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State Feedback Stabilization of Single Input Systems Through Actuators with Saturation and Deadzone Characteristics[1]

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Abstract

Actuators with the saturation and deadzone characteristics are common in control systems, and often have adverse effects on the system performance or stability. For single input systems equipped with such actuators, we propose methods for synthesizing state feedback gains that can stabilize the systems. The goals are to get a large stability region under the limitation of saturation, to minimize the effect of deadzone, and to ensure reasonable decay rates of state trajectories. Due to the adopted linear matrix inequality formulations, the proposed methods are easy to apply because effective computation tools are readily available.

1. Introduction

In practical control systems, the actuators or devices like power amplifiers always have inherent nonlinearities, among which saturation and deadzone are very common. Physical limitations or imperfect manufacturing are possible reasons for the existence of these nonlinearities. However, no matter what the reasons may be, they will deteriorate the overall performance, and even make the closed-loop system unstable. In the literatures, an important approach to the saturation problem is to view it as a kind of constrained control or bounded control problem. Many solution methods have been proposed accordingly, such as the polyhedral set plus linear programming method [1] [11] [13], eigenstructure assignment method [4], and linear matrix inequality (LMI) method [5] [8] [9]. As to the treatment of the deadzone problem, there are much less results.

Traditionally, describing functions are used to approximate the frequency domain effects of many kinds of nonlinearities, including the deadzone characteristic, yet the main purpose is to investigate the possible existence of periodic solutions such as limit cycles [10]. To ensure closed-loop system stability, more precise analysis is needed. In this paper, we study single input state space models, and show that the recently popular idea of finding positively invariant set in the state space [2] is useful for solving both saturation and deadzone problems. The ellipsoid invariant set is chosen because it is intimately related to the Lyapunov stability criteria, and it generates LMI conditions, which can be conveniently processed by existing effective computation tools, such as [6].

This paper is organized as follows. We state the problem formulation and introduce the positively invariant set in Section 2. The problem caused by the saturation nonlinearity is discussed in Section 3, which includes the saturation avoidance and allowance methods. In Section 4, the deadzone problem is studied, and it is shown how to stabilize the system so that all state trajectories converge to an ultimate boundedness set. Combining the results developed in Sections 4 and 5, we handle the saturation and deadzone problems simultaneously in Section 5. In addition to plain stability, the decay rates of state trajectories are considered in all Sections, and every newly proposed method is illustrated with a numerical example. Finally, conclusions are made in Section 6.

2. Problem Formulation

In Fig. 1, the system to be studied consists of three parts: the single-input plant

$$\dot{x}(t) = Ax(t) + bu(t), \quad x(0) = x_o, \tag{1}$$

with the state vector $x(t) \in \mathcal{R}^n$ and the control input $u(t) \in \mathcal{R}$, the actuator characterized by

$$u(t) = \phi[u_c(t)],$$

$u_c(t) \in \mathcal{R}$, and the state feedback controller

$$u_c(t) = kx(t).$$

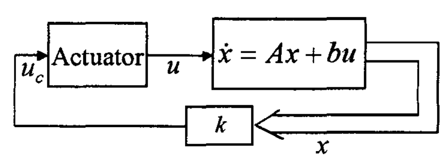

Fig. 1. A system with a nonlinear actuator.

Assume that the plant $\{A, b\}$ is controllable. Furthermore, assume that the actuator characteristic $\phi(\cdot)$ is described by the symmetric, piecewise linear curve shown in Fig. 2. Clearly, it includes both saturation

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($-\bar{u} \le u \le \bar{u}$) and deadzone ($-\delta \le u_c \le \delta$) characteristics. We want to find a state feedback gain $k \in \mathcal{R}^{1\times n}$ to stabilize, in the sense to be described below, the closed-loop system.

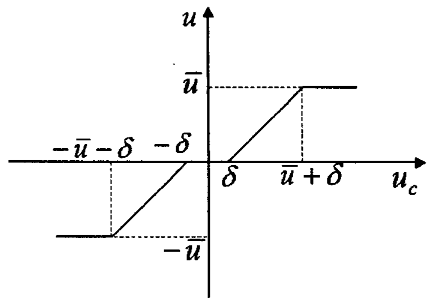

Fig. 2. Actuator input-output characteristics.

For $\delta = 0$ and $0 < \bar{u} < \infty$, in general it is impossible to stabilize the closed-loop system for all x_o unless the plant (1) is stable [12]. Therefore in this case we want to find an invariant set $\mathcal{P} = \{x \mid x^TPx < 1\}$, where $P \in \mathcal{R}^{n\times n}$ is positive definite, such that from every $x_o \in \mathcal{P}$, the state trajectory of the closed-loop system stays inside $\mathcal{P}$ and converges asymptotically to the origin.

For $0 < \delta < \infty$ and $\bar{u} = \infty$, there is no need to restrict x_o to a bounded invariant set, but due to the existence of deadzone, it is not possible to bring every state trajectory back to the origin, unless the plant itself is stable. Hence, we can only guarantee ultimate boundedness, which means all state trajectories will converge to a compact set $\mathcal{P}_\epsilon = \{x \mid x^TPx \le \epsilon < 1\}$, and ϵ will be minimized.

Of course when $0 < \delta < \infty$ and $0 < \bar{u} < \infty$, the combined problem must be considered. To facilitate presentation, first the above two problems are treated separately. Then we combine the results for systems containing actuators with both nonlinearities.

3. Saturation Only

Before we start, some notations are defined first. For $X, Y \in \mathcal{R}^{n\times n}$, $X \ge Y$ means that X, Y are symmetric and $X - Y$ is positive semi-definite. Similar notations apply to symmetric positive/negative definite matrices.

Consider the case with $\delta = 0$ and $0 < \bar{u} < \infty$. Facing the saturation we have two choices [7]: allowing the actuator to saturate or not. In practice it is often desirable not to saturate the actuator for various reasons, such as preventing the actuators from overheating. However, sometimes saturation is allowed in order to make full use of the actuator's capacity. In the sequel we develop state feedback gain synthesis methods for both choices. Let us start with the choice of saturation avoidance. If the actuator does not saturate in Fig. 1, then the closed-loop system is linear and described by

$$\dot{x} = (A + bk)x, \quad x(0) = x_o. \tag{2}$$

System (2) is quadratically stable [3] if and only if there exists an $n \times n$ matrix $P > 0$ such that

$$(A + bk)^TP + P(A + bk) < 0, \tag{3}$$

which also implies that $\mathcal{P}$ is an invariant set. Multiplying (3) on the left and right by P^{-1}, and defining new variables $Q = P^{-1} > 0$ and $y = kQ$, we may re-write (3) as

$$QA^T + AQ + y^Tb^T + by < 0. \tag{4}$$

Meanwhile, the input $u_c = kx = (yQ^{-1})x$ to the actuator should be confined to the interval $[-\bar{u}, \bar{u}]$. This can be guaranteed by requiring the ellipsoidal invariant set $\mathcal{P}$ to lie between the two hyper-planes $kx = \bar{u}$ and $kx = -\bar{u}$, which is equivalent to the LMI condition

$$\begin{bmatrix} \bar{u}^2 & y \\ y^T & Q \end{bmatrix} > 0. \tag{5}$$

Subject to the LMIs $Q > 0$, (4), and (5), it is possible to maximize the "size" of $\mathcal{P}$ by solving any one of the following convex optimization problems

$$\begin{array}{ll} (i) & \min\ \log\det Q^{-1} \\ (ii) & \min\ -\mathrm{tr}(Q) \\ (iii) & \min\limits_{0<\lambda I \le Q}\ -\lambda \end{array}$$

We select to solve (iii) with an attempt to make $\mathcal{P}$ more sphere-like. After some numerical experiments, we found that because of the sole objective of maximizing the invariant set "size", the solutions correspond to marginally stable systems, of which some eigenvalues are almost on the $\jmath\omega$-axis of the complex plane. Our remedy is to replace (4) by

$$QA^T + AQ + y^Tb^T + by < \alpha Q. \tag{6}$$

Then we solve the generalized eigenvalue problem (GEVP) $Q > 0$, (5), and (6) to find the smallest feasible $\alpha^* < 0$ first. Finally we solve the optimization problem (iii) subject to (5) and (6), with α in (6) fixed to a desirable value in the interval $[\alpha^*, 0)$. This ensures that the decay rate [3] of the resultant closed-loop system will be at least $|\alpha|/2$.

Example 1

Consider the system in Fig. 1 with $\delta = 0$, $\bar{u} = 4$,

$$A = \begin{bmatrix} 0.5 & -1 \\ 1 & 0.5 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}.$$

The above procedure allows us to use $\alpha = -1$,

$$k = [0.278 \ \ -2.139], \text{ and } P = \begin{bmatrix} 0.276 & -0.246 \\ -0.246 & 0.447 \end{bmatrix}.$$

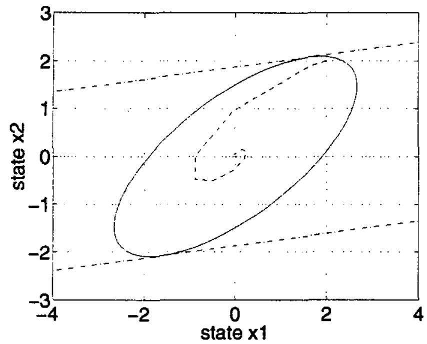

Fig. 3. Results of Example 1.

The invariant set $\mathcal{P}$, the two lines $kx = \bar{u}$ and $kx = -\bar{u}$, and a state trajectory starting from $x_o = [2 \;\; 2]^T$ are plotted in Fig. 3.

Next we turn to the choice of saturation allowance. Our basic conditions of the existence of a state feedback gain k and a Lyapunov function $V(x) = x^T P x$ to ensure the quadratic stability of the closed-loop system in the invariant set $\mathcal{P}$ are

$$P > 0,\; A^T P + PA + k^T b^T P^T + Pbk < 0, \text{ and} \quad (7)$$

$$x^T(A^T P + PA)x + \bar{u} b^T P x + x^T P b \bar{u} < 0 \quad (8)$$
$$\forall x \in \{x \mid x^T P x < 1, kx > \bar{u}\}.$$

Note that (7) is for the trajectory segments with unsaturated control, and (8) is for those with the control input u saturated at $\bar{u}$. Due to symmetry, (8) also covers the trajectory segments with u saturated at $-\bar{u}$ (similar remarks will not be repeated below unless absolutely necessary). Applying the S-procedure, we know that (8) holds provided there exist $\tau_1 > 0$ and $\tau_2 > 0$ such that

$$x^T(A^T P + PA)x + \bar{u} b^T P x + x^T P b \bar{u} - \tau_1(x^T P x - 1) - \tau_2(\bar{u} - kx) \le 0 \quad \forall x. \quad (9)$$

With $\hat{P} = P/\tau_2$, and $\tau_3 = \tau_1/\tau_2$, (9) is equivalent to

$$\begin{bmatrix} A^T\hat{P} + \hat{P}A - \tau_1\hat{P} & \hat{P}b\bar{u} + \frac{1}{2}k^T \\ \bar{u}b^T\hat{P} + \frac{1}{2}k & \tau_3 - \bar{u} \end{bmatrix} \le 0. \quad (10)$$

Defining $\hat{Q}^{-1} = \hat{P}, \hat{y} = k\hat{Q}$, and multiplying from both sides of (10) by diag$[\hat{P}^{-1}, 1]$, we get

$$\begin{bmatrix} \hat{Q}A^T + A\hat{Q} & b\bar{u} + \frac{1}{2}\hat{y}^T \\ \bar{u}b^T + \frac{1}{2}\hat{y} & \tau_3 - \bar{u} \end{bmatrix} \le \tau_1 \begin{bmatrix} \hat{Q} & 0 \\ 0 & 0 \end{bmatrix}. \quad (11)$$

We can similarly manipulate (7) to produce the LMIs

$$\hat{Q} > 0,\; \hat{Q}A^T + A\hat{Q} + \hat{y}^T b^T + b\hat{y} < 0. \quad (12)$$

Inequalities (11) and (12) together form a GEVP, for which any feasible solutions with positive τ_1 and τ_3 will give us an invariant set $\mathcal{P}$ and a stabilizing state feedback gain k. However, we are not able to form convex optimization problems to maximize the "size" of $\mathcal{P}$, as in the saturation avoidance case. Our approach is to solve the GEVP to find the smallest $\tau_1^* > 0$ first. Then τ_3 is maximized subject to (11) and (12), with τ_1 in (11) fixed to τ_1^*. Selecting τ_3 as the objective function is motivated by the relation $P = \tau_1^* \hat{Q}^{-1}/\tau_3$, and it is expected to "minimize" P by maximizing τ_3 (despite that $\hat{Q}$ may change).

Example 2

For the same system in Example 1, using the above approach, we get $\tau_1 = 2.04, \tau_2 = 0.51, \tau_3 = 4$,

$$k = [0.008 \;\; -1.2418], \text{ and } P = \begin{bmatrix} 0.063 & -0.032 \\ -0.032 & 0.095 \end{bmatrix}.$$

The invariant set $\mathcal{P}$, the two lines $kx = \bar{u}$ and $kx = -\bar{u}$, and a state trajectory starting from $x_o = [2 \;\; 3.5]^T$ are plotted in Fig. 4.

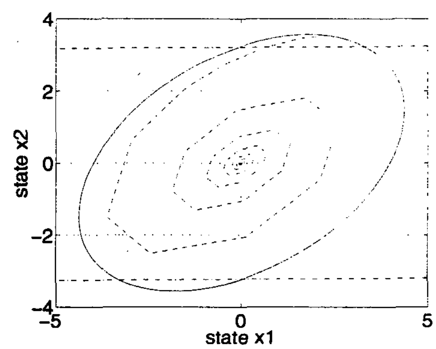

Fig. 4. Results of Example 2.

No matter from the saturation avoidance or allowance methods, when a pair $\{k, P\}$ is found for a given problem, we can try to enlarge the derived invariant set $\mathcal{P}$ by examining the Lyapunov function derivative on trajectories which start from the neighborhood of $\mathcal{P}$. Along every state trajectory $x(\cdot)$ starting from $x_o \in \{x \mid x^T P x < 1+\rho\}$, where $\rho > 0$, if $dx^T(t)Px(t)/dt < 0$ for all t, then the trajectory will converge to the origin. Since we already have (3) and (7), we simply need to check the Lyapunov function derivative along state trajectory segments driven by the saturating actuator. Thus it is enough to search for a $\rho > 0$ such that

$$x^T(A^T P + PA)x + \bar{u} b^T P x + x^T P b \bar{u} < 0 \quad (13)$$
$$\forall x \in \{x \mid x^T P x < 1 + \rho, kx > \bar{u}\}.$$

By the S-procedure, (13) is implied by the existence of $\tau_4 > 0$ and $\tau_5 > 0$ such that

$$x^T(A^T P + PA)x + \bar{u} b^T P x + x^T P b \bar{u} - \tau_4(x^T P x - 1 - \rho) - \tau_5(\bar{u} - kx) \le 0 \quad \forall x,$$

which is equivalent to

$$\begin{bmatrix} A^TP + PA - \tau_4 P & Pb\bar{u} + \frac{1}{2}\tau_5 k^T \\ \bar{u}b^TP + \frac{1}{2}\tau_5 k & -\tau_5\bar{u} + \tau_4 \end{bmatrix} \leq -\rho \begin{bmatrix} 0 & 0 \\ 0 & \tau_4 \end{bmatrix}.$$

This is a GEVP that can be used to find a maximum $\rho > 0$ subject to $\tau_4 > 0$ and $\tau_5 > 0$.

Example 3

Given the results of Example 1, we can expand the invariant set from $\mathcal{P}$ (dashed ellipse) to $\{x \mid x^TPx < 1.325\}$ (solid ellipse) as displayed in Fig. 5, where a state trajectory starting from $x_o = [2 \;\; 2.25]^T$ are plotted. It must be noted that for the state trajectory segment lying between the outer ellipse and the line $kx = \bar{u}$, the control input u is saturated.

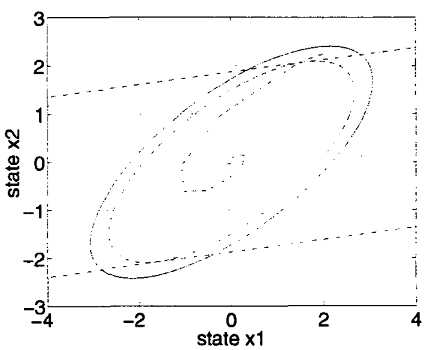

Fig. 5. Results of Example 3.

Example 4

From the results of Example 2, the enlargement procedure merely gives $\rho = 0.01$, which is not so significant. The main reason is that in Example 2, saturation is already allowed. However, because the S-procedure adopted may cause conservativeness, it is possible that there is a larger ρ satisfying (13).

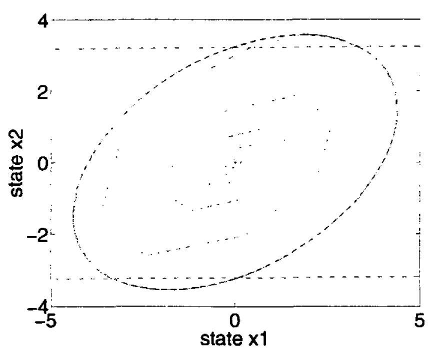

Fig. 6. Results of Example 4.

4. Deadzone Only

Now we consider the system in Fig. 1 with $0 < \delta < \infty$ and $\bar{u} = \infty$, which means that there are no saturation limitations, but the deadzone effect does exist. Clearly, the system dynamics are

$$\dot{x}(t) = \begin{cases} Ax + b(kx - \delta), & \delta \leq kx, \\ Ax, & -\delta < kx < \delta, \\ Ax + b(kx + \delta), & kx \leq -\delta. \end{cases}$$

Due to the deadzone, it is expected that we have no control over state trajectories inside some neighborhood of the origin in the state space. We try to confine this neighborhood with $\mathcal{P}_\epsilon = \{x \mid x^TPx \leq \epsilon < 1\}$, and want to find k and $P > 0$ making

$$x^T(A^TP + PA + k^Tb^TP + Pbk)x - x^TPb\delta - \delta b^TPx < 0 \quad \forall x \notin \mathcal{P}_\epsilon. \qquad (14)$$

The S-procedure tells us that (14) is implied by the existence of a $\tau_6 > 0$ such that

$$x^T(A^TP + PA + k^Tb^TP + Pbk)x - x^TPb\delta - \delta b^TPx - \tau_6(\epsilon - x^TPx) \leq 0 \quad \forall x. \qquad (15)$$

Note that if inequalities (14) and (15) hold, then we can replace all x's therein by $-x$'s. The results are exactly the inequalities for negative control input u. For example, (14) becomes

$$x^T(A^TP + PA + k^Tb^TP + Pbk)x + x^TPb\delta + \delta b^TPx < 0 \quad \forall x \notin \mathcal{P}_\epsilon. \qquad (16)$$

Introducing $\hat{P} = P/\tau_6 > 0$ lets one re-write (15) as

$$x^T(A^T\hat{P} + \hat{P}A + k^Tb^T\hat{P} + \hat{P}bk)x - x^T\hat{P}b\delta - \delta b^T\hat{P}x - (\epsilon - x^T\tau_6\hat{P}x) \leq 0 \quad \forall x.$$

Using the same skill as in the saturation allowance method, we set $\hat{Q} = \hat{P}^{-1} > 0$ and $\hat{y} = k\hat{Q}$. The above condition can be transformed into

$$\begin{bmatrix} \hat{Q}A^T + A\hat{Q} + \hat{y}^Tb^T + b\hat{y} & -b\delta \\ -\delta b^T & -\epsilon \end{bmatrix} \leq -\tau_6 \begin{bmatrix} \hat{Q} & 0 \\ 0 & 0 \end{bmatrix}. \qquad (17)$$

The inequalities $0 < \epsilon < 1$ and (17) form a GEVP for us to find the largest $\tau_6^* > 0$. Since τ_6 is related to the decay rate of the system, as suggested by (15), we can select a suitable value for it in the interval $(0, \tau_6^*]$. Then τ_6 is fixed in (17), which becomes an LMI in terms of $\hat{Q}, \hat{y}$, and $\epsilon \in (0, 1)$, and ϵ may be minimized to find a pair $\{\hat{y}, \hat{Q}\}$, or the corresponding $\{k, P\}$. To show that every state trajectory of the closed-loop system will eventually converge to the set $\mathcal{P}_\epsilon$, we consider the Lyapunov function derivative $dV[x(t)]/dt = dx^T(t)Px(t)/dt$. When $u = kx - \delta > 0$ or $u = kx + \delta < 0$, if $x^TPx > \epsilon$, then $dV[x(t)]/dt < 0$ by (14) and (16). When $u = 0$ and $x^TPx > \epsilon$, we see from (14) and (16) respectively that

$$\begin{aligned} \frac{dV[x(t)]}{dt} &= x^T(t)(A^TP + PA)x(t) \\ &< -2[b^TPx(t)][kx(t) - \delta] \end{aligned} \qquad (18)$$

and

$$\frac{dV[x(t)]}{dt} = x^T(t)(A^TP + PA)x(t)$$
$$< -2[b^TPx(t)][kx(t) + \delta]. \quad (19)$$

Thus $dV[x(t)]/dt < 0$ for all $x^T(t)Px(t) > \epsilon$ and $-\delta < kx(t) < \delta$ no matter $b^TPx(t) \geq 0$ or $b^TPx(t) \leq 0$.

Example 5

Suppose the actuator of the system in Example 1 is changed to one with $\delta = 0.4$ and $\bar{u} = \infty$. We can solve the GEVP (17) to select $\tau_6 = 1$. Then we minimize ϵ with respect to the selected τ_6, and get $\epsilon = 1.2 \times 10^{-10}$, $k = [12.18 \ \ -21.86]$, and

$$P = \begin{bmatrix} 0.333 & -0.313 \\ -0.313 & 0.42 \end{bmatrix} \times 10^{-7}.$$

Fig. 5 shows the ellipsoidal set $\mathcal{P}_\epsilon$ and the last part of a state trajectory starting from $x_o = [-10 \ \ 20]^T$, which eventually enters $\mathcal{P}_\epsilon$ and forms a limit cycle. The two dash-dotted parallel lines are $kx = \pm\delta$.

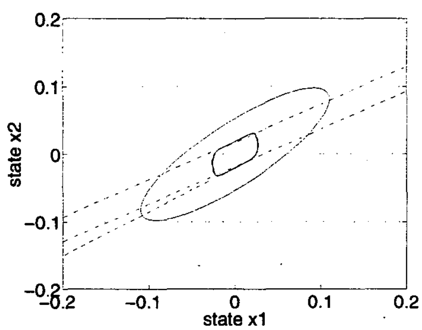

Fig. 7. Results of Example 5.

5. Saturation and Deadzone

Finally, let us consider the case with both saturation and deadzone nonlinearities. For $0 < \delta, \ \bar{u} < \infty$, results from Sections 3 and 4 may be combined to require

$$\begin{bmatrix} (\bar{u}+\delta)^2 & k \\ k^T & P \end{bmatrix} > 0,$$
$$P > 0, \quad 0 < \epsilon < 1, \quad \text{and}$$
$$x^T(A^TP + PA + k^Tb^TP + Pbk)x - x^TPb\delta - \delta b^TPx \leq \tau_7(\epsilon - x^TPx) \quad \forall x,$$

which ensure that all state trajectories starting from $x_o \in \mathcal{P}$ will converge to $\mathcal{P}_\epsilon$ without saturating the actuator. With the change of variables $Q = P^{-1} > 0, y = kQ$ and $\tau_8 = \tau_7\epsilon > 0$, the above conditions may be re-written as

$$\begin{bmatrix} (\bar{u}+\delta)^2 & y \\ y^T & Q \end{bmatrix} > 0,$$
$$Q > 0, \quad 0 < \tau_8 < \tau_7, \quad \text{and}$$
$$\begin{bmatrix} QA^T + AQ + y^Tb^T + by & -b\delta \\ -\delta b & -\tau_8 \end{bmatrix} \leq -\tau_7 \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}.$$

Ignoring the constraint $\tau_8 < \tau_7$ temporarily, we have a GEVP that enables us to determine if there is any usable $\tau_7 > 0$, which is related to the decay rate of the closed-loop system. Assuming that we have found a desirable τ_7, we can fix it in the above inequalities, bring back the constraint $\tau_8 < \tau_7$, replace τ_8 by $\tau_7\epsilon$, and form a set of LMIs involving variables Q, y, and ϵ. Subject to the LMI constraints, we can solve an optimization problem which simultaneously attempts to minimize and maximize the "sizes" of $\mathcal{P}_\epsilon$ and $\mathcal{P}$, respectively. For example, in Example 6 given below we minimize an objective function equal to $\epsilon - \lambda$, where λ is the smallest eigenvalue of Q.

If saturation is allowed, then the idea presented in Section 3 is also applicable here to enlarge the invariant set $\mathcal{P}$. For each solution $\{k, P, \epsilon\}$ found above, the following GEVP

$$\begin{bmatrix} A^TP + PA - \tau_9P & Pb\bar{u} + \frac{1}{2}\tau_{10}k^T \\ \bar{u}b^TP + \frac{1}{2}\tau_{10}k & -\tau_{10}(\bar{u}+\delta) + \tau_9 \end{bmatrix} \leq -\rho \begin{bmatrix} 0 & 0 \\ 0 & \tau_9 \end{bmatrix} \quad (20)$$

with positive variables τ_9, τ_{10}, and ρ can be solved to find a maximal ρ, and state trajectories starting from $x_o \in \{x \mid x^TPx < 1 + \rho\}$ will eventually converge to the set $\mathcal{P}_\epsilon$.

Example 6

If the actuator in Example 1 has both saturation and deadzone characteristics, where $\bar{u} = 4$, and $\delta = 0.4$, then we apply the first part of the results developed in this Section to obtain $\tau_7 = 0.69$, $\tau_8 = 0.028$, $\epsilon = 0.04$,

$$k = [0.93 \ \ -3.84], \text{ and } P = \begin{bmatrix} 0.689 & -0.506 \\ -0.506 & 0.922 \end{bmatrix}.$$

Moreover, the second part of the results gives $\rho = 1.28$. Fig. 8 below shows $\mathcal{P}_\epsilon$ (innermost ellipse), $\mathcal{P}$ (dashed ellipse), the enlarged invariant set (outermost ellipse), the lines $kx = \pm\bar{u}$, and a state trajectory starting from $x_o = [2 \ \ 1.8]^T$, which eventually enters $\mathcal{P}_\epsilon$ and forms a limit cycle.

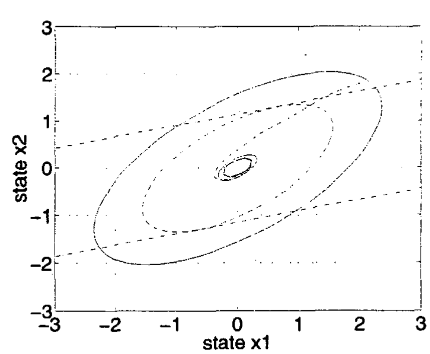

Fig. 8. Results of Example 6.

6. Conclusions

While actuator nonlinearities are common in control systems, to take into consideration their effects in the design of controllers is usually difficult. We deal with this problem in a direct fashion for single input state space models to be stabilized by state feedback. Two very often seen nonlinearities, saturation and deadzone, are handled, and the controller design methods proposed are easy to apply because of the LMI formulations adopted.

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