Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

NEURO-FUZZY-BASED DIRECT ADAPTIVE CONTROLLER DESIGN FOR A CLASS OF UNCERTAIN MULTIVARIABLE NONLINEAR SYSTEMS

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ABSTRACT This paper develops an approach for designing a direct adaptive MIMO fuzzy logic controller to overcome the interaction among the subsystems and facilitate robust properties. The proposed adaptive fuzzy controller requires no knowledge of the controlled nonlinear system. By employing fuzzy descriptions to the input applied to one subsystem affecting the other subsystem and using the Lyapunov stability theory, the overall adaptation scheme has been proved to be able to guarantee the tracking error residual set being uniform ultimate bounded. The bounds of the fuzzy modeling error are estimated adaptively using an estimation algorithm and the global asymptotic stability of the algorithm is established via H^∞ tracking performance index. Simulation results of a two-dimensional inverted pendulum confirm that the effect of both the fuzzy approximation error and external disturbance on the tracking error can be attenuated efficiently by the proposed method.

1. INTRODUCTION

Control of multivariable nonlinear systems has emerged as an exciting research area because of its widespread applications. Previous algorithms for solving the highly coupled nonlinear dynamic control problem require accurate mathematical model of the plant dynamics [1,2]. However there exists inevitable unmodelled nonlinearities and uncertain disturbance in their constructed model where conventional control strategies cannot be easily derived. Fuzzy logic and neural network based control has emerged to be practical and successful alternative in the control of complex or ill-defined systems [3]. Wang [4] has developed an important adaptive fuzzy control system to incorporate with the expert information systematically and has shown the stability of adaptive control algorithms. However, in the direct adaptive fuzzy control of MIMO (multi-input multi-output) nonlinear systems, the major problems are how to construct suitable fuzzy control rules and deal with the unknown coupling among the subsystems.

This paper will address the problem of controlling an unknown MIMO nonlinear affine system subject to unmodeled dynamics, bounded exogenous disturbances and measurement noise. The development of the direct adaptive fuzzy controller involves the design of a decoupling network and the cascaded multilayer fuzzy systems. The fuzzy rules are constructed in the form of "IF *situation* THEN *the control input*" [5,6]. By employing the fuzzy descriptions to the input applied to one subsystem affecting the other subsystem, the overall adaptation scheme has been proved to be able to guarantee the tracking error residual set to be uniform ultimate bounded through Lyapunov stability theory. Thus adjusting the parameters of the controller and the estimated parameters representing the true of the plant parameters can create the control rules. The control objective is obtained by tailoring a nominal adaptation process of weights to supply H^∞ control design with more intelligent and fuzzy control design with better performance. Finally, we evaluate the performance of the proposed controller by considering the model following control problem of a two-dimensional inverted pendulum.

2. PROBLEM FORMULATION

Consider an MIMO nonlinear system governed by

$$y^{(r)} = f(x) + G(x)u + d(x,t) \tag{1}$$

where $y=[y_1,\cdots,y_m]^T$ and $y^{(r)} \equiv [y_1^{(r_1)},\cdots,y_m^{(r_m)}]^T$ denote the output vector and its derivative, respectively, $r=[r_1,\cdots,r_m]$ with $\sum_{i=1}^m r_i = n$ is defined as the system relative degree, $u=[u_1,\cdots,u_m]^T$ is the system input, $x=[x_1,\cdots,x_n]^T=[y_1,\cdots,y_1^{(r_1-1)},y_2,\cdots,y_m,\cdots,y_m^{(r_m-1)}]^T$ is the state vector, $f(x)=[f_1(x),\cdots,f_m(x)]^T$, $G(x)=[g_1(x),\cdots,g_m(x)]$, $f_i(x)$ and $g_i(x)=[g_{i1}(x),\cdots,g_{im}(x)]^T$ are unknown smooth functions, $i=1,\cdots,m$, and $d(x,t)=[d_1(x,t),\cdots,d_m(x,t)]^T$ is the disturbance with the properties of standard smoothness and boundedness.

Let $\dot{y}_{di}=(y_{di},\dot{y}_{di},\cdots,y_{di}^{(r_i-1)})^T$ represents the known desired trajectories for $y_{di}(t)$, $i=1,\cdots,m$. Define the tracking error as $\underline{e}=[e_1^T,\cdots,e_m^T]^T \in R^n$ with

$$\begin{aligned} e_i &= [y_{di}-y_i,\dot{y}_{di}-\dot{y}_i,\cdots,y_{di}^{(r_i-1)}-y_i^{(r_i-1)}]^T \\ &= [e_i,\dot{e}_i,\cdots,e_i^{(r_i-1)}]^T \end{aligned} \tag{2}$$

and $\underline{\Lambda} = Block\ diag[\Lambda_1,\cdots,\Lambda_m]$ with $\Lambda_i=[\alpha_{i1},\cdots,\alpha_{ir_i}]^T \in R^{r_i}$ be such that all roots of the polynomial

$$h_i(s) = s^{(r_i)} + \alpha_{i1}s^{(r_i-1)} + \cdots + \alpha_{i(r_i-1)}\dot{s} + \alpha_{ir_i} \tag{3}$$

are in the open left-half plane. According to (1), if f and G are known, the optimal control is

$$u^* = G^{-1}(x)[-f(x) + y_d^{(r)} + \underline{\Lambda}^T \underline{e}]. \tag{4}$$

After some manipulations through (1) and (4), we obtain the tracking error dynamic equation

$$\dot{\underline{e}} = A_{n\times n}\cdot\underline{e} - B_{n\times m}\cdot[G(x)\cdot(u-u^*)+d] \tag{5}$$

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where $A = Block\ diag[A_1, A_2, \cdots, A_m]$, $B = Block\ diag[B_1, B_2, \cdots, B_m]$, and

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_{ir_i} & -\alpha_{i(r_i-1)} & \cdots & -\alpha_{i1} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

The aim of the control is to make each subsystem of the composite nonlinear system described by (1) asymptotically match a linear reference model of the form (3) in the presence of bounded disturbance $d(x,t)$. That is to determine a controller for the plant (1) so that the tracking error $\hat{e} = [e_1, e_2, \cdots, e_m]^T$ will be attenuated to an arbitrarily small residual tracking error set.

Conceptually, the control strategy consists of two parts. Firstly, the equivalent control $\hat{u}_{eq}$ is estimated by using direct adaptive fuzzy controller, and the parameters of the controller are directly adjusted to reduce some norm of the output error between the plant and the reference model toward a desired dynamics. Secondly, due to the existence of fuzzy approximation errors and external disturbances, simply an equivalent control term cannot ensure the stability of the closed-loop system. Therefore it is necessary to preserve a robust compensator to deal with the equivalent uncertainty. Thus the control law can be represented as

$$u = \hat{u}_{eq} + \hat{u}_r \tag{6}$$

where $\hat{u}_{eq}$ and $\hat{u}_r$ are, respectively, yielded through fuzzy and non-fuzzy design manner.

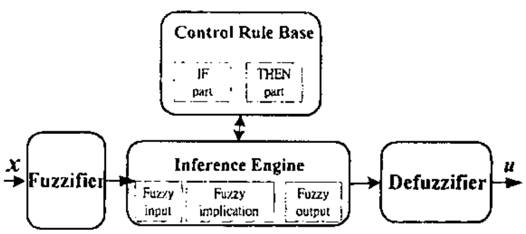

Fig. 1 The basic configuration of the fuzzy logic system.

3. DESCRIPTION OF THE FUZZY LOGIC SYSTEMS

Various fuzzy models and their control have been successfully applied in many fields [7,8]. The fuzzy logic systems can be constructed from the fuzzy IF-THEN rules using some specific fuzzification, inference and defuzzification strategies [4]. The basic configuration of the fuzzy logic system is shown in Fig. 1. The fuzzy control rules are the principal factor to determine the performance of a fuzzy controller. Thus linguistic information from human experts can be directly incorporated into controllers.

The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R^m$. Let $U = U_1 \times \cdots \times U_n$ and $V = V_1 \times \cdots \times V_m$ where $U_k \subset R$, $k = 1,2,\cdots,n$ and $V_i \subset R$, $i = 1,2,\cdots,m$, respectively. A multivariable system can be controlled by the following N linguistic rules

R^l: IF x_1 is A_1^l and $\cdots$ and x_n is A_n^l

THEN u_1 is B_1^l and $\cdots$ and u_m is B_m^l (7)

where $l = 1,\cdots,N$, x_k , $k = 1,2,\cdots,n$, are the input variables to the fuzzy system, u_i , $i = 1,2,\cdots,m$, are the output variables of fuzzy systems, and the antecedent fuzzy sets A_k^l in U_k and the consequent fuzzy sets B_i^l in V_i are linguistic terms characterized by the fuzzy membership functions $\mu_{A_k^l}(x_k)$ and $\mu_{B_i^l}(u_i)$, respectively. The fuzzy logic systems with center-average defuzzifier, product inference and singleton fuzzifier is defined as [4]

$$u_i(x) = \frac{\sum_{l=1}^{N} \mu^l(x) \cdot c_i^l}{\sum_{l=1}^{N} \mu^l(x)} \tag{8}$$

where $\mu^l(x) = \Pi_{k=1}^{n} \mu_{A_k^l}(x_k)$ is the matching degree of the lth rule, and c_i^l is the center of the consequent membership function of the lth rule. If c_i^l is chosen as the design parameter, the adaptive fuzzy system can be viewed as the type of neural network. Therefore, (8) can be rewritten as

$$u_i(x) = \phi_i^T \cdot \psi(x) \tag{9}$$

where $\phi_i = (c_i^1, \cdots, c_i^N)^T$ is a parameter vector, and $\psi(x) = (\xi_1, \cdots, \xi_N)^T$ is a regressor, and where the fuzzy basis function is defined as [4]

$$\xi_l = \frac{\Pi_{k=1}^{n} \mu_{A_k^l}(x_k)}{\sum_{l=1}^{N} \left(\Pi_{k=1}^{n} \mu_{A_k^l}(x_k) \right)} . \tag{10}$$

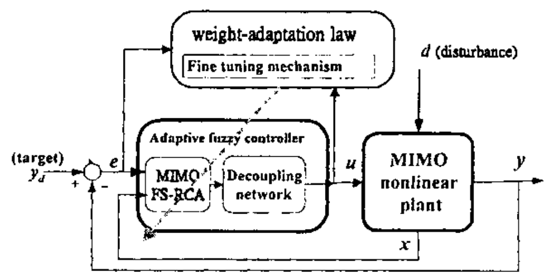

Fig. 2 The diagram of the adaptive fuzzy control system.

4. DESIGN OF THE ROBUST NEURO-FUZZY CONTROLLER

The proposed robust neuro-fuzzy controller (RNFC) is composed of the following three parts: a multi-layer fuzzy system with rule credit assignment, a fine-tuning mechanism on the consequent membership functions, and a decoupling network [9]. Fig. 2 shows the configuration of the MIMO fuzzy-set rule credit assignments (FS-RCA) and the interconnections compensating network of the robust adaptive fuzzy control system. The multi-layer fuzzy system and the decoupling network are nominal designs based on on-line approximation of the unknown nonlinear functions of the plant. The fine-tuning mechanism is designed to encounter the equivalent uncertainty resulted by the plant uncertainty, the function approximation error, or the external disturbances. Since nonlinear functions $G(x)$ and $f(x)$ of the nonlinear system (1) are unknown, we will introduce fuzzy systems, which are expressed as a series expansion of fuzzy basis functions, to model the uncertainties $G(x)$ and $f(x)$ by tuning the parameters of the corresponding fuzzy systems. According to these fuzzy models, a stable adaptive controller can be constructed to achieve the control objectives of effective tracking control architecture.

Let u_i^0 be the output of the system's ith MIMO FS-RCA and does not take the interconnections among subsystems into consideration. If the functions f and g_{ii} in (1) are known, $i=1,\cdots,m$, then the output response of the fuzzy controller becomes

$$u^0 = \hat{D}^{-1}(x)[-f(x)+y_d^{(r)}+\underline{\Lambda}^T \underline{e}] \quad (11)$$

where $\hat{D}=Diag[g_{ii}]$, $u^0=[u_1^0,\cdots,u_m^0]^T$. The proposed controller has a neural part to release the interaction among the subsystems and a fuzzy part to asymptotically cancels the non-linearity in the system. The output of the equivalent controller is combined with u^0 and its modification by decoupling network

$$u_{eq}(t)=u^0(t)+Mu^0(t) \quad (12)$$

To derive a stable weight adaptation in control matrix, the matrix M be chosen as

$$M=-(I_m+\hat{C}^{-1}\hat{D})^{-1} \quad (13)$$

where I_m denotes a $m\times m$ identity matrix and

$$\hat{C}=\begin{bmatrix} 0 & g_{12}(x) & \cdots & g_{1m}(x) \\ g_{21}(x) & 0 & \cdots & g_{2m}(x) \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}(x) & g_{m2}(x) & \cdots & 0 \end{bmatrix} \quad (14)$$

Using (11)-(14) and the Matrix Inversion Lemma [10]

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B(DA^{-1}B+C^{-1})^{-1}DA^{-1}$$

the analytical formulation of RNFC resolves into

$$u_{eq}=G^{-1}(x)[-f(x)+y_d^{(r)}+\underline{\Lambda}^T\underline{e}] \quad (15)$$

where $G=\hat{C}+\hat{D}$.

In this paper we use direct adaptive fuzzy controllers, therefore, the parameters of the controller are directly adjusted to reduce some norm of the output error between the plant and the reference model. Due to the existence of fuzzy approximation errors and external disturbances, simply an equivalent control term cannot ensure the stability of the closed-loop system. Therefore it is necessary to preserve a robust compensator to deal with them from (6). Suppose that the control u due to the direct adaptive control (DAFC) is the summation of a basic fuzzy logic system $\hat{u}(x\,|\,\underline{\phi})$ and a robust compensator ($\hat{u}_r=G^{-1}u_r$):

$$u=\hat{u}(x\,|\,\underline{\phi})-G^{-1}u_r \quad (16)$$

where $\underline{\phi}=[\varphi_1,\cdots,\varphi_m]$, $\hat{u}=(\hat{u}_1,\cdots,\hat{u}_m)^T$ with $\hat{u}_i(x\,|\,\varphi_i)=\varphi_i^T\cdot\xi_u(x)$, where $\xi_u(x)=(\xi_{u1},\cdots,\xi_{uN})^T$ is a vector of fuzzy bases, $\varphi_i=(\phi_{i1},\cdots,\phi_{iN})$ is the corresponding parameters of fuzzy logic systems, $i=1,\cdots,m$.

5. ADAPTIVE FUZZY CONTROL LAW

In direct adaptive fuzzy control (DAFC), linguistic fuzzy control rules can be directly incorporated into the controllers and the parameters of the controller are directly adjusted to reduce some norm of the output error between the plant and the reference model. As far as the adaptation of the controller parameters are concerned the input applied to one subsystem affecting the other subsystem. Thus the unknown functions $G(x)$ is estimated and the controller is chosen by assuming the estimated parameters being able to representing the true of the plant parameters. This is similar that the indirect adaptive fuzzy control (IAFC) uses the fuzzy system as approximator for the dynamic systems. The main objective in this section is to derive the proper direct adaptive fuzzy control law for the plant model whose structure is represented by exploiting the advantages of the DAFC and the IAFC into a single controller i.e. both the fuzzy control rules and the fuzzy descriptions can be incorporated into a single controller.

Firstly we determine the control as

$$u=\lambda u_D+(1-\lambda)u_I \quad (17)$$

where u_D and u_I are the control inputs contributed by the DAFC and IAFC, respectively, and $0\le\lambda\le 1$ is a weighting factor. If fuzzy control rules are more important than fuzzy descriptions, choose λ to be large; otherwise, choose λ to be small. Secondly the functions $f(x)$ and $G(x)$ of the system (1) are unknown, we replace $f(x)$ and $G(x)$ by the fuzzy logic system $\hat{f}(x\,|\,\underline{\theta})=[\hat{f}_i(x\,|\,\theta_i)]_m$ and $\hat{G}(x\,|\,\underline{w})=[\hat{g}_{ij}(x\,|\,w_{ij})]_{m\times m}$ where $\underline{\theta}=[\theta_1,\cdots,\theta_m]$, $\underline{w}=[w_{11},w_{12}\cdots,w_{1m},w_{21},\cdots,w_{mm}]$, $\hat{f}_i(x\,|\,\theta_i)=\theta_i^T\cdot\xi_f(x)$, $\hat{g}_{ij}(x\,|\,w_{ij})=w_{ij}^T\cdot\xi_g(x)$ with $\xi_f(x)=(\xi_{f1},\cdots,\xi_{fN})^T$ and $\xi_g(x)=(\xi_{g1},\cdots,\xi_{gN})^T$ are vectors of fuzzy bases, $\theta_i=(\theta_{i1},\cdots,\theta_{iN})$ and $w_{ij}=(w_{ij1},\cdots,w_{ijN})$ are the corresponding parameters of fuzzy logic systems, $i,j=1,\cdots,m$. The resulting error equation (5) due to the DAFC, given by (16), becomes

$$\underline{\dot{e}}=A\cdot\underline{e}-B\cdot[G(x)\cdot(\hat{u}_D-u_D^*)+d-u_r] \quad (18)$$

where u_D^* is the optimal control of the DAFC from (4). The resulting error dynamics (5) due to the IAFC, given by (15), becomes

$$\underline{\dot{e}} = A\cdot\underline{e} - B\cdot[(f-\hat{f}) + (G-\hat{G})u_I + d - u_r] \quad (19)$$

Since the overall controller of (17) consists of a weighted combination of DAFC and IAFC, the resulting errors is given by the weighting sum of (18) and (19) as

$$\underline{\dot{e}} = A\underline{e} + Bu_r - Bd + (1-\lambda)B[(\hat{f}-f) + (\hat{G}-G)u_I] - \lambda BG(\hat{u}_D - u_D^*) \quad (20)$$

Define the parameters $\underline{\phi}^*$, $\underline{\theta}^*$, $\underline{w}^*$ of the best function approximation to be

$$\underline{\phi}^* \equiv \arg\min_{|\underline{\phi}|\le M_{\phi_{\max}}}[\sup_{|x|\le M_{x_{\max}}} |u(x) - \hat{u}(x\,|\,\underline{\phi})|] \quad (21)$$

$$\underline{\theta}^* \equiv \arg\min_{\underline{\theta}\in\Omega_\theta}[\sup_{x\in\Omega_x} |f(x) - \hat{f}(x\,|\,\underline{\theta})|] \quad (22)$$

$$\underline{w}^* \equiv \arg\min_{\underline{w}\in\Omega_w}[\sup_{x\in\Omega_x} |G(x) - \hat{G}(x\,|\,\underline{w})|] \quad (23)$$

where Ω_θ and Ω_w are constraint sets for $\underline{\theta}$ and $\underline{w}$, respectively, defined as $\Omega_\theta = \{\underline{\theta} : |\underline{\theta}| \le M_{\theta_{\max}}\}$ and $\Omega_w = \{\underline{w} : |\underline{w}| \le M_{w_{\max}}\}$ where $M_{\theta_{\max}}$, $M_{w_{\max}}$ and $M_{\phi_{\max}}$ are specified by the designer. Applying (21)-(23) to (20), the modified error dynamic equation is

$$\begin{aligned}\underline{\dot{e}} &= A\underline{e} + (1-\lambda)B[\hat{f}(x|\underline{\theta}) - \hat{f}(x|\underline{\theta}^*) + (\hat{G}(x|\underline{w}) - \hat{G}(x|\underline{w}^*))u_I] \\ &\quad - \lambda BG(x)[(\hat{u}_D(x|\underline{\phi}) - \hat{u}_D(x|\underline{\phi}^*)] + Bu_r - Bd + B\underline{\zeta} \\ &= A\underline{e} + (1-\lambda)B\begin{bmatrix}\tilde{\theta}_1^T\xi_f + \sum_{j=1}^m \tilde{w}_{1j}^T\xi_g u_{Ij} \\ \tilde{\theta}_2^T\xi_f + \sum_{j=1}^m \tilde{w}_{2j}^T\xi_g u_{Ij} \\ \vdots \\ \tilde{\theta}_m^T\xi_f + \sum_{j=1}^m \tilde{w}_{mj}^T\xi_g u_{Ij}\end{bmatrix} - \lambda BG(x)\begin{bmatrix}\tilde{\varphi}_1^T\xi_u \\ \tilde{\varphi}_2^T\xi_u \\ \vdots \\ \tilde{\varphi}_m^T\xi_u\end{bmatrix} \\ &\quad + Bu_r + B\underline{\zeta}\end{aligned} \quad (24)$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$, $\tilde{w}_{ij} = w_{ij} - w_{ij}^*$, $\tilde{\varphi}_i = \varphi_i - \varphi_i^*$ and $\underline{\zeta} = (1-\lambda)[\hat{f}(x|\underline{\theta}^*) - f(x) + (\hat{G}(x|\underline{w}^*) - G(x))u_I] - \lambda G(x)[\hat{u}_D(x|\underline{\phi}^*) - u_D^*(x)] - d$ is the minimum approximation errors. Our design objective involves specifying the control u_r and adaptive laws for $\underline{\theta}$, $\underline{w}$, and $\underline{\phi}$ such that H^∞ tracking performance is achieved.

Theorem 1: The tracking error by (24) allows us to use the following parameter adaptation law in [4,11]

$$u_r = -R^{-1}B^T P\underline{e} \quad (25)$$

$$\dot{\varphi}_i = \alpha_i k_i^T P\underline{e}\xi_u \quad (26)$$

$$\dot{\theta}_i = -\beta_i b_i^T P\underline{e}\xi_f \quad (27)$$

$$\dot{w}_{ij} = -\gamma_{ij} b_i^T P\underline{e}\xi_g u_{Ij} \quad (28)$$

where $R = Diag[r_1, \cdots, r_m]$ with r_i, α_i, β_i and γ_{ij} are positive scalar values, $i, j = 1, \cdots, m$, $B = [b_1, \cdots, b_m]$, $BG = K = [k_1, \cdots, k_m]$, $P = P^T \ge 0$ is the solution of the following Riccati-like equation

$$PA + A^T P + Q - 2PBR^{-1}B^T P + \frac{1}{\rho^2}PBB^T P = 0 \quad (29)$$

with the design parameters $Q > 0$, $\rho > 0$, then the H^∞ tracking performance is achieved for a prescribed attenuation level ρ.

Proof: We choose the Lyapunov function candidate as

$$\begin{aligned}V &= \tfrac{1}{2}\underline{e}^T P\underline{e} + \tfrac{\lambda}{2}(\alpha_1^{-1}\tilde{\varphi}_1^T\tilde{\varphi}_1 + \cdots + \alpha_m^{-1}\tilde{\varphi}_m^T\tilde{\varphi}_m) \\ &\quad + \tfrac{(1-\lambda)}{2}[(\beta_1^{-1}\tilde{\theta}_1^T\tilde{\theta}_1 + \cdots + \beta_m^{-1}\tilde{\theta}_m^T\tilde{\theta}_m) \\ &\quad + (\textstyle\sum_{j=1}^m \gamma_{1j}^{-1}\tilde{w}_{1j}^T\tilde{w}_{1j} + \cdots + \sum_{j=1}^m \gamma_{mj}^{-1}\tilde{w}_{mj}^T\tilde{w}_{mj})]\end{aligned} \quad (30)$$

Taking the derivative of V by the fact $\dot{\tilde{\varphi}}_i = \dot{\varphi}_i$, $\dot{\tilde{\theta}}_i = \dot{\theta}_i$, $\dot{\tilde{w}}_{ij} = \dot{w}_{ij}$, (24) and (25), we obtain

$$\begin{aligned}\dot{V} &= \frac{1}{2}\underline{e}^T[A^T P + PA - 2PBR^{-1}B^T P]\underline{e} + \frac{1}{2}(\underline{\zeta}^T B^T P\underline{e} + \underline{e}^T PB\underline{\zeta}) \\ &\quad - \lambda[\tilde{\varphi}_1^T(k_1^T P\underline{e}\xi_u - \alpha_1^{-1}\dot{\tilde{\varphi}}_1) + \cdots + \tilde{\varphi}_m^T(k_m^T P\underline{e}\xi_u - \alpha_m^{-1}\dot{\tilde{\varphi}}_m)] \\ &\quad + (1-\lambda)[\tilde{\theta}_1^T(b_1^T P\underline{e}\xi_f + \beta_1^{-1}\dot{\tilde{\theta}}_1) + \cdots + \tilde{\theta}_m^T(b_m^T P\underline{e}\xi_f + \beta_m^{-1}\dot{\tilde{\theta}}_m)] \\ &\quad + (1-\lambda)[\textstyle\sum_{j=1}^m \tilde{w}_{1j}^T(b_1^T P\underline{e}\xi_g u_{Ij} + \gamma_{1j}^{-1}\dot{\tilde{w}}_{1j}) + \cdots + \sum_{j=1}^m \tilde{w}_{mj}^T(b_m^T P\underline{e}\xi_g u_{Ij} + \gamma_{mj}^{-1}\dot{\tilde{w}}_{mj})] \\ &= \frac{1}{2}\underline{e}^T[A^T P + PA - 2PBR^{-1}B^T P + \frac{1}{\rho^2}PBB^T P]\underline{e} \\ &\quad - \frac{1}{2}(\frac{1}{\rho}B^T P\underline{e} - \rho\underline{\zeta})^T(\frac{1}{\rho}B^T P\underline{e} - \rho\underline{\zeta}) + \frac{1}{2}\rho^2\underline{\zeta}^T\underline{\zeta} \\ &\quad - \lambda[\tilde{\varphi}_1^T(k_1^T P\underline{e}\xi_u - \alpha_1^{-1}\dot{\varphi}_1) + \cdots + \tilde{\varphi}_m^T(k_m^T P\underline{e}\xi_u - \alpha_m^{-1}\dot{\varphi}_m)] \\ &\quad + (1-\lambda)[\tilde{\theta}_1^T(b_1^T P\underline{e}\xi_f + \beta_1^{-1}\dot{\theta}_1) + \cdots + \tilde{\theta}_m^T(b_m^T P\underline{e}\xi_f + \beta_m^{-1}\dot{\theta}_m)] \\ &\quad + (1-\lambda)[\textstyle\sum_{j=1}^m \tilde{w}_{1j}^T(b_1^T P\underline{e}\xi_g u_{Ij} + \gamma_{1j}^{-1}\dot{w}_{1j}) + \cdots + \sum_{j=1}^m \tilde{w}_{mj}^T(b_m^T P\underline{e}\xi_g u_{Ij} + \gamma_{mj}^{-1}\dot{w}_{mj})]\end{aligned}$$

If we choose the adaptive law as $\dot{\varphi}_i = \alpha_i k_i^T P\underline{e}\xi_u$, $\dot{\theta}_i = -\beta_i b_i^T P\underline{e}\xi_f$, $\dot{w}_{ij} = -\gamma_{ij} b_i^T P\underline{e}\xi_g u_{Ij}$, $i, j = 1, \cdots, m$, and the Riccati-like equation (29), we get

$$\dot{V} \le -\frac{1}{2}\underline{e}^T Q\underline{e} + \frac{1}{2}\rho^2\underline{\zeta}^T\underline{\zeta} \quad (31)$$

We integrate (31) from $t = 0$ to $t = T$ yields

$$V(T) - V(0) \le -\frac{1}{2}\int_0^T \underline{e}^T Q\underline{e}dt + \frac{1}{2}\rho^2\int_0^T \underline{\zeta}^T\underline{\zeta}dt \quad (32)$$

From (30) and $V(T) \ge 0$, we obtain the control objective of H^∞ tracking performance index as

$$\begin{aligned}\int_0^T \underline{e}^T Q\underline{e}dt &\le \underline{e}^T(0)P\underline{e}(0) + \rho^2\int_0^T \underline{\zeta}^T\underline{\zeta}dt \\ &\quad + \lambda(\alpha_1^{-1}\tilde{\varphi}_1^T(0)\tilde{\varphi}_1(0) + \cdots + \alpha_m^{-1}\tilde{\varphi}_m^T(0)\tilde{\varphi}_m(0)) \\ &\quad + (1-\lambda)[(\beta_1^{-1}\tilde{\theta}_1^T(0)\tilde{\theta}_1(0) + \cdots + \beta_m^{-1}\tilde{\theta}_m^T(0)\tilde{\theta}_m(0)) \\ &\quad + \textstyle\sum_{j=1}^m \gamma_{1j}^{-1}\tilde{w}_{1j}^T(0)\tilde{w}_{1j}(0) + \cdots + \sum_{j=1}^m \gamma_{mj}^{-1}\tilde{w}_{mj}^T(0)\tilde{w}_{mj}(0)]\end{aligned}$$

Claim 1: The solvability of H^∞ tracking performance by parameter adaptation law (26)-(28) is on the existence of a solution $P = P^T \ge 0$ of the Riccati-like equation (29). The Riccati-like equation has a positive semi-definite solution $P = P^T \ge 0$ if and only if [12]

$$2R^{-1} - \tfrac{1}{\rho^2}I \ge 0 \text{ or } 2\rho^2 I \ge R \quad (33)$$

Therefore, for a prescribed ρ in H^∞ tracking control, the weighting factor R on robust compensator (25) must satisfy the above inequality.

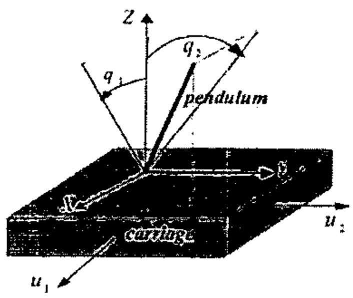

Fig. 3 A two-dimensional inverted pendulum system.

6. SIMULATION

Here, we demonstrate the geometric configuration of an inverted pendulum manipulator with 2 degrees of freedom in the rotational angles described by Euler angles q_1 and q_2 in X and Y direction [13], respectively, as shown in Fig. 3. On the assumption that the rotational angles of the two planar pendulums are small, motions of the two planar pendulums can be considered independent of each other. The external forces, u_1 and u_2, are applied to keep the pendulum at the upright position in X and Y direction, respectively, m_0 and m_l are the mass of the carriage and pendulum.

The dynamic equations describing the motion of an inverted pendulum system are derived by the Lagrange scale function $L(q,\dot{q})$ [14]. Variable q represents the states q_1 and q_2, l is the half-length of the pendulum. After some manipulation, the dynamic equations are of the following form

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \end{bmatrix}+Z(q)=N\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}+\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where

$$H_{11}=m_1 l^2(1/3+\cos^2 q_2+\sin^2 q_1 \sin^2 q_2) - (m_1^2 l^2/(m_0+m_1))(\cos^2 q_1 \cos^2 q_2+\sin^2 q_1 \sin^2 q_2)$$

$$H_{12}=H_{21}=[(2m_1^2 l^2/(m_0+m_1))-m_1 l^2]\cos q_1 \sin q_1 \cos q_2 \sin q_2$$

$$H_{22}=m_1 l^2(1/3+\sin^2 q_2+\cos^2 q_1 \cos^2 q_2) - (m_1^2 l^2/(m_0+m_1))(\sin^2 q_1 \sin^2 q_2+\cos^2 q_1 \cos^2 q_2)$$

$$\begin{aligned} C_1 &= (m_1^2 l^2/(m_0+m_1))(\dot{q}_1^2+\dot{q}_2^2)\cos q_1 \sin q_1(\cos^2 q_2-\sin^2 q_2) \\ &+ (2m_1^2 l^2/(m_0+m_1))\dot{q}_1\dot{q}_2(\cos^2 q_1-\sin^2 q_1)\cos q_2 \sin q_2 \\ &+ m_1 l^2(\dot{q}_1^2+\dot{q}_2^2)\cos q_1 \sin q_1 \sin^2 q_2 \\ &- 2m_1 l^2 \dot{q}_1\dot{q}_2 \cos^2 q_1 \cos q_2 \sin q_2 \end{aligned}$$

$$\begin{aligned} C_2 &= (m_1^2 l^2/(m_0+m_1))(\dot{q}_1^2+\dot{q}_2^2)(\cos^2 q_1-\sin^2 q_1)\cos q_2 \sin q_2 \\ &+ (2m_1^2 l^2/(m_0+m_1))\dot{q}_1\dot{q}_2 \cos q_1 \sin q_1(\cos^2 q_2-\sin^2 q_2) \\ &+ m_1 l^2(\dot{q}_1^2+\dot{q}_2^2)\sin^2 q_1 \cos q_2 \sin q_2 \\ &- 2m_1 l^2 \dot{q}_1\dot{q}_2 \cos q_1 \sin q_1 \cos^2 q_2 \end{aligned}$$

$$Z=-m_1 g l[\sin q_1 \cos q_2, \cos q_1 \sin q_2]^T$$

$$N=\frac{m_1 l}{m_0+m_1}\begin{bmatrix} \cos q_1 \cos q_2 & -\sin q_1 \sin q_2 \\ -\sin q_1 \sin q_2 & \cos q_1 \cos q_2 \end{bmatrix}$$

The kinematics and inertial parameters of the pendulum system are chosen as $m_0=1kg$, $m_1=0.5kg$, $l=0.5m$, and initial states $q_1(0)=q_2(0)=0.2\,rad$, $\dot{q}_1(0)=\dot{q}_2(0)=0\,rad/s$. The trajectories to be followed are described by two decoupled linear systems, the desired coefficients are specified to be $\alpha_{i1}=2$, $\alpha_{i2}=1$, $i=1,2$. The pendulum is given the following target joint rotations:

$$q_{d1}=(\pi/15)\sin t, \quad q_{d2}=(\pi/15)\cos t+(\pi/30)\cos t$$

The membership functions of states q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ (represented by generic variable x_i) for the qualitative statements are defined as

$NB:\exp(-0.5(x_i+0.4)^2)$, $NS:\exp(-0.5(x_i+0.2)^2)$,
$PB:\exp(-0.5(x_i-0.4)^2)$, $PS:\exp(-0.5(x_i-0.2)^2)$,
$ZE:\exp(-0.5x_i^2)$.

In (26)-(28), the design parameters are given by $Q=10I_{4\times4}$, $\alpha_i=1$, $\gamma_{ij}=0.01$, $i,j=1,\cdots,m$. Consider three cases of prescribed attenuation levels $\rho=0.04,\ 0.08,\ 0.12$ and set $R=2\rho^2 I$, respectively.

In this simulation the combined effects of friction and the external disturbances are given as

$$d_1=\sin(\dot{q}_1)+1.25\sin(\dot{q}_2)+0.25\sin(t)$$
$$d_2=2.5\sin(\dot{q}_1)+2\sin(\dot{q}_2)+0.2\sin(t)$$

The curves of q_1 and q_2 under different attenuation levels are given in Fig. 4 and Fig. 5, respectively. Notice the important feature that under low attenuation level (ρ is large, e.g., $\rho=0.15$) the H^∞ tracking performance is often poor. The simulation result in Fig. 6 indicates that the integrals of error signals under different prescribed attenuation levels are decreased obviously. The effect of both the fuzzy approximation error and external disturbance on the tracking error has been attenuated efficiently. Thus we see that our robust adaptive fuzzy controller can control the inverted pendulum to follow the desired trajectory without using any linguistic information.

7. CONCLUSION

The goal of this work is the development and implementation of a multi-layer fuzzy system for direct adaptive fuzzy control to operate a two-dimensional inverted-pendulum system. The self-tuning mechanism on the membership function has been proposed to encounter the equivalent uncertainty resulted by function approximation error, external disturbances, and measurement noise. By employing the fuzzy descriptions to the input applied to one subsystem affecting the other subsystem, the overall adaptation scheme has been proved to be able guarantee the tracking error residual set to be uniform ultimate bounded.

Although only the inverted-pendulum system has been studied in this paper, the proposed control scheme can also be used to address the other class of MIMO nonlinear systems. Further works are still under investigation to apply the proposed adaptive fuzzy algorithm to build automatically fuzzy IF-THEN rules.

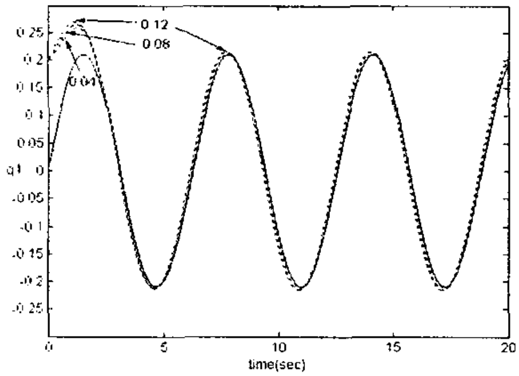

Fig. 4 The tracking curves of the q_1 (dashed line) and q_{d1} (solid line) with $\rho = 0.04,\ 0.08,\ 0.12$.

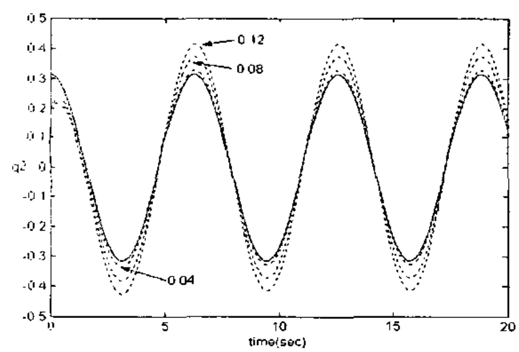

Fig. 5 The tracking curves of the q_2 (dashed line) and q_{d2} (solid line) with $\rho = 0.04,\ 0.08,\ 0.12$.

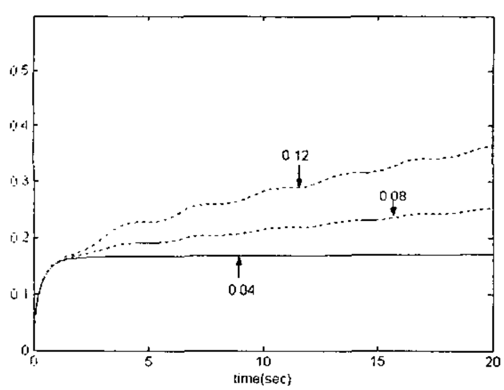

Fig. 6 The curves of the $\int_0^T e^2(t)dt$ with $\rho = 0.04, 0.08, 0.12$.

8. ACKNOWLEDGEMENT

The financial support for this research from the National Science Council of Taiwan, R. O. C. under NSC90-2212-E002-216 is gratefully acknowledged.

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