ANALYSIS OF DISCONTINUITIES IN DIELECTRIC SLAB WAVEGUIDES

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Discontinuities in dielectric slab waveguides have been analyzed by many authors due to their significance in designing various optical and millimeter wave components, such as transformers, grating couplers, antenna feeds and others. The unbounded structure which leads to continuous spectrum makes the analysis of these discontinuity problems in dielectric waveguides more difficult than those in metallic waveguides. Mahmoud and Beal [1] first used a complete set of "good functions" to discretize the continuous spectrum, and solved the problem by matching the tangential fields at the junction of a step discontinuity. Rozzi [2] solved an integral equation containing the junction fields by the Ritz-Galerkin approach. These methods can tackle the discontinuities with regular shape, namely the step-type discontinuities. Recently, Chung and Chen [3] treated the problems of arbitrary discontinuities in an otherwise uniform slab waveguide. In the present study, we will discuss the problem with arbitrary discontinuities between two different slab guides (Fig.1).

Consider the structure in Fig.1, which is uniform in the y-direction. The discontinuity region Ω with refractive index $n(x,z)$ is enclosed by three artificial boundaries Γ_1, Γ_2, Γ_3, and the magnetic wall Γ_0 which is introduced due to symmetry. The regions I ($0\leq x<\infty$, $z\leq 0$) and II ($0\leq x<\infty$, $z\geq \ell$) are slab waveguides I and II, respectively, whose refractive indices are n_1 and n_2, and the region III ($x\geq X_0$) is free space. Note the overlapping of regions I and III as well as II and III. Symmetric guided TE modes, whose electric fields are y-polarization, are incident upon the discontinuities.

From the partial variational principle [4], a partial variational equation is obtained for the unknown fields (E_y, H_x, H_z) :

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$$\delta^a I^a = 0$$

$$I^a = \frac{j}{\omega\mu_0}\int_\Omega dv \left[\frac{\partial E_y^a}{\partial x}\frac{\partial E_y}{\partial x} + \frac{\partial E_y^a}{\partial z}\frac{\partial E_y}{\partial z} - k_o^2 n^2 E_y^a E_y\right] + \int_\Gamma ds\ \hat{n}\cdot[\hat{z}H_x^a(\Gamma^+) - \hat{x}H_z^a(\Gamma^+)][E_y(\Gamma^-) - E_y(\Gamma^+)] - \int_\Gamma ds\ E_y^a(\Gamma^-)\hat{n}\cdot[\hat{x}H_z(\Gamma^+) - \hat{z}H_x(\Gamma^+)] \quad (1)$$

where $\Gamma=\Gamma_1+\Gamma_2+\Gamma_3$, whose outward normal is $\hat{n}$. The inner and outer sides of Γ are represented by Γ^- and Γ^+, respectively. (E_y^a, H_x^a, H_z^a) are the test fields, which may be regarded as a set of weighting functions.

Eq.(1) contains the fields both interior and exterior to the boundary Γ. In the finite element solution, the interior field is represented by the nodal values ϕ_i and the corresponding bases B_i,

$$E_y(\Omega) = \sum_i \phi_i B_i(\Omega) \quad . \quad (2)$$

The electric fields in regions I and II are expanded by the modes of the corresponding waveguides,

$$E_y^\alpha = \sum_m [A_m^\alpha \exp(\mp j\beta_m^\alpha z) + a_m^\alpha \exp(\pm j\beta_m^\alpha z)]u_m^\alpha(x) + \sum_p b_p^\alpha \int_0^\infty d\rho\ f_p^\alpha(\rho)\ u^\alpha(\rho;x)\ \exp(\pm j\beta z) \ , \quad (3)$$

where α=I or II and the upper signs are used when α=I, otherwise the lower ones are used. A_m and a_m are the coefficients of the incident (known) and scattered (unknown) mth guided mode whose modal function and propagation constant are $u_m(x)$ and β_m. $u(\rho;x)$ and $\beta=\sqrt{k_o^2-\rho^2}$ are the modal function and propagation constant of the radiation mode, respectively, with ρ being the wavenumber in the x-direction. For numerical computation, the spectrum $\mathbb{R}(\rho)$ of the radiation modes is discretized by a set of "good functions" $f_p(\rho)$, i.e., $\mathbb{R}(\rho)= \sum_p b_p f_p(\rho)$, where b_p is the corresponding (unknown) coefficient.

The electric field in region III is related to the one at the boundary $x=X_0$

$$E_y(x,z)=-\int_{-\infty}^{\infty} dz'\ G(x-X_0,z-z')\ \frac{\partial E_y}{\partial x'}(X_0,z') \ , \quad (x,z) \in \text{III} \ , \quad (4)$$

using the Green's function G in free space

$$G(x-X_0,z-z') = -\frac{j}{2} H_o^{(2)}(k_o\sqrt{(x-X_0)^2+(z-z')^2}\ ) \ . \quad (5)$$

$\frac{\partial E_y}{\partial x'}$ in (4) is calculated from (2) when $0<z'<\ell$, and from (3) when $z'<0$ or $z'>\ell$, thus E_y in region III can finally be written as

$$E_y(III) = \sum_\alpha [\sum_m (F_A^\alpha A_m^\alpha + F_a^\alpha a_m^\alpha) + \sum_p F_b^\alpha b_p^\alpha] + \sum_i F_i \phi_i \qquad (6)$$

where the F's are known functions of x and z.

In (1) the exterior fields associated with Γ_1 and Γ_2 are calculated from (3), and those with Γ_3 are calculated from (6). Casting these into (1), and using the Ritz-Galerkin approach, one finally gets a matrix equation of the form

$$\bar{\bar{A}}\bar{\Psi} = \bar{s} \quad , \qquad (7)$$

where $\bar{\bar{A}}$ is a known matrix, and $\bar{\Psi}$ $(=[\phi_i, a_m^I, a_m^{II}, b_p^I, b_p^{II}]^T)$ and $\bar{s}$ are vectors associated with the unknown coefficients and the incident fields, respectively.

To check the validity of the present method, we consider a step discontinuity shown in Fig.2. The finite-element region Ω is chosen to enclose the junction. After obtaining the coefficients a_m's and b_p's, one may calculate the tangential fields at the junction. Fig.2 shows the field distributions at the planes z=0- and z=0+. The agreement of the curves ensures the continuation of the fields, and ensures the validity of the present method. A typical result for reflection and transmission coefficients is presented in Fig.3.

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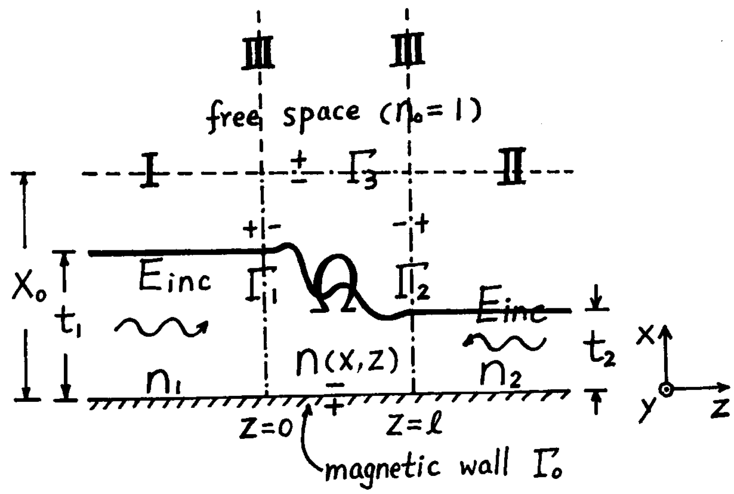

Fig.1 Arbitrary discontinuities between two slab guides.

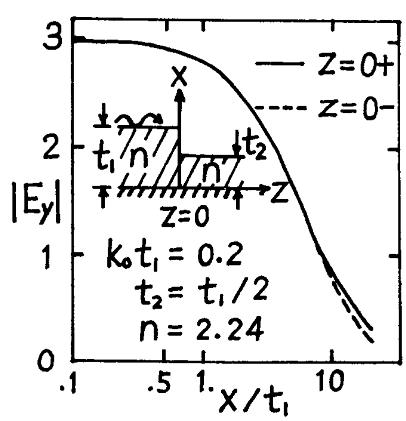

Fig.2 Tengential electric field at junction of step discontinuity.

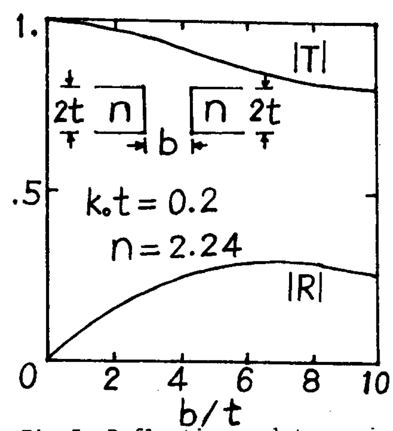

Fig.3 Reflection and transmission coefficients of a gap.