with flexible joints. As both static and dynamic feedback linearization techniques require the exact knowledge of the robot parameters, robustness study of all feedback linearization techniques are necessary. In addition, such kind of control technology normally requires the use of joint acceleration and/or jerk feedback which may be impractical from the implementation point of view. Even if not impossible, it would be very difficult. To circumvent the difficulty of obtaining acceleration and jerk signals, Bortoff *et al* [5] have considered the design of an observer to estimate the link jerk and acceleration, however, stability of the overall control system when such observers are used has not been investigated.

Contributions in the area of adaptive control for flexible joint robots include [6]-[9]. Chen and Fu [6] used a concept of two-stage design for devising an adaptive control scheme in which the link acceleration was still assumed to be available. Ahmad and Mrad [7] also developed an adaptive strategy which used only the link and motor variables, but this methodology contained a switching mechanism that would cause the chattering phenomenon. Lozano and Brogliato [9] designed an adaptive controller based upon the Lyapunov analysis, but the controller design required the prior knowledge on the upper bounds of system parameters to insure asymptotical stability of the overall system.

As the joint stiffness is relatively high compared to the actuator inertia and joint inertia, the singular perturbation model of the flexible joint dynamics may be taken for controller design ([8], [10]-[11])

In that formulation, the control torque is a composite of a fast control term and another slow control term. The fast control torque is used to stabilize the fast subsystem, mainly the motions of the actuators, whereas the slow control one is used to stabilize the slow subsystem, mainly the motion of the links. Several design strategies are possible in such a context , including the integral manifold approach [8], [10], adaptive control approach [8], robust control approach [7]. The stability of the overall system is established given the stability of the local individual subsystem and the fact that the joint stiffness is high enough.

Recently, [14], [15] have proposed an observer-based set-point control for robots with flexible joints. In these schemes, only the measurements on motor positions, and the information on gravity terms and joint stiffness matrix are needed. However, they can not be applied to the trajectory tracking problems.

In this paper, we present two nonlinear Lyapunov-based schemes for the trajectory tracking control of flexible-joint robots. It is not required to measure the acceleration and jerk of links in the developed approaches. The first scheme is designed under the assumption that all the system parameters are known *a prior*, and the asymptotical stability of the overall system can be concluded by the Lyapunov theory. In the design procedure of the second scheme, no knowledge of system parameters is required but the structural information of the model is assumed available. The asymptotical stability can also be achieved when the system undergoes this adaptive control actuation.

This paper is organized as follows : Section 1 gives the brief introduction to the flexible-joint robots and some related works in this area. Section 2 presents the dynamic model of flexible-joint robots under some fundamental assumptions. In section 3, a nonlinear control approach is proposed and the corresponding stability results are also analyzed. Besides, an adaptive version of the control scheme derived in Section 3 is also presented in Section 4. Section 5 shows the computer simulation for the both cases, in which the tracking performance has demonstrated the effectiveness of the proposed controllers. Finally, some conclusions are given in section 6.

2 Dynamic Model

The robot manipulator considered here is composed of a serial chain of rigid links with elasticity lumped at each joint, and can be derived as follows:

$$
\begin{aligned}
M(q_l)\ddot{q}_l + C(q_l,\dot{q}_l)\dot{q}_l + G(q_l) &= K(q_m - q_l) \quad (1)\\
J\ddot{q}_m + K(q_m - q_l) &= \tau \quad (2)
\end{aligned}
$$

The dynamic model derived above possesses some well-known properties which may be useful for the controller design and the corresponding stability proof.

(P1) The inertia matrix $M(q_l)$ is positive definite for all $q_l \in R^n$.

(P2) There exists a parameterization $C(q_l, \dot{q}_l)$ such that

$$x^T(\dot{M} - 2C)x = 0, \ \forall x \in R^n,$$

which is the well-known passivity property of mechanical systems [22].

(P3) All the system parameters can be arranged so that the so-called linear-in-parameter property is satisfied, namely,

$$M(q_l)\ddot{u} + C(q_l,\dot{q}_l)\dot{u} + G(q_l) = \bar{W}(\dot{u},\ddot{u},q_l,\dot{q}_l)\theta$$

where θ is a constant parameter vector and $\bar{W}(\cdot)$ is a matrix function independent of system parameters.

3 Nonadaptive Controller Design

In this section, we will develop a nonlinear model-based controller for the flexible-joint manipulator whose parameters are assumed to be known *a prior*.

Consider the dynamic equations (1) and (2), and let the control objective be to force the link angles to track some prespecified trajectories, i.e.,

$$q_l(t) \rightarrow q_{ld}(t) \qquad as \qquad t \rightarrow \infty,$$

where the desired trajectory $q_{ld}(t)$ and its time derivatives $q_{ld}^{(i)}(t)$, $i = 1,2,3,4$, are bounded time functions. As the link variables track their desired trajectories, i.e., $q_l \rightarrow q_{ld}$, $\dot{q}_l \rightarrow \dot{q}_{ld}$, $\ddot{q}_l \rightarrow \ddot{q}_{ld}$, as $t \rightarrow \infty$, the motor angles should also track the desired motor trajectories q_{md}, i.e., $q_m(t) \rightarrow q_{md}(t)$, as $t \rightarrow \infty$, where $q_{md}(t)$ can be obtained as follows :

$$
\begin{aligned}
q_{md}(t) &= K^{-1}[M(q_{ld}(t))\ddot{q}_{ld}(t) + C(q_{ld}(t),\dot{q}_{ld}(t))\\
&\quad \dot{q}_{ld}(t) + G(q_{ld}(t))] + q_{ld}(t)\\
&= K^{-1}[(W(q_{ld}(t),\dot{q}_{ld}(t),\ddot{q}_{ld}(t))\theta] + q_{ld}(t)\\
&\equiv K^{-1}W_d\theta + q_{ld} \quad (3)
\end{aligned}
$$

with

$$W_d = W(q_{ld},\dot{q}_{ld},\ddot{q}_{ld})$$

In the sequel, the nonlinear model-based controller will be designed for the flexible-joint robot system. First, some auxiliary signals must be defined, i.e.,

$$
\begin{aligned}
s_l &= (\dot{q}_l - \dot{q}_{ld}) + \Lambda_l(q_l - q_{ld}) = \dot{e}_l + \Lambda_l e_l \quad (4)\\
s_m &= (\dot{q}_m - \dot{q}_{md}) + \Lambda_m(q_m - q_{md}) = \dot{e}_m + \Lambda_m e_m \quad (5)
\end{aligned}
$$

where $e_l = q_l - q_{ld}$, $e_m = q_m - q_{md}$, and Λ_l, Λ_m are their corresponding constant gain matrices which are designed to be positive definite. The meaning of $s_l = 0$ and $s_m = 0$ can be referred to as the sliding surfaces which are well-known in the context of variable structure system [12], i.e., as the solution trajectories reach their associated sliding surfaces, they will slide along the surface and approach to the origin. Under this definition, the original dynamics (1), (2) can be reformulated as

$$
\begin{aligned}
M(q_l)\dot{s}_l + C(q_l,\dot{q}_l)s_l &= W_l(\dot{q}_{rl},\ddot{q}_{rl},q_l,\dot{q}_l)\theta + K(q_m - q_l)\\
J\dot{s}_m &= J(-\ddot{q}_{md} + \Lambda_m\dot{e}_m) - K(q_m - q_l)\\
&\quad + \tau, \quad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
W_l(\dot{q}_{rl},\ddot{q}_{rl},q_l,\dot{q}_l)\theta &= M(q_l)\ddot{q}_{rl} + C(q_l,\dot{q}_l)\dot{q}_{rl} - G(q_l)\\
\dot{q}_{rl} &= -\dot{q}_{ld} + \Lambda_l e_l\\
\ddot{q}_{rl} &= -\ddot{q}_{ld} + \Lambda_l \dot{e}_l
\end{aligned}
$$

Note that the definition of W_l when u is identified with q_{rl} is slightly different from that of $\bar{W}$ originally defined in (P3) for the opposite sign before the vector $G(q_l)$. Now, let the proposed nonadaptive control law be designed as:

$$\tau = -\tau_m - \frac{(1+k)s_m}{\|s_m\|^2 + \epsilon}\tau_l \quad (7)$$

with τ_m and τ_l being respectively defined as

$$
\begin{aligned}
\tau_m &= J(-\ddot{q}_{md} + \Lambda_m\dot{e}_m) + K_m s_m + K(q_m - q_l)\\
\tau_l &= s_l^T W_l(\dot{q}_{rl},\ddot{q}_{rl},q_l,\dot{q}_l)\theta + s_l^T K_l s_l - s_l^T K(q_m - q_l)
\end{aligned}
$$

and k satisfying

$$\dot{k} = \begin{cases} \frac{1}{k}\left(\frac{k\|s_m\|^2-\epsilon}{\|s_m\|^2+\epsilon}\right)\tau_l & , k \neq 0 \\ \delta & , k = 0 \end{cases} \quad (8)$$

where K_m, K_l are some positive definite matrices, ϵ is some small positive constant, and δ is a nonzero constant. Then, we can guarantee the asymptotical stability of s_m, s_l, and the following theorem will give the detailed statement.

Theorem 1 *Consider the dynamical systems defined by equations (1)-(2). If the nonadaptive control law (7) is applied to such a system, then all signals in the closed-loop system will remain bounded and the tracking errors e_l of the links converge to zero asymptotically.*

Proof:

Let the Lyapunov function be chosen as

$$V(s_l, s_m, k) = \frac{1}{2}s_l^T M(q_l)s_l + \frac{1}{2}s_m^T J s_m + \frac{1}{2}k^2$$

and, then, take the time derivative of V along the dynamics (6)-(8) to obtain

$$\dot{V} = \frac{d}{dt}V = -s_m^T K_m s_m - s_l^T K_l s_l \leq 0, \quad when \quad k \neq 0$$

Since $V(t)$ is a continuous function of k, $V(t)$ is nonincreasing in t, which along with the fact that $M(\cdot)$ is always bounded and $M(\cdot)$, J are uniformly positive definite matrices implies that all signals s_m, s_l, and k in the closed-loop system are bounded. This, in turn, implies boundedness of q_l, $\dot{q}_l$, q_m, $\dot{q}_m$, and τ. So far, this proves the uniform boundedness of all signals in the closed-loop system as claimed in the theorem.

On the other hand, since K_l and K_m are positive definite matrices, we can conclude from the above that s_m and s_l belong to L_2-space. Combining this fact and the fact that $\dot{s}_m$ and $\dot{s}_l$ are bounded from equation (6), we can directly infer that s_m, s_l converge to zero asymptotically via Barbalat's Lemma [12]. Finally, from stable filter theory [12], we have e_l and e_m both converge to zero asymptotically. This conclude our proof.

Q.E.D.

4 Adaptive Controller Design

So far, we have assumed that the parameters of the manipulator system are known *a priori*, but there may exist some uncertainties in a general system, in which case the nonlinear model-based control law proposed above will fail to exactly cancel the system nonlinearity so that the controlled performance may deteriorate or even become unstable. Hence, in this section, an adaptive control law will be devised to cope with the parametric uncertainties.

Define the " virtual " estimates of these variables as

$$\begin{aligned} \hat{q}_{md} &= \frac{1}{r}W_d\hat{\theta} + q_{ld} \\ \dot{\hat{q}}_{md} &= \frac{1}{r}W_{da}\hat{\theta} + \dot{q}_{ld} \\ \ddot{\hat{q}}_{md} &= \frac{1}{r}W_{db}\hat{\theta} + \ddot{q}_{ld}, \end{aligned}$$

respectively, where r is some large positive constant replacing the stiffness level of the joints. Accordingly, from the equations, we can write

$$\begin{aligned} s_l &= (\dot{q}_l - \dot{q}_{ld}) + \Lambda_l(q_l - q_{ld}) \\ \hat{s}_m &= (\dot{q}_m - \dot{\hat{q}}_{md}) + \Lambda_m(q_m - \hat{q}_{md}) \\ &= (\dot{q}_m + \Lambda_m q_m) - \frac{1}{r}(W_{da} + \Lambda_m W_d)\hat{\theta} - (\dot{q}_{ld} + \Lambda_m q_{ld}) \end{aligned}$$

Similarly, $\hat{s}_m$ also denotes the " virtual " estimate of s_m. Under these definitions, the original dynamics (1), (2) can be rewritten as

$$\begin{aligned} M(q_l)\dot{s}_l + C(q_l, \dot{q}_l)s_l &= W_l(\dot{q}_{rl}, \ddot{q}_{rl}, q_l, \dot{q}_l)\theta \\ &\quad + K(q_m - q_l) \qquad (9) \\ J\dot{\hat{s}}_m + K(q_m - q_l) &= J\left[\Lambda_m\dot{q}_m - (\ddot{q}_{ld} + \Lambda_m\dot{q}_{ld})\right. \\ &\quad - \frac{1}{r}\frac{d}{dt}(W_{da} + \Lambda_m W_d)\hat{\theta} \\ &\quad \left.- \frac{1}{r}(W_{da} + \Lambda_m W_d)\dot{\hat{\theta}}\right] + \tau \\ &= J \cdot h_m + \tau \qquad (10) \end{aligned}$$

where

$$\begin{aligned} h_m &= \Lambda_m\dot{q}_m - (\ddot{q}_{ld} + \Lambda_m\dot{q}_{ld}) \\ &\quad - \frac{1}{r}\frac{d}{dt}(W_{da} + \Lambda_m W_d)\hat{\theta} - \frac{1}{r}(W_{da} + \Lambda_m W_d)\dot{\hat{\theta}} \\ &= [h_{m1}, h_{m2}, \cdots, h_{mn}]^T \in R^n \end{aligned}$$

and $\dot{\hat{\theta}}$ is yet to be specified. Equivalently, equations (9), (10) can also be written as :

$$\begin{aligned} M(q_l)\dot{s}_l + C(q_l, \dot{q}_l)s_l &= W_l(\dot{q}_{rl}, \ddot{q}_{rl}, q_l, \dot{q}_l)\theta + Q_{ml} \cdot K_e \\ J\dot{\hat{s}}_m + Q_{ml} \cdot K_e &= H_m \cdot J_e + \tau \end{aligned}$$

where

$$\begin{aligned} Q_{ml} &= diag[q_{m1} - q_{l1}, q_{m2} - q_{l2}, \cdots, q_{mn} - q_{ln}] \in R^{n \times n} \\ K_e &= [k_1, k_2, \cdots, k_n]^T \in R^n \\ H_m &= diag[h_{m1}, h_{m2}, \cdots, h_{mn}] \in R^{n \times n} \\ J_e &= [J_1, J_2, \cdots, J_n]^T \in R^n \end{aligned}$$

Now, let the adaptive control law be designed as :

$$\tau = -\tau_m - \frac{(1+k)\hat{s}_m}{\|\hat{s}_m\|^2 + \epsilon}\tau_l \quad (11)$$

with τ_m and τ_l being respectively defined as :

$$\begin{aligned}\tau_m &= -Q_{ml}\hat{K}_e + H_m\hat{J}_e + K_m\hat{s}_m \\ \tau_l &= s_l^T W_l(\dot{q}_{rl}, \ddot{q}_{rl}, q_l, \dot{q}_l)\hat{\theta} + s_l^T Q_{ml}\hat{K}_e + s_l^T K_l s_l\end{aligned}$$

and parameter estimates satisfying the following adaptation law :

$$\begin{aligned}\dot{\hat{K}}_e &= \Gamma_K Q_{ml}^T(s_l - \hat{s}_m) \\ \dot{\hat{J}}_e &= \Gamma_J H_m^T \hat{s}_m \\ \dot{\hat{\theta}} &= \Gamma_\theta W_l^T(\dot{q}_{rl}, \ddot{q}_{rl}, q_l, \dot{q}_l)s_l \\ \dot{k} &= \begin{cases} \frac{1}{k}\left(\frac{k\|\hat{s}_m\|^2-\epsilon}{\|\hat{s}_m\|^2+\epsilon}\right)\tau_l, & k \neq 0 \\ \delta, & k = 0\end{cases}\end{aligned}$$

where Γ_K, Γ_J and Γ_θ are some constant positive definite matrices, and ϵ and δ are defined as previously. By applying such control algorithm, all the signals in the closed-loop system will be uniformly bounded and the link tracking error will converge to zero asymptotically. This is stated in the following theorem.

Theorem 2 *Consider the manipulator dynamics described by equations (1), (2) again but subject to the assumption of the existence of parametric uncertainty. If the control law (11) is applied, then the following properties will hold.*

(1) *All signals in the closed-loop system are uniformly bounded.*

(2) $\|\hat{s}_m\| \rightarrow 0$, $\|s_l\| \rightarrow 0$ *asymptotically and, hence,* $\|e_l\|$, $\|\dot{e}_l\| \rightarrow 0$ *asymptotically.*

Proof:

(1) To establish the stability property of the closed-loop system, let the Lyapunov function be chosen as :

$$\begin{aligned}V &= \frac{1}{2}s_l^T M(q_l)s_l + \frac{1}{2}\hat{s}_m^T J\hat{s}_m + \frac{1}{2}(\hat{\theta}-\theta)^T\Gamma_\theta^{-1}(\hat{\theta}-\theta) \\ &\quad \frac{1}{2}(\hat{J}_e - J_e)^T\Gamma_J^{-1}(\hat{J}_e - J_e) \\ &\quad \frac{1}{2}(\hat{K}_e - K_e)^T\Gamma_K^{-1}(\hat{K}_e - K_e) + \frac{1}{2}k^2\end{aligned}$$

The time derivative of V along the system dynamics (9), (10) and the control law (11), we can readily obtain

$$\dot{V} = -s_l^T K_l s_l - \hat{s}_m^T K_m \hat{s}_m, \qquad k \neq 0$$

Since $\dot{V} \leq 0$ for $k \neq 0$, $t \geq t_0$, following the similar arguments as in Theorem 1 we can conclude that all signals including the input τ in the closed-loop system are uniformly bounded and, hence, prove property (1).

(2) To prove the second property of the theorem, we see that signals s_l and $\hat{s}_m$ are L_2 signals and $\dot{s}_l$, $\dot{\hat{s}}_m$ are bounded from equations (9), (10). Thus, again by Barbalat's Lemma, we can conclude that $\|s_l\|$, $\|\hat{s}_m\| \rightarrow 0$ as $t \rightarrow \infty$, which completes the proof.

Q.E.D.

5 Computer Simulation

In this section, the performance of the proposed controller, as it is applied to a two-link flexible-joint robots, will be demonstrated by several simulation examples. The flexible-joint robot used for simulation consists of two rigid uniform beams making up the links, and each link is directly actuated by a D.C. electrical motor. The joints connecting the actuators to the links are assumed flexible, and are modeled by linear torsional springs. In addition, we assume that the load carried by the end-effector is a part of the last link. In order to analyze the controlled performance, we let the flexible-joint robot track a trajectory described by

$$\begin{aligned}q_{l1d}(t) &= \frac{180}{\pi}(1 - 0.5sin(t))\ (degree) \\ q_{l2d}(t) &= \frac{180}{\pi}(1.2 - 0.4sin(t))\ (degree)\end{aligned}$$

Case I-Nonadaptive Control : In this case, all the system parameters are assumed to be known *a priori*, and the controller gains are selected as : $\Gamma_l = 2I_{2\times2} \in R^{2\times2}, \Gamma_m = 3I_{3\times3} \in R^{3\times3}, r = 150, \epsilon = 0.5$

The response of the joint tracking errors are shown in Fig.(1)-Fig.(2).

Case II-Adaptive Control : In this case, we apply the proposed adaptive control scheme to the same dynamical model without the prior knowledge of the model parameters. The controller gains are chosen the same as before. In addition, the parameter adaptation gains are selected as : $\Gamma_\theta = 0.5I_{5\times5} \in R^{5\times5}, \Gamma_J = 0.1I_{2\times2} \in R^{2\times2}, \Gamma_K = 0.1I_{2\times2} \in R^{2\times2}$,
where $I_{i\times i}$ is the identity matrix with dimension i. Similarly, the response of the joint tracking errors are shown in Fig.(3)-Fig.(4).

6 Conclusion

In this paper, we have presented the design of a nonlinear model-based control scheme and an adaptive nonlinear control scheme for the manipulators with compliant joints by using only the positions and velocities of motors and links. The control algorithms consist of two major parts, namely, motor part τ_m and link part τ_l. The motor part contributes to the asymptotical tracking of motor trajectories while the link part controlling the link dynamics in a desired manner. Asymptotical stability of the overall system has also been guaranteed via Lyapunov analysis. Moreover, in the adaptive nonlinear control scheme, a parameter updated law is incorporated to circumvent the performance degradation due to parametric uncertainty. Numerical simulation were also provided to demonstrate the effectiveness of the proposed controllers.

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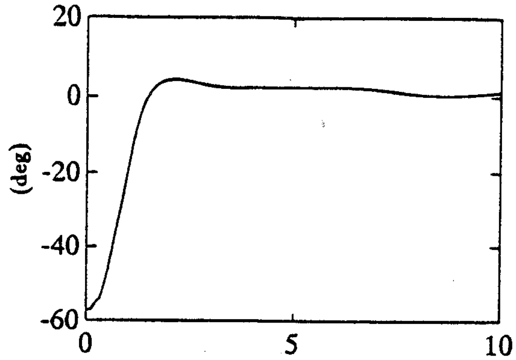

Fig.(1) Position tracking error of link 1

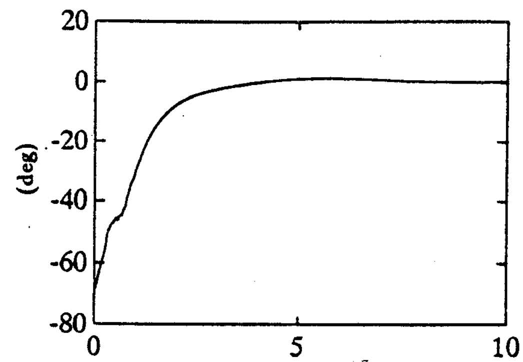

Fig.(2) Position tracking error of link 2

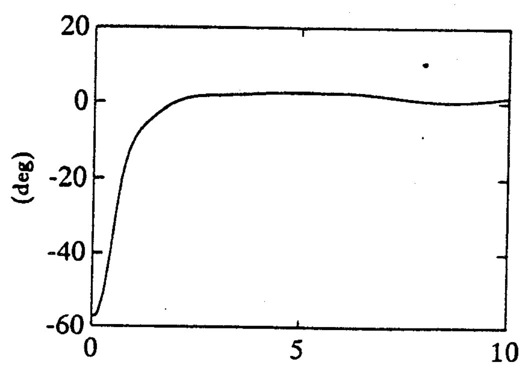

Fig.(3) Position tracking error of link 1

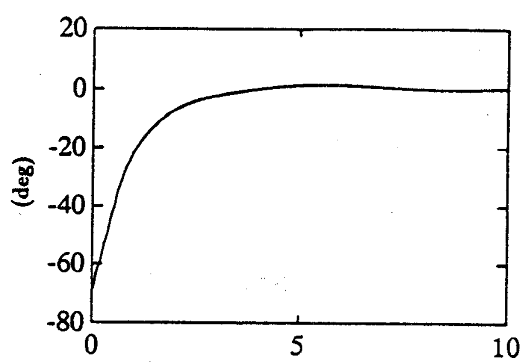

Fig.(4) Position tracking error of link 2