行政院國家科學委員會專題研究計畫 期中進度報告

利用自動機理論探討離散事件系統之可控制性(2/3)
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行政院國家科學委員會專題研究計劃成果報告

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一、摘要

本年度之研究計畫中，我們研究在派區網路(Petri Nets)中如何來計算其向上封閉集合(Upward-Closed Sets)的最小元素。一般而言，在派區網路上這樣的最小元素個數是有限的(finite)，但是這樣的最小元素有限集合不一定是可以有效地被計算(effectively computable)出來。在這裡我們對派區網路設計了一個一致策略來有效地計算向上封閉集合的最小元素大小。由於派區網路已被廣泛地應用於描述離散事件系統，本研究將可直接用在此類系統可控制性的研究上。

關鍵字：派區網路，向上封閉集合，最小元素。

Upward-closed sets of integer vectors enjoy the merit of having a finite number of minimal elements, which is behind the decidability of a number of Petri net related problems. In general, however, such a finite set of minimal elements may not be effectively computable. In here, we develop a unified strategy for computing the sizes of the minimal elements of certain upward-closed sets associated with Petri nets.

Keyword: Petri Nets，Upward-Closed Sets，Minimal Elements.

二、計畫緣由與目的

Evidence has suggested that upward -closed sets play a key role in a number of decidability results in automated verification of infinite state systems. In the analysis of Petri nets, the notion of upward-closed sets is closely related to the so-called property of monotonicity which serves as the foundation for many decision procedures for Petri net problems. What the monotonicity property says is that if a sequence σ of transitions of a Petri net is executable from a marking $\mu \in \mathbb{N}^k$, then the same sequence is legitimate at any marking greater than or equal to μ. That is, all the markings enabling σ form an upward-closed set.

In spite of the fact that the set of all the minimal elements of an upward–closed set is always finite, such a set may not be effectively computable in general. There are, however, certain interesting upward-closed sets for which their minimal elements are effectively computable. A notable example is the set of initial markings of a Petri net from which a designated final marking is coverable. More recent work demonstrated decidability to compute, from a given upward-closed set of final states, the set of states that are backward reachable from the final states.

Given the importance of upward-closed sets, it is of interest theoretically and practically to be able to characterize the class of upward-closed sets for which their minimal elements are computable. Along this line of research, Valk and Jantzen presented a sufficient and necessary condition under which the set of minimal elements of an upward-closed set is guaranteed to be effectively computable. Supposed U is an upward -closed set over $\mathbb{N}^k$ and ω is a symbol representing something being arbitrarily large. Valk and Jantzen hown that the set of minimal elements of U is effectively computable iff the question '$reg(v) \cap U \neq \varnothing$?' is decidable for every $v \in (\mathbb{N} \cup \{\omega\})^k$, where $reg(v) = \{x \mid x \in \mathbb{N}^k, x \leq v\}$. However, there is no complexity bounds for the sizes of the minimal elements in the result of Valk and Jantezen. As knowing the size of minimal elements might turn out to be handy in many cases, the following question arises naturally. If more is known about the query '$reg(v) \cap U \neq \varnothing$? ', could the size of the minimal elements be measured? In fact, answering the question in the affirmative is the main contribution of our work.

三、計畫方法

It is well known that every upward-closed set over $\mathbb{N}^k$ has a finite number of minimal elements. However, such a finite set may not be effectively computable in general. In an article by Valk and Jantzen, the following result was proven which suggests a sufficient and necessary condition under which the set of minimal elements of an upward-closed set is effectively computable:

Theorem 1. *For each upward-closed set $K (\subseteq \mathbb{N}^k)$, min(K) is effectively computable iff for every $v \in \mathbb{N}^k_\omega$, the problem '$reg(v) \cap U \neq \varnothing$?' is decidable.* (*Recall that* $reg(v) = \{x \mid x \in \mathbb{N}^k, x \leq v\}$)

In what follows, we show that for every $v \in \mathbb{N}^k_\omega$, should we be able to compute the size of a witness for $reg(v) \cap U \neq \varnothing$ (if one exists), then an upper bound can be placed on the size of all minimal elements.

Theorem 2. *Given an upward-closed set $U (\subseteq \mathbb{N}^k)$, if for every $v \in \mathbb{N}^k_\omega$, a witness $\hat{w} \in \mathbb{N}^k$ for every '$reg(v) \cap U \neq \varnothing$' (if one exists) can be computed with*

(i) $\|\hat{w}\| \leq b$, *for some* $b \in \mathbb{N}$ *when* $v = (\omega, \ldots, \omega)$,

(ii) $\|\hat{w}\| \leq f(\|v\|)$ *when* $v \neq (\omega, \ldots, \omega)$, *for some monotone function f, then* $\|min(U)\| \leq f^{(k-1)}(b)$.

We examine some upward-closed

sets defined and discussed in Valk and Jantezen. Given a PN=(P,T,φ), a vector $\mu \in \mathbb{N}^k$ is said to be

(i) $\hat{T} - blocked$, for $\hat{T} \subseteq T$ if $\forall \mu' \in R(\mathcal{P},\mu)$, $\neg(\exists t \in \hat{T}, \mu' \xrightarrow{t})$. For each case when $\hat{T} = T$, μ is said to be a *total deadlock*.

(ii) *dead* if $F(\mathcal{P},\mu)$ is finite.

(iii)*bounded* if $R(\mathcal{P},\mu)$ is finite; otherwise, it is called *unbounded*.

(iv) $\hat{T} - continual$, for $\hat{T} \subseteq T$, if there exists a $\sigma \in T^{\omega}$, $\mu \xrightarrow{\sigma}$ and $\hat{T} \subseteq In(\sigma)$.

For a PN $(\mathcal{P},\mu_0)$, consider the following four sets defined in Valk and Jantezen.

(i) $NOTBLOCKED(\hat{T}) = \{\mu \in \mathbb{N}^k \mid \mu$ is not $\hat{T} - blocked\}$.

(ii) $NOTDEAD = \{\mu \in \mathbb{N}^k \mid \mu$ is not *dead* $\}$.

(iii) $UNBOUNDED = \{\mu \in \mathbb{N}^k \mid \mu$ is *unbounded* $\}$.

(iv) $CONTINUAL(\hat{T}) = \{\mu \in \mathbb{N}^k \mid \mu$ is $\hat{T} - continual\}$.

It has been shown in Valk and Jantezen that for each of the above four upward-closed sets, the '$reg(v) \cap K \neq \varnothing$?' query of Theorem 1 is decidable; as a consequence, the set of minimal elements is effectively computable. We now show how to use Theorem 2 to estimate the bound of the minimal elements for each of the four sets. To this end, we show that if '$reg(v) \cap K \neq \varnothing$?', where K is any of the above four upward–closed sets, then there is a witness whose max-value is bounded by $n^{2^{d \times k \times logk}}$, where d is constant, n is the maximum number of tokens that can be added to or subtracted from a place in the k-dimensional PN and n is independent of v.

Theorem 3. Given a k-dimensional PN (P,T,φ) and a $\hat{T} \subseteq T$, $\|min(NOTBLOCKED(\hat{T}))\|$, $\|min(UNBOUNDED)\|$, $\|min(NOTDEAD)\|$, $\|\min(CONTINUAL(\hat{T})\| \leq n^{2^{d \times k \times logk}}$, where $n = \|\bar{T}\|$ and d is a constant.

Now we consider a problem that arises frequently in *automated verification*. Given a system S with initial state q, and a designated set of states Q, it is often of interest and importance to ask whether some state in Q can be reached from q, which constitutes a question related to the analysis of a safety property. Instead of using the forward-reachability analysis, an equally useful approach is to use the so-called backward-reachability analysis. In the latter, we compute the set $pre^*(S,Q)$ which consists of all the states from which some state in Q is reachable, and then decide whether

$q \in pre^*(S,Q)$. In general, $pre^*(S,Q)$ may not be computable for infinite state systems.

For PNs, we define the *backward-reachability* (BR, for short) problem as follows:

-Input：A PN $\mathcal{P}$ and a set U of markings

-Output：The set $pre^*(\mathcal{P},U) = \{\mu \mid R(\mathcal{P}, u) \cap U \neq \emptyset\}$

In words, the problem is to find the set of initial markings from which a marking in U can be reached. Now suppose U is upward-closed, then $\{\mu \mid R(\mathcal{P},u) \cap U \neq \emptyset\}$ is upward-closed as well, and is, in fact, equivalent to $\bigcup_{v \in min(U)} \{\mu \mid \exists \mu' \in R(\mathcal{P},\mu), \mu' \geq v\}$. The latter is basically asking about coverability issues of PNs. Hence, the max-value of the minimal elements can be derived along the same line as that for the set *NOTBLOCKED*.

四、結論與未來展望

We have developed a unified strategy for computing the sizes of the minimal elements of certain upward-closed sets associated with Petri nets. Our approach can be regarded as a refinement of Valk and Jantezen in the sense that complexity bounds become available (as opposed to merely decidability as was the case in Valk and Jantezen), as long as the size of a witness for a key query is known. Several upward-closed sets that arise in the theory of Petri nets as well as in backward-reachability analysis in automated verification have been derived in this project. It would be interesting to seek additional applications of our technique.

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