

## ANTENNA POLARIMETRIC CALIBRATION USING MULTI-MODE TRL CALIBRATION METHOD AND ITS EXTENSION

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This paper presents two antenna polarimetric calibration methods. One is to use a multi-mode TRL calibration method. This method is generalized from the conventional TRL method, hence its operation bandwidth is limited. The other one is the "TRR (thru-rotation-reflector)" calibration method. This method uses one antenna to rotate along the propagation axis with an unknown angle. It has no limitation on the operation bandwidth.

### 1 Introduction

The TRL (thru-reflection-line) calibration method [1] and its related methods [2] are widely used in two-port microwave circuit measurement. In [3], the TRL calibration is generalized to a multi-mode TRL calibration method. This method is developed for the circuits with transmission lines containing more than one propagating mode. In this paper, we will show that the multi-mode TRL calibration method and its extension can be used for antenna polarimetric calibration.

The wave propagation in free space can be decomposed into two linearly independent polarizations. These two polarizations can be considered as two modes propagating in free space. Hence the multi-mode TRL can be applied to the antenna polarimetric calibration. However, the TRL calibration has a limited operating bandwidth caused by the line calibrator.

In the antenna polarimetric calibration, one has an advantage that is not readily available in the circuit calibration to rotate antenna along the axis of propagation. This will then give a new calibrator which is not bandwidth limited. As can be shown, the rotation angle can be unknown as a self-consistent parameter. This new type of antenna polarimetric calibration method is given as the "TRR (thru-rotation-reflector)" calibration method and will be described in the following section.

### 2 Formulation

#### 2.1 Generalized transfer and scattering matrices

In general, the incident and reflected waves in free space can be decomposed into horizontal and vertical polarization components. For example at port  $j$ , they can be denoted by wave vectors as

$$\begin{bmatrix} A_j \end{bmatrix} = \begin{bmatrix} a_{h,j} \\ a_{v,j} \end{bmatrix} \text{ and } \begin{bmatrix} B_j \end{bmatrix} = \begin{bmatrix} b_{h,j} \\ b_{v,j} \end{bmatrix}. \quad (1)$$

The transfer matrices between port 1 and 2 are then given as

$$\begin{bmatrix} B_1 \\ A_1 \end{bmatrix} = T \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}. \quad (2)$$

In (2),  $T$  is a  $4 \times 4$  square matrix and each  $T_{ij}$  is a  $2 \times 2$  square submatrix. Similarly, one can define the scattering matrix and its submatrices as

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = S \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}. \quad (3)$$

The four port scattering matrix in (3) can be directly measured by a four-port network analyser or indirectly by a two-port network analyser with method proposed in [4] or [5]. Each measured transfer matrix  $M$  is related to the DUT (device under test) transfer matrix  $N$  by

$$M = ANB^{-1} \quad (4)$$

where  $A$  and  $B$  are the transfer matrices of the transitions  $A$  and  $B$  as described in Fig. 1.

## 2.2 Multi-mode TRL for antenna polarimetric calibration

Each DUT of generalized transfer matrix  $N$  will give a measured matrix  $M$  as:

$$M = ANB^{-1} \text{ and } N = A^{-1}MB. \quad (5)$$

As the two antennas are placed face to face and separated by an appropriate distance, this gives the “thru” calibrator as in the TRL calibration method. The reference plane is then at the midpoint of two antennas separation. Hence by locating two antennas apart by a distance  $l$ , it becomes the “line” calibrator. The transfer matrix of thru ( $N_1$ ) and line ( $N_2$ ) are given as

$$N_1 = \begin{bmatrix} I & \bar{0} \\ \bar{0} & I \end{bmatrix} \text{ and } N_2 = \begin{bmatrix} \exp(-\gamma_h l) & 0 & 0 & 0 \\ 0 & \exp(-\gamma_v l) & 0 & 0 \\ 0 & 0 & \exp(\gamma_h l) & 0 \\ 0 & 0 & 0 & \exp(\gamma_v l) \end{bmatrix}, \quad (6)$$

where  $I$  denotes an identity matrix and  $\bar{0}$  is a null matrix.  $\gamma_h$  and  $\gamma_v$  are the propagation constants for horizontal and vertical polarization. As in [3], two matrices are defined as

$$P = N_2 N_1^{-1}, Q = M_2 M_1^{-1} \text{ and } P = A^{-1}QA. \quad (7)$$

$P$  and  $Q$  are similar matrices, so they have identical eigenvalues. One can define an eigenvalue matrix  $\Lambda$  as

$$\text{eig}(P) = \text{eig}(Q) = \lambda_i, \Lambda = \text{diag}(\lambda_i), i=1\sim 4. \quad (8)$$

Since  $P$  is equal to  $N_2$ ,  $N_2$  and  $Q$  have identical eigenvalues. This characteristics can be used to find the unknown distance  $l$ .

As derived in [3], the reflection (R) calibrator must be reciprocal and at least one of its non-diagonal elements must be non-zero. The dihedral corner reflector is commonly used in the polarimetric radar calibration. The scattering matrix of a dihedral corner reflector rotated by an angle  $\phi$  is given as

$$R = \eta \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}, \quad (9)$$

where  $\eta$  is related to the location of the reflector and the reference plane. One can clearly see that dihedral reflector satisfies the R calibrator requirement in the multi-mode TRL when  $\phi \neq 0$ . Since the reflection coefficients of the calibrator can be calculated after the calibration, the angle  $\phi$  can be unknown and solved from the calibration process.

## 2.3 TRR calibration method

As described in the previous section, the line calibrator in TRL calibration is now replaced by a rotated calibrator. By rotating one of the two antennas by an angle  $\theta$  along the direction of wave propagation, the transfer matrix  $N_2$  of this rotation calibrator becomes

$$N_2 = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \\ 0 & 0 & \sin \theta & \cos \theta \end{bmatrix}, \quad (10)$$

From the thru and rotation measurement results, one can calculate the eigenvector matrix  $V$  and eigenvalue matrix  $\Lambda$  of  $P$  as

$$V = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ and} \quad (11)$$

$$\Lambda = \begin{bmatrix} \cos\theta + \sin\theta & 0 & 0 & 0 \\ 0 & \cos\theta - \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta + \sin\theta & 0 \\ 0 & 0 & 0 & \cos\theta - \sin\theta \end{bmatrix} \quad (12)$$

Since the eigenvalues can be calculated from the measured transfer matrices  $Q$ , the rotation angle can be solved. A rotated corner reflector is then served as the reflection calibrator in the described TRR calibration method.

As compared with the line calibrator used in the multimode TRL method, the rotation of calibrator is not frequency dependent. Therefore TRR becomes a wideband calibration method.

### 3 Conclusion

In this paper, we have presented two antenna polarimetric calibration methods. One is the direct implement using the multi-mode TRL method. The other is the TRR method, in which a rotation calibrator is given by rotating one of two antennas along the axis of propagation at a certain angle. This rotation angle can be solved during the calibration as a self-consistent parameter. In addition, the rotation calibrator has no limitation in bandwidth as compared with the line calibrator in TRL method.

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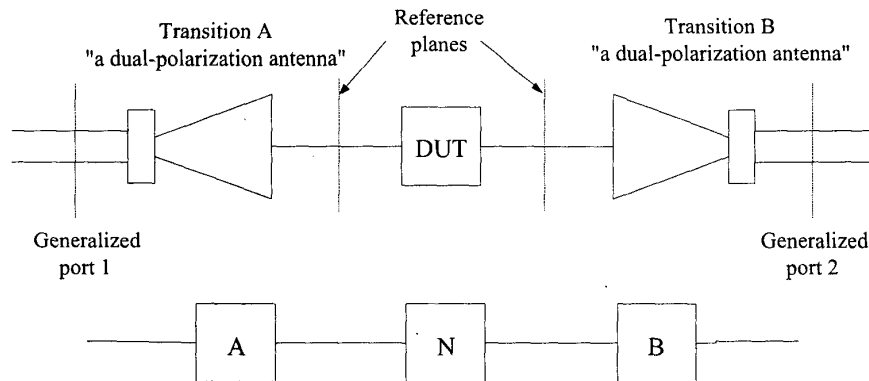


Fig.1 Basic arrangement for antenna polarimetric calibration.