Adaptive Fuzzy Control of Bank-To-Turn Missiles

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Abstract -**An adaptive fuzzy autopilot is developed for bank-to-turn (BTT) missiles. The BTT missile is a complex multi-input and multi-output (MIMO) system which has a non-minimum phase phenomenon. Fuzzy control is suitable for such system since BTT missiles can be seen as an uncertain system with unmodeled dynamics. One feature of the proposed autopilot is to incorporate the terminal attractor controller into the autopilot to speed up the convergence rate. The stability analysis shows that the states and tracking errors of the BTT missile are uniformly bounded. Two simulation examples demonstrate the effectiveness of the proposed autopilot. The first example is to present the significant contribution of the terminal attractor controller to the convergence rate. Another simulation result is illustrated in our scenarios and shows the superior tracking performance under various flight conditions.**

I. INTRODUCTION

In the past decade, the autopilot design for bank-to-turn (BTT) missiles has received a considerable amount of attention. High maneuverability requirement and large aerodynamic coupling make the design of autopilot for BTT missiles a challenging task. High maneuverability will require high roll rates which will in turn introduce cross-coupled nonlinear dynamics to affect the motion of pitch and yaw. Moreover, because the cross section of a BTT missile is asymmetric, the high acceleration capability in pitch plane will restrict the acceleration in the yaw plane. Other limitations of the autopilot design are keeping attack angles positive and sideslip angles small.

Many approaches have been introduced to do the autopilot design. Lin and Yueh [9] neglected the nonlinear terms so that classical SISO methods can be applied to the pitch and yaw channels, which are simplified to be independent. The conventional approach for designing BTT autopilot based on linear approximation of the cross-coupling at each design point has been widely investigated [10]. Many researches apply optimal control theory to the autopilot design [11][12][13]. Recently, adaptive robust control approaches based on the nonlinear geometric theory have been presented to achieve the satisfactory tracking performance [14], [15].

Fuzzy logic control is generally applied to plants that are hard to be described by mathematical models and/or can successfully be controlled by human experts. In the past, fuzzy control has not been viewed as a rigorous approach due to a lack of formal synthesis techniques that can guarantee the very basic requirements of global stability and acceptable performance [6]. Recently, adaptive fuzzy controllers, which are proved to be globally stable, have been developed in [6], [7]. The fuzzy controllers, used to approximate an optimal controller or unknown part of the plant, are adjusted by an adaptive law based upon a Lyapunov synthesis approach.

Motivated by the work in [4], we are inspired to apply the fuzzy control theory to perform the attitude control for BTT aircraft. In this paper, fuzzy basis function expansion is used to approximate partial unknown parts of the plant. The nominal system of the BTT missiles containing nominal part of aircraft dynamics and the first-order actuator dynamics is derived in [14]. The unmodeled dynamics can be viewed as the uncertainties. On the other hand, a tactical missile should track target as soon as possible. Therefore, the terminal attractor, which is introduced by Zak [3], is added to the controller to improve the convergence rate.

The remainder of this paper is organized as follows. In section II, the mathematical structure of the autopilot design problem is described. Section III is dedicated to the introduction of the architecture of the controller in detail. A global stable adaptive fuzzy autopilot in a constructive manner based on the Lyapunov synthesis technique is developed. In section IV, the faster convergence rate of autopilot with terminal attractor is demonstrated in the first simulation example. Another simulation example is to demonstrate that the proposed fuzzy adaptive autopilot can operate at various flight conditions. At last, section V concludes the paper.

II. PROBLEM STATEMENT

The state notations of BTT missiles can be referred to the Appendix. The overall BTT missile dynamic system including actuators has been shown to be not an affine system [14]. The output signals Φ, A_y and A_z are chosen to track the desired trajectories Φ_c, A_{yc} and A_{zc} respectively. In addition, BTT missiles have significant non-minimum phase phenomenon. In [14], the undesirable non-minimum phase property is avoided by output-redefinition method such that the nominal model of a BTT missile can be viewed as a weakly non-minimum phase system. The new output signals Φ, $\dot{V}$ and $\dot{W}$ are chosen according to the profile of the desired trajectories.

The nominal plant of BTT missiles can be rewritten by the input-output feedback linearization technique [2] in the following form:

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$$\begin{bmatrix} y_1^{(n_1)} \\ y_2^{(n_2)} \\ y_3^{(n_3)} \end{bmatrix} = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ f_3(\underline{x}) \end{bmatrix} + \begin{bmatrix} g_{11}(\underline{x}) & g_{12}(\underline{x}) & g_{13}(\underline{x}) \\ g_{21}(\underline{x}) & g_{22}(\underline{x}) & g_{23}(\underline{x}) \\ g_{31}(\underline{x}) & g_{32}(\underline{x}) & g_{33}(\underline{x}) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{2.1}$$

$$= f(\underline{x}) + G(\underline{x})\underline{u}$$

where $y_1=\Phi$, $y_2=\dot{V}$, $y_3=\dot{W}$, $[u_1 \ \ u_2 \ \ u_3]^T=[\delta_p \ \delta_q \ \delta_r]^T$ and $\underline{x}=[P \ Q \ R \ \Phi \ \Theta \ \Psi \ U \ V \ W]^T$. The relative degree (n_1, n_2, n_3) is found to be (3, 1, 1). In the above equation, $f(\underline{x})$ is an unknown function vector, $G(\underline{x})$ is a known gain matrix as given in the Appendix and $\underline{u}$ is the input vector.

The control objective is to force the plant states, Φ, $\dot{\Phi}$, $\ddot{\Phi}$, $\dot{V}$ and $\dot{W}$, to follow the specified trajectories, Φ_c, $\dot{\Phi}_c$, $\ddot{\Phi}_c$, $\dot{V}_c$ and $\dot{W}_c$, and output signals, y_1, y_2, y_3, to follow the desired output trajectories, y_{1d}, y_{2d}, y_{3d}. For the simplicity of presentation, define the following error vectors:

$$\underline{e}_1 = \begin{bmatrix} y_1 \\ \dot{y}_1 \\ \ddot{y}_1 \end{bmatrix} - \begin{bmatrix} y_{1d} \\ \dot{y}_{1d} \\ \ddot{y}_{1d} \end{bmatrix} = \begin{bmatrix} \Phi \\ \dot{\Phi} \\ \ddot{\Phi} \end{bmatrix} - \begin{bmatrix} \Phi_c \\ \dot{\Phi}_c \\ \ddot{\Phi}_c \end{bmatrix},$$

$$\underline{e}_2 = [y_2 - y_{2d}]^T = [\dot{V} - \dot{V}_c]^T,$$

$$\underline{e}_3 = [y_3 - y_{3d}]^T = [\dot{W} - \dot{W}_c]^T. \tag{2.2}$$

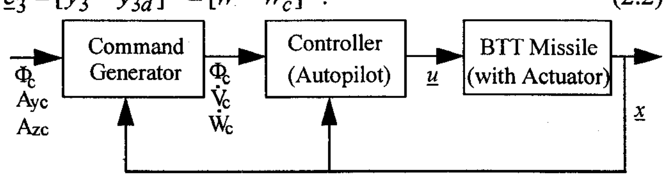

Fig. 1. The block diagram of the closed loop system

Since the outputs are redefined, it is necessary to generate the corresponding redefined tracking signals by a command generator as shown in Fig. 1. If $f(\underline{x})$ is known and the known gain matrix $G(\underline{x})$ is invertible, then the ideal control law is adopted:

$$\underline{u}(t) = G^{-1}\left(\begin{bmatrix} y_{1d}^{(3)} \\ \dot{y}_{2d} \\ \dot{y}_{3d} \end{bmatrix} - \underline{v}_{lf}(t) - \underline{f}(\underline{x})\right), \tag{2.3}$$

where $\underline{v}_{lf}(t)=[\underline{v}_{lf1}(t) \ \underline{v}_{lf2}(t) \ \underline{v}_{lf3}(t)]^T=[\underline{k}_1^T\underline{e}_1 \ \ \underline{k}_2^T\underline{e}_2 \ \ \underline{k}_3^T\underline{e}_3]^T$ is a linear feedback control law. The control law (2.3) will lead to the following error dynamics

$$e_i^{(n_i)} + k_{i1}e_i^{(n_i-1)} + \cdots + k_{in_i} = 0. \tag{2.4}$$

If we choose $\underline{k}_i = [k_{in_i} \ \ \ldots \ \ k_{i1}]^T$ such that all roots of the polynominal $p_i(s) = s^{n_i} + k_{i1}s^{n_i-1} + \cdots + k_{in_i}$ are in the open left half of the complex plane, the closed loop system will be asymptotically stable. Unfortunately, $f(\underline{x})$ is unknown and the control law (2.3) should be modified. For theoretical and practical reasons stated in [6], we can use a fuzzy basis function network to approximate $f(\underline{x})$. Suppose that the control law is modified as:

$$\underline{u}(t) = G^{-1}\left(\begin{bmatrix} y_{1d}^{(3)} \\ \dot{y}_{2d} \\ \dot{y}_{3d} \end{bmatrix} - \underline{v}_{lf}(t) - \underline{v}_{FBF}(t)\right), \tag{2.5}$$

where $\underline{v}_{FBF}(t)=[\underline{v}_{FBF1}(t) \ \underline{v}_{FBF2}(t) \ \underline{v}_{FBF3}(t)]^T$ is the fuzzy basis function expansion used to model the unknown nonlinear function $f(\underline{x})$. With (2.5), the closed loop system becomes

$$\dot{\underline{e}}_i = A_i\underline{e}_i + \underline{b}_i(f_i(\underline{x}) - v_{FBFi}(t)), \ i = 1, 2, 3 \tag{2.6}$$

where $\underline{b}_1=[0 \ 0 \ 1]^T$, $\underline{b}_2=\underline{b}_3=1$, and

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_{11} & -k_{12} & -k_{13} \end{bmatrix}, \ A_2 = -k_2, \text{ and } A_3 = -k_3$$

are matrices in the canonical form, with eigenvalues at the roots of the Hurwitz polynomials which are determined by the linear feedback control law $\underline{v}_{lf}(t)$ [1].

However, the approximation errors and disturbances are unavoidable. One approach to solve the control problem is to define an error metric $\underline{s}$ as:

$$\underline{s}(t) = \begin{bmatrix} (\frac{d}{dt} + \lambda_1)^2(y_1 - y_{1d}) \\ \lambda_2\underline{e}_2 \\ \lambda_3\underline{e}_3 \end{bmatrix} = \begin{bmatrix} \underline{\lambda}_1^T\underline{e}_1 \\ \lambda_2\underline{e}_2 \\ \lambda_3\underline{e}_3 \end{bmatrix} \tag{2.7}$$

where $\underline{\lambda}_1=[\lambda_1^2 \ \ 2\lambda_1 \ \ 1]^T$, λ_1, λ_2, λ_3 are constants. The equation $\underline{s}(t)=0$ defines three time-varying hyperplanes on which the tracking error vectors decay exponentially to zero, so that perfect tracking can be asymptotically obtained by maintaining this condition [1]. From (2.6) and (2.7), the time derivative of the metric can be then written as:

$$\dot{s}_i(t) = -k_{D_i}s_i(t) + f_i(t) - v_{FBFi}(t), \ i = 1, 2, 3, \tag{2.8}$$

where k_{Di} satisfies $k_{Di}^n - k_{in}k_{Di}^{n-1} - k_{i\,n-1}k_{Di}^{n-2} - \cdots - k_{i1} = 0$. Deadzones can be incorporated into error metrics by defining continuous functions $s_{i\Delta}$ as:

$$s_{i\Delta}(t) = s_i(t) - \Delta_i sat(s_i(t)/\Delta_i) \tag{2.9}$$

where *sat* is the saturation function:

$$sat(z) = \begin{cases} 1, & z > 1 \\ z, & |z| \le 1 \\ -1, & z < -1 \end{cases}. \tag{2.10}$$

Deadzone functions, which are specified around the zero of their corresponding error metrics, will be used in the adaptation law to tolerate the parameter errors, unknown dynamics approximation errors and disturbances.

Equation (2.4) implies that the tracking errors will asymptotically converge to zero, that is, they approach zero at infinite time. However, the missile response time should be as small as possible. For this control objective, the concept of terminal attractor is used to reduce the tracking errors to zero in finite time. The detailed controller architecture and stability analysis will be discussed in the following section.

III. ADAPTIVE FUZZY AUTOPILOT DESIGN

A. Fuzzy Basis Function Expansion

Fuzzy systems perform a mapping from $U \subset R^n$ to R. Assume that the fuzzy rule base consists N linguistic rules in the IF-THEN form as follows:

R_j: If x_1 is A_{j1} and x_2 is A_2 and ... and x_n is A_{jn} then y is B_j,

where j=1, 2, ..., N, x_i (i=1, 2, ..., n) are the input variables to the fuzzy system, y is the output variable of the fuzzy system, and A_{ji} and B_j are linguistic terms characterized by their corresponding fuzzy membership functions $\mu_{A_{ji}}(x_i)$ and $\mu_{B_j}(y)$, respectively.

The output of the fuzzy logic system with center-average defuzzification, product inference, and singleton fuzzification is of the following form:

$$y = \frac{\sum_{j=1}^{N} c_j \left(\prod_{i=1}^{n} \mu_{A_{ji}}(x_i)\right)}{\sum_{j=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_{ji}}(x_i)\right)}, \text{ or} \tag{3.1}$$

$$y = \underline{c}^T \underline{\phi}, \tag{3.2}$$

where $\underline{c}=[c_1\ c_2\ \dots\ c_N]^T$ is weight vector, $\underline{\phi}=[\phi_1\ \phi_2\ \dots\ \phi_N]^T$ consists of normalized firing strength ϕ_j, which is referred to as fuzzy basis function and defined as:

$$\phi_j = \frac{\prod_{i=1}^{n} \mu_{A_{ji}}(x_i)}{\sum_{j=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_{ji}}(x_i)\right)}. \tag{3.3}$$

In this paper, $\mu_{A_{ji}}(x_i)$'s are fixed and c_j's are adjustable.

For any given real function h over U, there exists a fuzzy system in the fuzzy basis function expansion form of (3.3) such that it can uniformly approximate h on the compact set U to arbitrary accuracy [5]. Accordingly, we have the following assumption.

Assumption 1: There exist vectors $\underline{c}_1^*$, $\underline{c}_2^*$, and $\underline{c}_3^*$ such that $\hat{f}_1$, $\hat{f}_2$, and $\hat{f}_3$ approximate f_1, f_2, and f_3 with arbitrary accuracy ε_1, ε_2, and ε_3, respectively over a compact set U, i.e.

$$\exists\ \underline{c}_i^* \ s.t.\ |f_i(\underline{x}(t)) - \hat{f}_i(\underline{c}_i^*, \underline{x}(t))| \le \varepsilon_i, \quad i = 1, 2, 3.$$

Hence (2.8) can be rewritten as:

$$\dot{s}_i(t) = -k_{Di} s_i(t) + \hat{f}_i(\underline{c}_i^*, \underline{x}(t)) - v_{FBFi}(t) + d_i(t), \quad i = 1, 2, 3, \tag{3.4}$$

where the disturbance $d_i(t) = f_i(\underline{x}(t)) - \hat{f}_i(\underline{c}_i^*, \underline{x}(t))$ satisfies $|d_i(t)| \le \varepsilon_i$.

It seems reasonable that the approximation can be made arbitrarily good by choosing a sufficient number of rules. However, in practical applications, the size of fuzzy rule base should keep small. Therefore, an adaptive compensator [1], [7] is used to compensate the approximator error of fuzzy approximator, that is, the disturbance is estimated by $\hat{d}_i$.

B. Controller Architecture

The output-redefinition method is applied such that the new command signals should be transformed from the original commands, or trajectories. A fuzzy command generator is used to perform the task. The command Φ_c is not changed and $\dot{V}_c$=0; as to $\dot{W}_c$, it is generated from fuzzy rules in the following form:

If *height* is H_j and *Velocity* is V_j and A_{zc} is *NM* then $\dot{W}_c$ is *NS*,

where H_j, V_j, *NM*, and *NS* are linguistic terms of missile's height, missile's velocity, A_{zc}, and $\dot{W}_c$, respectively. Such rules are determined by analysis of data.

The architecture of the controller consists of three components: linear feedback controller, terminal attractor controller and fuzzy controller as shown in Fig. 2. Then, the overall control law is to combine the output of the three controllers as follows:

$$\underline{u} = G^{-1}(-\underline{v}_{lf} - \underline{v}_{TA} - \underline{v}_{FBF}) \tag{3.5}$$

where

$$\underline{v}_{lf} = \begin{bmatrix} v_{lf1} \\ v_{lf2} \\ v_{lf3} \end{bmatrix} = \begin{bmatrix} -y_{1d}^{(3)} + k_{11}(y_1 - y_{1d}) + k_{12}(\dot{y}_1 - \dot{y}_{1d}) + k_{13}(\ddot{y}_1 - \ddot{y}_{1d}) \\ -\dot{y}_{2d} + k_2(y_2 - y_{2d}) \\ -\dot{y}_{3d} + k_3(y_3 - y_{3d}) \end{bmatrix}, \tag{3.6}$$

$$\underline{v}_{TA} = \begin{bmatrix} v_{TA1} \\ v_{TA2} \\ v_{TA3} \end{bmatrix} = \begin{bmatrix} k_{ta1} s_{1\Delta}^{\frac{1}{\alpha_1}} \\ k_{ta2} s_{2\Delta}^{\frac{1}{\alpha_2}} \\ k_{ta3} s_{3\Delta}^{\frac{1}{\alpha_3}} \end{bmatrix}, \text{ and} \tag{3.7}$$

$$\underline{v}_{FBF} = \begin{bmatrix} v_{FBF1} \\ v_{FBF2} \\ v_{FBF3} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{N_1} \hat{c}_{1j}\phi_{1j} + \hat{d}_1 \\ \sum_{j=1}^{N_2} \hat{c}_{2j}\phi_{2j} + \hat{d}_2 \\ \sum_{j=1}^{N_3} \hat{c}_{3j}\phi_{3j} + \hat{d}_3 \end{bmatrix}, \tag{3.8}$$

where α_1, α_2, and α_3 are positive integers, k_{ta1}, k_{ta2}, and k_{ta3} are positive constants. In the numerical simulation in next section, we choose $\alpha_1=\alpha_2=\alpha_3=3$.

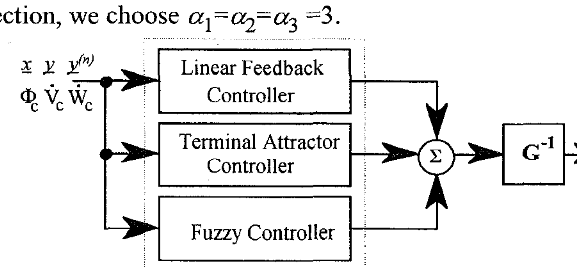

Fig. 2. The block diagram of autopilot.

The parameters $\hat{c}_{ij}$'s and $\hat{d}_i$'s are updated by the following adaption law:

$\dot{\hat{c}}_{ij} = k_{ci} s_{i\Delta} \phi_j$

$$\dot{\hat{d}}_i = k_{di} s_{i\Delta}, \quad i = 1, 2, 3, \quad j = 1, \ldots, N_i, \tag{3.9}$$

where k_{ci} and k_{di} are positive constants determining the adaptation rate. We also denote the parameters' error as: $\tilde{c}_{ij} = \hat{c}_{ij} - c^*_{ij}$ and $\tilde{d}_i = \hat{d}_i - d_i$. Hence, the time derivative of error metric (3.4) can be rewritten as:

$$\dot{s}_i(t) = -k_{Di} s_i(t) - \sum_{i=1}^{N_i} \tilde{c}_{ij}\phi_j - k_{tai} s_{i\Delta}^{\frac{1}{\alpha_i}} - \tilde{d}_i(t), \quad i = 1, 2, 3, \tag{3.10}$$

Before proceeding to do the stability analysis, we discuss that the choice of the coefficients of Hurwitz polynomial. The coefficients are not only confined in the left of complex plane but should also satisfy some other restrictions. Assume that A_i in (2.6) is an nxn canonical matrix whose eigenvalues are all in the open left half plane by choosing the last row vector $\underline{k}_i = [k_{i1}, k_{i2}, \ldots, k_{in}]$ suitably. However, $\underline{k}_i$ and k_{Di} must satisfy some relationship as stated below.

$$\underline{\lambda}_i^T A_i \underline{e}_i = -k_{Di} \underline{\lambda}_i^T \underline{e}_i \quad \text{or} \tag{3.11}$$

$$\begin{aligned}
-k_{Di}\,\lambda_{i1} &= -k_{i1}\,\lambda_{in} \\
-k_{Di}\,\lambda_{i2} &= -k_{i2}\,\lambda_{in} + \lambda_{i1} \\
-k_{Di}\,\lambda_{i3} &= -k_{i3}\,\lambda_{in} + \lambda_{i2} \\
&\vdots \\
-k_{Di}\,\lambda_{in} &= -k_{in}\,\lambda_{in} + \lambda_{in-1}
\end{aligned} \tag{3.12}$$

From (3.12), it can be shown that $\underline{k}_i$ and k_{Di} should satisfy the following relationship:

$$k_{Di}^n - k_{in} k_{Di}^{n-1} - k_{i\,n-1} k_{Di}^{n-2} - \cdots - k_{i1} = 0. \tag{3.13}$$

C. Stability Analysis

Theorem 1: Consider the dynamic equations of BTT missiles (2.1) with the control law (3.5)-(3.9), and then all states in the (2.1) will remain bounded and the tracking errors will approach zero.

proof: Consider the Lyapunov function candidate $V_i(t) = \frac{1}{2}(s_{i\Delta}^2 + \frac{1}{k_{ci}} \sum_{j=1}^{N_i} \tilde{c}_{ij}^2 + \frac{1}{k_{di}} \tilde{d}_i^2)$. Since $\frac{d}{dt} s_{i\Delta}^2 = 2 s_{i\Delta} \dot{s}_i$ and $\dot{s}_i$ is continuous everywhere, $\frac{d}{dt} s_{i\Delta}^2$ is well defined and continuous everywhere.

Differentiating $V_i(t)$ with respect to time, then $\dot{V}_i = 0$ when $|s_i| \le \Delta_i$. When $|s_i| > \Delta_i$, using (3.10) and $s_{i\Delta} sat(s_i / \Delta_i) = |s_{i\Delta}|$, one has

$$\dot{V}_i = s_{i\Delta}\dot{s}_{i\Delta} + \frac{1}{k_{ci}} \sum_{j=1}^{N_i} \tilde{c}_{ij}\dot{\tilde{c}}_{ij} + \frac{1}{k_{di}} \tilde{d}_i \dot{\tilde{d}}_i$$

$$= s_{i\Delta}(-k_{Di}s_i - \sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j - \tilde{d}_i - k_{tai} s_{i\Delta}^{\frac{1}{\alpha_i}}) + \sum_{j=1}^{N_i} \tilde{c}_{ij} s_{i\Delta}\phi_j + \tilde{d}_i s_{i\Delta}$$

$$= -k_{Di} s_{i\Delta}^2 - k_{Di}|s_{i\Delta}|\Delta_i - k_{tai} s_{i\Delta}^{\frac{1+\alpha_i}{\alpha_i}} < 0.$$

Therefore, if $s_{i\Delta}$ and all $\tilde{c}_{ij}$'s are bounded at initial time t=0, they will remain bounded for all time t>0. If $\underline{\tilde{y}}_i(0)$ is bounded, then $\underline{\tilde{y}}_i(t)$ is also bounded for all time t, and since $\underline{y}_{id}(t)$ is specified, $\underline{y}_i(t)$ is bounded as well. Next, we will show that $s_{i\Delta} \to 0$ as $t \to \infty$. It is easy to show by Barbalat's lemma [2]:

$$V_{1i}(t) = V_i(t) - \int_0^t [\dot{V}_i(\tau) + (k_{Di} s_{i\Delta}^2 + k_{Di}|s_{i\Delta}|\Delta_i + k_{tai} s_{i\Delta}^{\frac{1+\alpha_i}{\alpha_i}})] d\tau$$

with $\dot{V}_{1i}(t) = -(k_{Di} s_{i\Delta}^2 + k_{Di}|s_{i\Delta}|\Delta_i + k_{tai} s_{i\Delta}^{\frac{1+\alpha_i}{\alpha_i}})$. Thus, every term in (3.10) is bounded. Hence, $s_{i\Delta}$ is bounded and s_i is bounded as well. This implies that $\dot{V}_{1i}(t)$ is a uniformly continuous function of time. Since V_{1i} is bounded by 0, and $\dot{V}_{1i} \le 0$ for all time t, Barbalat's lemma can be applied to prove that $\dot{V}_{1i} \to 0$ and hence $s_{i\Delta} \to 0$ as $t \to \infty$. Q.E.D.

D. Terminal Attractors

So far, we have discussed the autopilot (controller) architecture and the stability issue. However, the terminal attractors have not mentioned yet. The concept of terminal attractors can be easily explained by the equation $\dot{x} = ax^b$. An equilibrium point at x=0 will be approached by transients in finite time under the conditions: a<0 and 0<b<1, and repellers appear when a>0. In this section, we will use a theorem to state the existence of terminal attractors in our autopilot. Based upon Theorem 1, we can make the following assumption:

Assumption 2: If there are K rules whose fuzzy basis function value is greater than zero, then $|\tilde{c}_{ij}| \le M_c$, and $|\sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i| \le M_c \cdot K + \varepsilon_i$.

Theorem 1 stated that $\tilde{c}_{ij}$'s are bounded and $0 \le \phi_j \le 1$, so Assumption 2 is reasonable. Define the squared error $E = \frac{1}{2} s_{i\Delta}^2(t)$ as the error measure for the control task.

Theorem 2: Applying the control law (3.5)-(3.8) to (2.1), then $s_{i\Delta} = 0$ is a terminal attractor of $\dot{E}$.

proof: If $|s_i| \le \Delta_i$ or $s_{i\Delta} = 0$, then $\frac{d}{dt} s_{i\Delta}^2 = 2\dot{s}_i s_{i\Delta} = 0$. Next, we will show that when $|s_i| > \Delta_i$, $\frac{d}{dt} s_{i\Delta}^2 = -\eta_i (s_{i\Delta}^2)^{\frac{1}{2}} - k_{tai} (s_{i\Delta}^2)^{\frac{1+\alpha_i}{2\alpha_i}}$.

When $|s_i| > \Delta_i$ or $s_{i\Delta} \ne 0$, Assumption 1 is satisfied and there exist some k_{D_i} and Δ_i such that

$$|k_{D_i} s_i| > k_{D_i}\Delta_i > M_c \cdot K + \varepsilon_i > |\sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j| + |\tilde{d}_i| \ge |\sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i| .$$

The equation $|k_{D_i} s_i| > |\sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i|$ implies that sgn(s_i)= sgn($s_{i\Delta}$)=sgn($k_{D_i} s_i + \sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i$),

where sgn is a sign function. If the signs are different, then the terminal attractor becomes a repeller.

$$\frac{d}{dt} s_{i\Delta}^2 = 2 s_{i\Delta} \dot{s}_i$$

$$= -s_{i\Delta}(k_{D_i} s_i + \sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i) - k_{tai} s_{i\Delta}^{\frac{1+\alpha_i}{\alpha_i}}$$

$$= -\eta_i (s_{i\Delta}^2)^{\frac{1}{2}} - k_{tai} (s_{i\Delta}^2)^{\frac{1+\alpha_i}{2\alpha_i}} < 0,$$

where $\eta_i = k_{D_i} s_i + \sum_{j=1}^{N_i} \tilde{c}_{ij}\phi_j + \tilde{d}_i$. Therefore, $s_{i\Delta} = 0$ is a terminal attractor of $\dot{E}$. Q.E.D.

IV. SIMULATIONS

In this section, two examples are used to evaluate the performance of the proposed autopilot design.

Example 1: The main purpose of this example is to show the power of the terminal attractor controller. In this example, we compare three autopilots: the proposed autopilot, the proposed autopilot without the terminal attractor controller and the Gaussian neural network controller. The Gaussian neural network controller is a similar approach to the proposed method; however, the Gaussian neural network controller is not capable of incorporating the expert's knowledge. Another disadvantage is that the size of the Gaussian neural network is always larger than that of the proposed controller. In this simulation, during the interval between 0 second and 4 second, the desired output signals are Φ_c=135°, A_{yc}=0G, and A_{zc}=-15G, that is, the missile rolls 135° and is given a force to climb. The design specifications are rise time ≤ 0.5 sec and overshoot ≤ 10%. The simulation results are shown in Fig. 3. The number of linguistic terms of Φ, $\dot{\Phi}$, $\ddot{\Phi}$, $\dot{V}$ and $\dot{W}$ are 11, 11, 3, 11 and 11, respectively. Thus the number of rules in the fuzzy rule base is 385. As to the Gaussian neural network controller, there are 6795 hidden neurons are used. Therefore, in comparison with the Gaussian neural network controller, the proposed autopilot has the nice feature of using smaller rule base to achieve desired performance.

Example 2: In traditional gain scheduling controller, the controller is highly dependent on the LTI controller at each fixed operating point. In our approach, complexity is reduced for only one autopilot is used. Another autopilot design in [15], the design methodology is highly dependent upon the geometric parameters and the aerodynamics conditions. Our approach is more simple and easy to design.

In this example, the desired commands of the four flight conditions are:

I. Φ_c=0°, A_{yc}=0G, and A_{zc}=-50G
II. Φ_c=135°, A_{yc}=0G, and A_{zc}=-10G
III. Φ_c=90°, A_{yc}=0G, and A_{zc}=-3G
IV. Φ_c=0°, A_{yc}=0G, and A_{zc}=0G

The fuzzy rule base is the same as example 1; that is, the number of rules is also 385. Fig. 4 shows the traces of the missile meets the specified trajectory.

V. CONCLUSIONS

Concepts from terminal attractors have been used to derive an alternative adaptive autopilot for BTT missiles resulting in fast convergence of tracking errors. One advantage of the autopilot is capable of incorporating fuzzy IF-THEN rules into autopilot. The stability analysis which takes the unmodeled dynamics into account guarantees the autopilot is globally stable. Simulation results show that superior tracking performance is achieved by the simple adaptive fuzzy autopilot.

VI. APPENDIX

The complete 6-DOF missile dynamic equations have been derived in [14]. Fig. A-1 shows the diagram of BTT missiles. The state notations are as follows.

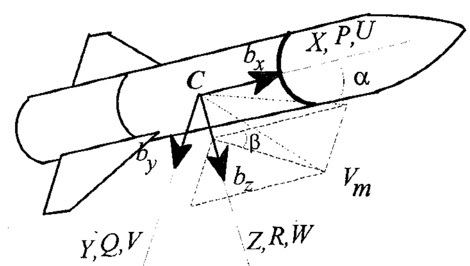

Fig. A-1. The BTT missile diagram.

Notations:

- $\{b_x, b_y, b_z\}$: a right-handed orthonormal basis of body coordinate frame which is attached to the center of mass, C, of the missile, where b_x, b_y are on the longitudinal and lateral axis, respectively;
- P,Q,R: roll rate, pitch rate and yaw rate corresponding to the axes b_x, b_y and b_z, respectively (clockwise), (rad/sec);
- Φ:, Θ, Ψ: roll angle, pitch angle, and yaw angle (rad);
- $(X, Y, Z)^T$: position vector of the center of mass of the missile (m);
- $(U, V, W)^T$: velocity vector of the missile (m/sec);
- δ_p, δ_q, δ_r: aileron deflation angle, elevator deflation angle and rudder deflation angle, respectively, (rad);

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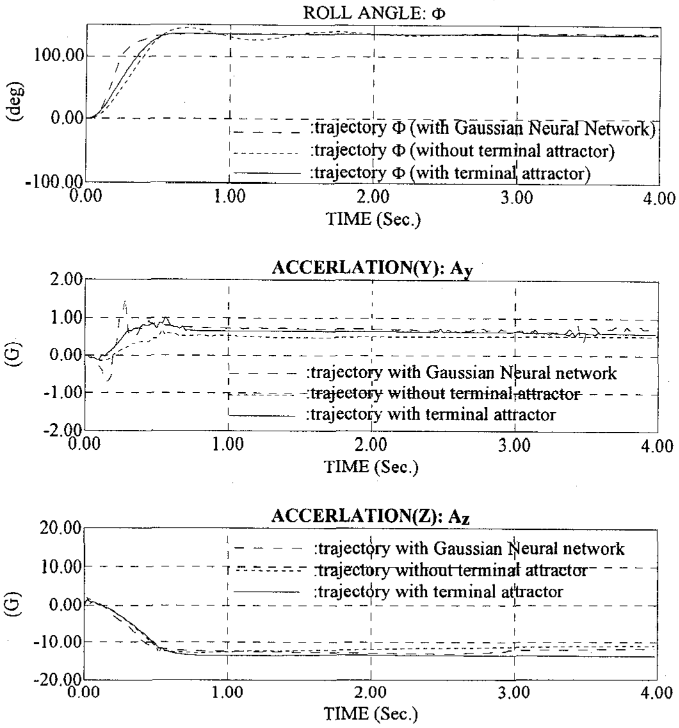

Fig. 3 Simulation results of example 1

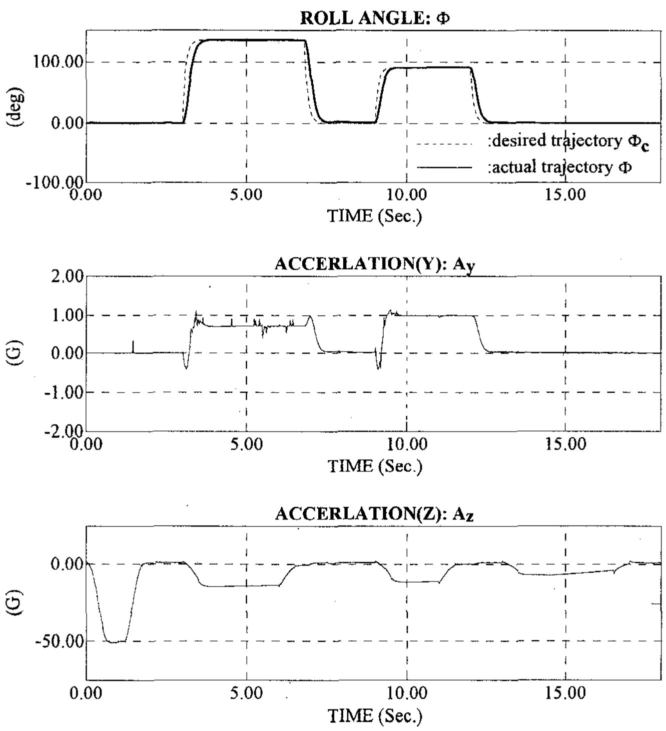

Fig. 4 Simulation results of example 2