A Ring-Structured Adaptive Notch Filter

Wei-Ji Shyu and Jenho Tsao

Department of Electrical Engineering
National Taiwan University
Taipei,Taiwan, R. O. C.

ABSTRACT

In this paper, the optimum structure for cascade adaptive filter is discussed. Different forms to implement the optimum structure are studied. For the tracking of noisy sinusoids, we propose a ring structure to implement it. The use of ring structure reduces the required N^2 cells of notches in the optimum structure to be N cells only. Necessary conditions that allow this reduction are given.

INTRODUCTION

Over a long period of time, adaptive filtering was an active area of research[1]. Some important application are linear prediction, echo cancellation and channel equalization[1]. Two types of filter used frequently in filtering are finite impulse response (FIR) and infinite impulse response (IIR) filters. Several realization forms can be utilized, such as direct, parallel, lattice and cascade forms. In this paper, we will discuss the optimum cascade structure.

Using adaptive filters to track and enhance narrow band signals also received substantial attention [2-5], which is known as adaptive notch filter (ANF). Frequently, the ANF is implemented by the direct form IIR filter[2]. Its disadvantage is that the tracked frequency must be obtained by roots finding and it is not easy to check the stability. Therefore, a cascade form is recommended. The parameters of cascade form can be adapted by two ways. First, each notch cell is adapted individually. Second, each notch cell is adapted simultaneously. The former structure yields biased frequency estimtes even in the absence of noise[3]. This is due to the coupling of different frequency components via nonoptimum computation of gradient components. Therefore, simultaneous adaption is better than individual adaption[3].

After discussion of the optimum cascade structure, we propose a ring structure to implement it.

* This work was supported by the National Science Council, R. O. C., under project CS80-0210-D002-39.

THE OPTIMUM STRUCTURE FOR ADAPTIVE CASCADE FILTER

A commom criterion used in adaptive filtering is to minimize the mean square error (MSE) ξ defined as follows:

$$\xi = \lim_{M\to\infty} \frac{1}{M} \sum_{i=0}^{M} e^2(n) \qquad (1)$$

where $e(n) = d(n) - x(n)$ and $d(n)$ is the desired signal. The filter's coefficients are updated according to some adaptive algorithms, such as stochastic gradient (SG) algorithms or Gauss-Newton algorithms. No matter which algorithm is used, the computation of gradient components which are the derivatives of ξ with respect to the filter's coefficients is necessary.

It is convenient to interpret the adaptive filtering in frequency domain. Assuming that $E(z)$ denote the z-transform of $e(n)$ and $X(z)$ the z-transform of $x(n)$, then $E(z)$ can be expressed as

$$E(z) = H(z)X(z), \qquad (2)$$

where $H(z)$ denotes the transfer function of the filter to be adapted.

The cascade form can be generated by factoring $H(z)$ into the product of N least order filters, i.e.,

$$H(z) = \prod_{i=1}^{N} H_i(z). \qquad (3)$$

The derivatives of $e(n)$ with respect to $H_i(z)$'s coefficients can be obtained by filtering the input signal $x(n)$ through a filter with the following transfer function[9],

$$\frac{\partial E(z)}{\partial a_{il}} = \prod_{k=1}^{i-1} H_k(z) \cdot S_i(z) \cdot \prod_{k=i+1}^{N} H_k(z), \qquad (4)$$

where $S_i(z)$ known as sensitivity filter is shown as

$$S_i(z) = \frac{\partial H_i(z)}{\partial a_{il}} \quad l = 1, \cdots, L, \qquad (5)$$

with L being the number of coefficients in $H_i(z)$ to be adapted.

From Tellegen's theorem[5], the sensitivity filter can be shown to be a cascade of two sets of filters. Transfer function of the first filter is equal to the transfer function from the input of $H_i(z)$ to the node just before the parameter, a_{il}. The second one is just after a_{il} to the output of $H_i(z)$. A total number of N^2 filter cells are required to compute the overall partial derivatives with respect to one a_{il}. Assuming $N = 4$, the resulting structure is show in Fig.1, which is named matrix form hereafter. The coefficient to be a adapted is inside the $i-th$ cell marked S_i. The required filtering before S_i is defined as prefiltering and that after S_i is defined as postfiltering.

To fully implement the matrix form in hardware, the number of filter cells are large. However, reduncies in this form enable us to implement it with tree structure[6] as well as ring structure. The implementation of ring structure for the special case of adaptive notch filter is given in the next section.

RING STRUCTURE FOR ADAPTIVE NOTCH FILTER

For tracking sinusoids corrupted by white noise, the input signal is

$$x(n) = \sum_{i=1}^{M} A_i sin(2\pi f_i n + \phi_i) + w(n) \qquad (6)$$

where $w(n)$ is the additive Gaussian white noise, f_i, A_i and ϕ_i are the frequency , amplitude and phase of the ith sinusoid.

Becasuse the notch filter posseses the characteristic of unit gain and zero phase, it is suitable to use it as $H_i(z)$. Here, we adopt the minimum parameter structure proposed in [7]:

$$H_i(z) = \frac{1 + \alpha_i z^{-1} + z^{-2}}{1 + \rho\alpha_i z^{-1} + \rho^2 z^{-2}}, \qquad (7)$$

where ρ is the radius of pole. For stability , ρ has to be smaller than 1. The frequency under tracking can be obtained by

$$f_i = -\frac{1}{2\pi} Cos^{-1}(\frac{\alpha_i}{2}) \qquad (8)$$

The 3dB rejection bandwidth of $H_i(z)$ is given by

$$B.W._{3db} = \pi(1-\rho) \qquad (9)$$

The implementation of the matrix form for tracking sinusoids with notch filters can be approximated by a rotated (right-shift) matrix form given in Fig.2. The main difference from Fig.1 is that the data input points and gradient components output points are changed. This change makes the required prefiltering and postfiltering in the matrix form be replaced by prefiltering only in the rotated matrix form. The rotated matrix form still has N^2 notches which can be reduced easily to N notches as the ring structure given in Fig.3., if we reuse the notches with time-sharing schemes. That is, the rotated matrix form is just the temperally decomposed form of the ring structure.

The ring structure can be used to implement the optimum matrix form even postfiltering is not replaced by prefiltering. If this replacement does not cause problem, the rotated matrix form will be a good choice since it has a much simpler control structure than the optimum matrix form. This simplicity could be an important concern in hardware implementation of the ring structure.

In general, if the cells are FIR filters, there is no difference between the matrix form and the rotated form. If the cells are IIR filters, theoretically the rotation is not allowed since an adaptive IIR filter is a time-varying system with memory and rotation will cause permutation of cells which will mix up the time-varying memory. However, for the case of sinusoid tracking with IIR notch filters, it can be shown that its Hessian matrix will be asymptotically diagonal[3], therefore there is no problem in permuting the notches.

For the case of adaption by Newton-Raphson algorithm the coefficient updating equation is

$$\theta_{n+1} = \theta_n - A_n^{-1} e(n) G(n), \qquad (10)$$

whereθ_n is the current coefficient vector:

$$\theta_n = (\alpha_1, \alpha_2, \cdots, \alpha_N)^T, \qquad (11)$$

A_n is the Hessian matrix of ξ at θ_n, and G_n is the gradient vector of ξ at θ_n, i.e.,

$$G_n = (g_1(n), g_2(n), \cdots, g_N(n))^T. \qquad (12)$$

From equation (10), there are interactions among gradient components and coefficients through the Hessian Matrix. However, just as said before that the Hessian Matrix will reduced to a diagonal matrix asymptotically. So that, the Newton-Raphson algorithm can be approximated by a normalized least mean square (NLMS) algorithm. If the diagonal terms are set equal to a scalar μ, one gets the LMS algorithm. No matter whether LMS or NLMS algorithm is used, the update of $H_i(z)$'s coefficient doesn't use the information of gradient components from other cells. This will simplify the operation of A_i appearing in Fig.3.

In reality, the tracking errors $e_i(n)$ at the output of each rows in Fig.2 are not the same, in order to avoid the situation that different row minimize different error function, we average $e_i(n)$ to form a new error $e_t(n)$, i.e.,

$$e_t(n) = \frac{1}{N}\sum_{k=1}^{N} e_i(n) \tag{13}$$

The alternative structure is shown in Fig.4, which is named summed matrix form.

COMPUTER SIMULATION

In order to show the performance of the rotated and summed matrix form, we carry out the following simulations. We adopt the LMS algorithm[8] to adjust α_i because it does not require any matrix inversions or divisions. The update of $i-th$ filter's coefficient is according to

$$\alpha_i(n+1) = \alpha_i(n) - \mu e(n) g_i(n) \tag{14}$$

In the following example, we demonstrate the capability of the structure when the rotated matrix form is implemented to track three close sinusoids. The given sinusoidal frequencies are 0. 10, 0.12 and 0.14 and all sinusoids have a SNR of 0 dB. In addition, after 1500 iterations, the given frequencies are changed to 0.14, 0.17 and 0.20 respectively. The notches' initial frequences are set at 0.03 , 0.05 and 0.16 and ρ is set to 0.95. The step size, μ is 0.030. When computing the gradient components, we adopt the pseudo linear assumption[4]. Referring to Fig.5, it is the learning curve of the direct cascade structure[1] which adapts each notch's coefficient individually. It is obvious that the tracking behaviour is not good. The variances of three tracked frequencies are obviously different. Furthermore, there are severe coupling among frequencies tracked by each notch. Fig.6 is the learning curve for the structure shown in Fig. 2. The three frequencies are decoupled well. From the learning curve, the tracking rate are almost identical for these three notches, thus proposed structure has the ability to track abrupt change in frequency. Fig.7 is the learning curve for the structure shown in Fig.3. Comparing to Fig.6, their performance are almost identical.

CONCLUSION

Adaptive cascade structures for tracking multiple sinusoids has been studied and they can be implemented by the ring structure. Some necessary conditions to allow the ring-structure implememtation are analyzed. From computer simulations, it is found that the ring structure might not degrade the performance of the optimum structure while reducing the number of cells from N^2 to N.

References

[1] B. Widrow and S. D. Stearns, **Adaptive Signal Processing**, Prentice-Hall 1985.

[2] D. V. B. Rao and S. Y. Kung, " Adaptive notch filtering for the retrival of sinusoids in noise," IEEE ASSP-32, pp. 791-802, Aug. 1984.

[3] T. Kwan and Ken Martin, " Adaptive Detection and Enhancement of Multiple Sinusoids Using a Cascade IIR Filter," IEEE CAS-36, pp. 937-947, July, 1989.

[4] T. S. Ng, " Some Aspects of an adaptive digital notch filter with constrained poles and zeros," IEEE Trans. Acoust. speech, signal processing, Vol. ASSP-35, pp. 158-161, Feb. 1987.

[5] R. Seviora and M. Sablatash, "A Tellegen's theorem for digital filters," IEEE Trans. Circuit Theory, Vol. CT-18, pp. 201-203, 1971.

[6] L. B. Jackson and S. L. Wood, " Linear Prediction in Cascade Form," IEEE ASSP-26, pp. 518-528, Dec. 1978.

[7] A. Nehorai, " A minimal parameter adaptive notch filter with constrained poles and zeros," IEEE ASSP-32, pp. 983-996, Aug. 1985.

[8] P. L. Feintuch, " An adaptive recursive LMS filter, " Proc. IEEE vol. 64, no. 11, pp. 1622-1624, Nov. 1976.

[9] L. S. Willimann, " Computation of the Response-Error Gradient of Linear Discrete Filters," IEEE Trans. Acoust. Speech, Signal Processing, Vol. ASSP-22, NO.1, Feb. 1974.

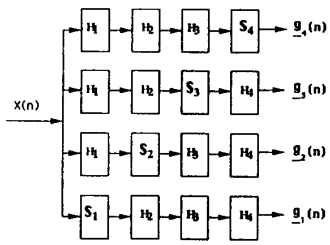

Fig.1 The matrix form for computing the gradient components of optimum cascade structure

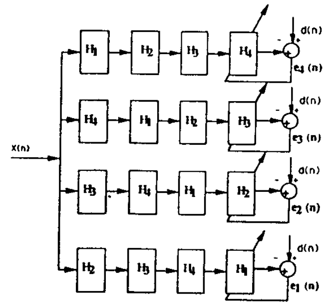

Fig. 2 The rotated matrix form for computing the gradient components.

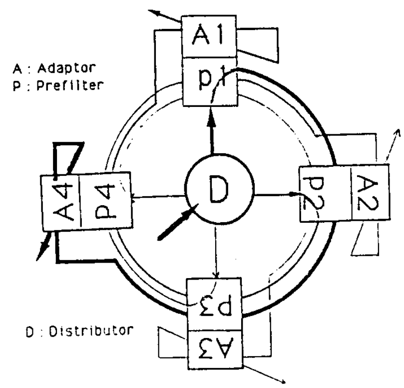

Fig.3 The ring structure for implementation of Fig.2.

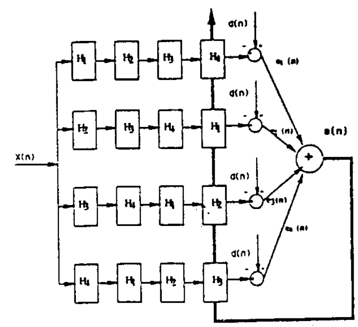

Fig.4 The structure minimizing the summed error.

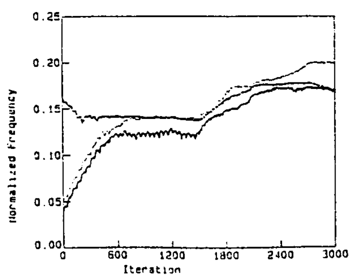

Fig.5 The tracking curve of direct cascade form

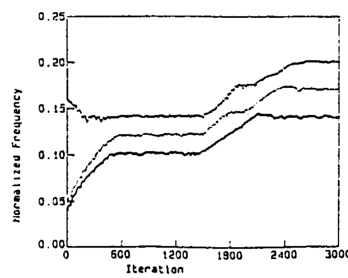

Fig.6 The tracking curve of the rotated matrix structure

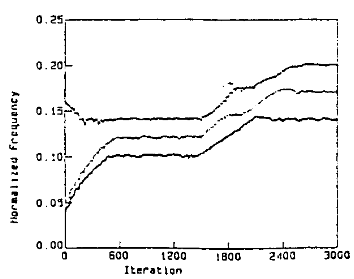

Fig.7 The tracking curve of Fig.4