Proceedings of the 1998 IEEE
International Conference on Control Applications
Trieste, Italy 1-4 September 1998

WP06

Robust Neuro-Fuzzy Model-Following Control of Robot Manipulators

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Abstract

A robust neuro-fuzzy model-following control system is proposed for robot control with torque disturbance and measurement noise. The control objective is obtained by tailoring a nominal adaptation process of weights and a fine tuning mechanism to overcome the equivalent uncertainty. The major difference comparing with previous approaches is that a novel fuzzy system is introduced such that the fuzzy rules are in the form of "IF *situation* THEN *the control input*" rather than "IF *situation* THEN *the value of some nonlinear function*". Using Lyapunov stability method, the uniform ultimate boundedness of tracking error has been proved.

Keywords: Neuro-fuzzy, Robust, Model following control, Robot manipulators.

1. Introduction

In the proposed neuro-fuzzy logic controller (NFLC) for the robust model-following control of robot manipulators, the parameters of the controlled plant are not assumed to be linear as in standard adaptive control techniques. No prior knowledge about the parameters and system matrix including its size is required. The proposed scheme has been inspired from the previous works [1-4], and we extend the application field to n-link robot manipulators with torque disturbance and measurement noise. The major difference comparing with previous works is that a novel Fuzzy System with Rule Credit Assignment (FS-RCA) is introduced such that the fuzzy rules are in the form of "IF *situation* THEN *the control input*" rather than "IF *situation* THEN *the value of some nonlinear function*". Thus, the major superiority of the fuzzy logic control is preserved, since the knowledge of "*the value of some nonlinear function*" exists inherently in the plant but hard to obtain from human expert knowledge. Besides, the proposed scheme can specifically deal with the measurement noise and considerably reduce the size of tracking error residual set. In order to dealing with the effect of uncertain modeling error and shrink the tracking error residual set, an on-line fine tuning mechanism for the consequent membership functions of the neuro-fuzzy system is constructed. This gives the neuro-fuzzy system some degree of "adaptability" in the sense that the fine tuning mechanism can reflect to different disturbances and plant uncertainties. The NFLC can be trained in two different ways. If the rough mathematical model or parameter of the robot manipulator is available, then *a priori* model knowledge is utilized to train the NFLC off-line. Otherwise the weights are tuned by using an on-line robust adaptation scheme. Using Lyapunov method, it is shown that the weight adaptation has some degree of robustness and the output tracking error can converge to a residual set ultimately.

2. The control problem

The dynamics of an n-degree-of-freedom rigid manipulator can be described in general form as follows,

$$u+\Delta(q,\dot{q})=H(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q) \qquad (1)$$

or

$$\ddot{q}=f(q,\dot{q})+G(q)u+d(q,\dot{q},t) \qquad (2)$$

with

$$f(q,\dot{q})=H^{-1}(q)(-C(q,\dot{q})\dot{q}-G'(q)),$$

$$G(q)=H^{-1}(q),\ d(q,\dot{q},t)=H^{-1}(q)\Delta(q,\dot{q},t),$$

where all the terms have the same meaning as chosen in [5] and Δ is the actuator input noise. Taking the measurement noise into account, we have $q_n=q+n_p$, $\dot{q}_n=\dot{q}+n_v$ where the vector noise n_p and n_v, as well as Δ are supposed to be bounded random perturbations and $n_p\in C^2$. Let q_{Mi} and v_i denote the reference output and input, respectively. The control strategy is to conduct the behavior of the ith joint asymptotically following a linear reference model of the following form

$$\ddot{q}_{Mi}=\alpha_{i1}q_{Mi}+\alpha_{i2}\dot{q}_{Mi}+v_i \qquad (3)$$

in the presence of bounded disturbance and measurement noise. The constants α_{i1},α_{i2} are selected such that an asymptotically stable reference model with desirable properties is obtained. Fig. 1 shows the architecture of the neuro-fuzzy logic controlled robotic system. Fig. 2 shows the proposed FS-RCA, which is an ith fuzzy system for the ith joint of the robot manipulator.

3. The neuro-fuzzy logic controller

3.1 The multi-layer fuzzy system

Considering the request of numerical input and output of the fuzzy system, a particular class of fuzzy system with the singleton fuzzified, algebraic product T-norm, the sup star compositional operator [2] and the local mean-of-maximum [6] method are used.

Fuzzy Rule Base Let $x=[q,\dot{q}]^T\in R^{2n}$, $\bar{x}=[q_n,\dot{q}_n]^T\in R^{2n}$, a multivariable system can be controlled by the following $N+1$ linguistic rules

R^j : IF x_1 is A_1^j $AND \cdots AND$ x_{2n} is A_{2n}^j
THEN u_1 is B_1^j $AND \cdots AND$ u_n is B_n^j for $j = 1, \cdots, N+1$.
The fuzzy sets A_k^j and B_i^j are linguistic terms characterized by the fuzzy membership functions

$$\mu_{A_k^j}(x_k) = \exp(-(x_k - m_k^j)^2 / a_k^j) \tag{4}$$

and

$$\mu_{B_i^j}(u_i) = \begin{cases} (1 + ((c_i^j - u_i)/a_{Li})^2)^{-1}, & if\ u_i \le c_i^j \\ (1 + ((u_i - c_i^j)/a_{Ri})^2)^{-1}, & if\ u_i > c_i^j \end{cases} \tag{5}$$

where $\{a_k^j, m_k^j\}$ and $\{a_{Li}, a_{Ri}, c_i^j\}$ are referred to the premise and consequence parameters, respectively.

Rule Credit Assignment: The basic idea of the rule credit assignment is to reward good rules by increasing the confidence of the consequent fuzzy sets and the recommendation fuzzy output of this rule. Denote $\omega_{ii}^j > 1$ (or $\omega_{ii}^j < 1$) as a reward (or a punishment) offered to the jth rule in the ith knowledge rule base, then the consequent membership function (5) can be reshaped into

$$\mu_{\tilde{B}_i^j}(u_i) = \begin{cases} (1 + (\omega_{ii}^j (c_i^j - u_i)/a_{Li})^2)^{-1}, & if u_i \le c_i^j \\ (1 + (\omega_{ii}^j (u_i - c_i^j)/a_{Ri})^2)^{-1}, & if u_i > c_i^j \end{cases} \tag{6}$$

and the recommendation fuzzy output of each rule is determined in singleton form as follows,

$$\begin{aligned} &\omega_{ii}^j \cdot I(\mu^j(\bar{x}), \mu_{\tilde{B}_i^j}(u_i)) \\ &= \begin{cases} \omega_{ii}^j \cdot \mu^j(\bar{x}), & for\ u_i = \tilde{c}_i^j \\ 0, & otherwise \end{cases} \end{aligned} \tag{7}$$

where "." is the multiplication operation, I is the implication function [2], $\mu^j(\bar{x}) = \mu_{A_1^j}(\bar{x}_1)\mu_{A_2^j}(\bar{x}_2)\cdots\mu_{A_{2n}^j}(\bar{x}_{2n})$ denotes the matching degree, respectively, $\tilde{c}_i^j$ denotes the location of the singleton implication fuzzy set and is defined as

$$\tilde{c}_i^j = \text{the centroid of the set } \{u_i : \mu_{\tilde{B}_i^j}(u_i) \ge \mu^j(\bar{x})\} \tag{8}$$

Using (6), (8) can be resolved into

$$\tilde{c}_i^j = c_i^j - a_{LRi}\mu^j \sqrt{(\mu^j)^{-1} - 1} \Big/ \omega_{ii}^j \tag{9}$$

where $a_{LRi} = (a_{Li} - a_{Ri})/2$.

Fine-tuning mechanism: Physically, the parameter a_{LRi} represents the difference between the left and right spread of the consequent membership functions. In this paper, this term is employed as a robust control component and a robust adaptive law for it is proposed in the next section.

Analytical formulation of the multi-layer fuzzy system: Using the center average defuzzification, the output response of the fuzzy controller is

$$u_i^0(t) = F_i(\bar{x}, \omega_{ii}^j, a_{LRi}) = \frac{\sum_{j=1}^{N+1} \omega_{ii}^j \cdot \mu^j \cdot \tilde{c}_i^j}{\sum_{j=1}^{N+1} \omega_{ii}^j \cdot \mu^j} \tag{10}$$

In the rule base, the (N+1)th rule is chosen to be of Takagi-Sugeno type and its consequent fuzzy set B_i^{N+1} is singleton with support represented as the form of the synthesis input

$$c_i' = \alpha_{i1} q_{ni} + \alpha_{i2} \dot{q}_{ni} + v_i \tag{11}$$

The curvature control parameter, a_k^{N+1}, of its antecedent membership functions assumed to approach to infinity so that this rule will be fired whatever $\bar{x}$ is. The credit assignment takes place in rules R^j, $j = 1, \cdots, N$ but assigned to be 1 for R^{N+1}. Accordingly, using (9) and (11), the analytical formulation of the multi-layer fuzzy system in equation (10) resolves into

$$u^0 = \hat{D}^{-1}(-\theta'^T \mu' + c' - a_{LR}\phi) \tag{12}$$

where $\hat{D} = Block\ diag(\omega_{11}^T \mu, \cdots, \omega_{nn}^T \mu)$, ω_{ii} and μ are $(N+1) \times 1$ column vectors composed of ω_{ii}^j and μ^j, $\theta' = [\theta_1', \cdots, \theta_n'] \in R^{N \times n}$, θ_i and μ' are $N \times 1$ column vectors composed of $\omega_{ii}^j c_i^j$ and μ^j, $c' = [c_1', \cdots, c_n']^T$, $a_{LR} = [a_{LR1}, \cdots, a_{LRn}]^T$, and $\phi = \sum_{j=1}^{N+1} \mu^j \sqrt{(\mu^j)^{-1} - 1}$.

3.2 The decouping network and the overall control law

As shown in Fig. 1, the output of the robust neuro-fuzzy controller, u, is a combination of u^0 and its modification by the decoupling network as shown below.

$$u = u^0 + Mu^0 \tag{13}$$

The decoupling network does not aim at complete decoupling by exact modeling to the interconnection effect among subsystems. Instead, it compensates the non-diagonal terms of G in (2). The matrix M is chosen as

$$M = -(I_n + \hat{C}^{-1}\hat{D})^{-1} \tag{14}$$

where I_n denotes a $n \times n$ identity matrix and

$$\hat{C} = \begin{bmatrix} 0 & \omega_{12}^T \mu & \cdots & \omega_{1n}^T \mu \\ \omega_{21}^T \mu & 0 & \cdots & \omega_{2n}^T \mu \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{n1}^T \mu & \omega_{n2}^T \mu & \cdots & 0 \end{bmatrix} \tag{15}$$

The Non-Singularity Supervisor: Since such a weight matrix M in (14) will fail to be obtained as $\text{rank}(\hat{C}) < n$, a non-singularity supervisor is introduced to overcome this difficulty. If $\hat{C}$ becomes to be singular, it is then perturbed with small component values, $[\delta_{ij}]_{n \times n}$, so that $\hat{C} + [\delta_{ij}]_{n \times n}$ is guaranteed to have full rank.

Using (12), (13), (14) and the Matrix Inversion Lemma $(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}$, the analytical formulation of the robust neuro-fuzzy controller resolves into

$$\begin{aligned} u &= (I_n - (I_n + \hat{C}^{-1}\hat{D})^{-1})\hat{D}^{-1}(-\theta'^T \mu' + c' - a_{LR}^T \phi) \\ &= \hat{G}^{-1}(-\theta'^T \mu' + c' - a_{LR}^T \phi) \end{aligned} \tag{16}$$

where $\hat{G} = \hat{C} + \hat{D}$.

4. Learning algorithm and performance

Let $\theta_i = [\theta_i'^T, \omega_{i1}^T, \cdots, \omega_{in}^T]^T$ being bounded by $M_{\theta_i} = \{\theta_i : |\theta_i| \le \theta_{i,Max}\}$, and define the parameters of the best function approximation to be

$$\begin{aligned} \theta_i'^* &\equiv \arg\min_{\theta_i' \in M_{\theta_i}}[\sup|f_i - \theta_i'^T \mu'|] \\ \omega_{ij}^* &\equiv \arg\min_{\omega_{ij} \in M_{\theta_i}}[\sup|g_{ij} - \omega_{ij}^T \mu|] \end{aligned} \tag{17}$$

Then equation (2) can be rewritten in terms of the measured output q_n and the ith component can be expressed as

$$\begin{aligned} \ddot{q}_{ni} &= f_i(x) + \sum_{j=1}^{n} g_{ij}(x) u_j + d_i(x,t) + \ddot{n}_{pi} \\ &= \theta_i'^{*T} \mu'(x) + \zeta_i^f + \sum_{j=1}^{n} (\omega_{ij}^{*T} \mu(x) + \zeta_{ij}^g) u_j + d_i(x,t) + \ddot{n}_{pi} \end{aligned} \tag{18}$$

where

$$\begin{aligned} \zeta_i^f &= f_i(x) - \theta_i'^{*T} \mu'(x) - \theta_i'^{*T} \Delta\mu'(x, n_p, n_v) \\ \zeta_{ij}^g &= g_{ij}(x) - \omega_{ij}^{*T} \mu(x) - \omega_{ij}^{*T} \Delta\mu(x, n_p, n_v) \end{aligned} \tag{19}$$

and

$$\begin{aligned} \Delta\mu(x, n_p, n_v) &= \mu(x) - \mu(\bar{x}) \\ \Delta\mu'(x, n_p, n_v) &= \mu'(x) - \mu'(\bar{x}) \end{aligned} \tag{20}$$

are measure of sensitivity of the nominal model $(d(q,\dot{q},t) \equiv n_p \equiv n_v \equiv 0)$ with respect to the measurement noise n_p and n_v. It is then possible to derive the error equation from (18) and (3) as follows:

$$\begin{aligned} \ddot{q}_{ni} - \ddot{q}_{Mi} &= -\alpha_{i1} q_{Mi} - \alpha_{i2} \dot{q}_{Mi} - v_i \\ &\quad + \theta_i'^{*T} \mu'(\bar{x}) + \sum_{j=1}^{n} \omega_{ij}^{*T} \mu(x) u_j + \zeta_i \end{aligned} \tag{21}$$

where

$$\zeta_i = \zeta_i^f + \sum_{j=1}^{n} \zeta_{ij}^g u_j + d_i(x,t) + \ddot{n}_p$$

By (16), subtracting $\sum_{j=1}^{n} \omega_{ij}^T \mu(x) u_j$ and adding $-\theta_i'^T \mu' + c_i' - a_{LRi}^T \phi$ to the right hand side of (21), we obtain

$$\begin{aligned} \ddot{q}_{ni} - \ddot{q}_{Mi} &= \alpha_{i1}(q_{ni} - q_{Mi}) + \alpha_{i2}(\dot{q}_{ni} - \dot{q}_{Mi}) \\ &\quad + (\theta_i'^{*T} - \theta_i'^T) \mu'(\bar{x}) + \sum_{j=1}^{n} (\omega_{ij}^{*T} - \omega_{ij}^T) \mu(x) u_j + \zeta_i - a_{LRi} \phi \end{aligned} \tag{22}$$

or

$$\dot{e}_i = A_i e_i - b_i w_i^T \tilde{\theta}_i + b_i(\zeta_i - a_{LRi}\phi) \tag{23}$$

where $e_i = [\bar{q}_i - q_{Mi}, \dot{\bar{q}}_i - \dot{q}_{Mi}]^T$ and $\tilde{\theta}_i = \theta_i - \theta_i^*$ denote the tracking error vector and parameter estimation error, respectively, and

$$A_i = \begin{bmatrix} 0 & 1 \\ -a_{i1} & -a_{i2} \end{bmatrix}, \quad b_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad w_i = \begin{bmatrix} \mu' \\ \mu u_1 \\ \vdots \\ \mu u_n \end{bmatrix} \tag{24}$$

To counteract the equivalent uncertainty, the fine-tuning mechanism $a_{LRi}\phi$ is employed. The parameter a_{LRi} is chosen as $a_{LRi}(\vartheta) = \vartheta_i \tanh(\frac{b_i^T P_i e_i \phi}{\varepsilon})$ where ϑ_i is an auxiliary adjustable parameter and ε is a small positive constant. Using the following assumption:

Assumption 1: There exists the smallest non-negative parameter values $\vartheta_i^* \ge 0$ such that for all $x \in \Re^n$ and $t \in \Re_+$

$$|\zeta_i| \le \vartheta_i^* \phi \tag{25}$$

And let $M_{\vartheta_i} = \{\vartheta_i : |\vartheta_i| < \vartheta_{i,\max}\}$ be the bound of ϑ_i, $M_{\vartheta_i}^{\varepsilon}$ be the union of M_{ϑ_i} and its boundary layer of thickness ε_ϑ. We propose the following smooth robust weight adaptation scheme

$$\dot{\theta}_i(t) = \begin{cases} 0, & \text{if } e^T P b b^T P e \le d_0^2 \\ (I - d_{\theta_i} \theta_{i\perp} \theta_{i\perp}^T) R_i^{-1} [b_i^T P_i e_i w - \sigma_1(\theta_i - \theta_{i0})], & \text{otherwise} \end{cases} \tag{26}$$

with

$$d_{\theta_i} = \begin{cases} 0, & \text{if } \theta_{i\perp}^T [b_i^T P_i e_i w - \sigma_1(\theta_i - \theta_{i0})] \le 0 \\ \min[1, dist(\theta_i, M_{\theta_i}) / \varepsilon_\theta], & \text{otherwise} \end{cases} \tag{27}$$

and

$$\dot{\vartheta}_i(t) = \begin{cases} 0, & \text{if } e^T P b b^T P e \le d_0^2 \\ (1 - d_{\vartheta_i}) r_{\vartheta i}^{-1} [w_i' b_i^T P_i e_i - \sigma_2(\vartheta_i - \vartheta_{i0})], & \text{otherwise} \end{cases} \tag{28}$$

with

$$d_{\vartheta_i} = \begin{cases} 0, & \text{if } \vartheta_i [b_i^T P_i e_i w_i' - \sigma_2(\vartheta_i - \vartheta_{i0})] \le 0 \\ \min[1, dist(\vartheta_i, M_{\vartheta_i}) / \varepsilon_\vartheta], & \text{otherwise} \end{cases} \tag{29}$$

$$w_i' = \phi \tanh(\tfrac{b_i^T P_i e_i \phi}{\varepsilon}) \tag{30}$$

where $e = [e_1, \cdots, e_n]^T$, R_i is a diagonal matrix with positive diagonal elements, P_i is a symmetric positive-definite matrix satisfying the Lyapunov equation $A_i^T P_i + P_i A_i = -Q_i$, with the design parameters $Q_i > 0$, and σ_1 and σ_2 are chosen small but positive constant to keep θ_i and ϑ_i from growing unbounded.

Theorem 1. Consider the robotic system (1) with unknown but bounded f_i, g_{ij}, d_i, and n_p and n_v. Let assumption 1 hold. Take the Neuro-Fuzzy Logic Controller (16) and parameter adaptation laws (26) and (28). Then in the bounded state $(q,\dot{q}) \in \Omega = \{(q,\dot{q}) : |(q,\dot{q})| \le \gamma\}$, we obtain

(1) θ_i, ϑ_i and the control input u are uniformly ultimately bounded.

(2) Given any ρ satisfying $\rho^* < \rho \le \gamma$ where

$$\rho^* = \frac{\sum_{i=1}^{m} [\sigma_1(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \sigma_2(\vartheta_i^* - \vartheta_{i0})^2 + 2\kappa\vartheta_i^M \varepsilon]}{\min_i \min\{\frac{\lambda_{\min}(Q_i')}{\lambda_{\max}(P_i)}, \frac{\sigma_1}{\lambda_{\max}(R_i)}, \frac{\sigma_2}{\gamma_\vartheta}\}} \tag{31}$$

with $\vartheta_i^M \equiv \max\{\vartheta_i^*, \vartheta_{i0}\}$ and κ being a constant that satisfies $\kappa = e^{-(\kappa+1)}$, i.e., $\kappa = 0.2785$, there exists T such that for $T \le t \le \infty$ the tracking error e converges to the residual set

$$\{e : e^T P e \le \rho \text{ or } e^T P b b^T P e \le d_0^2\} \tag{32}$$

Proof. Omitted due to limitation of space.

Remark 2. In the NFLC some degree of "adaptability" is achieved by the fine tuning mechanism, $a_{LRi}\phi$, which is able to deal with different plants with different disturbances and control efforts.

Remark 3. In view of (31), if the design constants ε, σ_1, σ_2, γ_ϑ, Q_i, P_i and R_i are appropriately chosen, tracking to a small neighborhood around $e = 0$ can be obtained.

Remark 4. The initial design of parameters θ_{i0} and ϑ_{i0} in the NFLC can be considered as initial estimates of the best parameters, θ_i^* and ϑ_i^*, respectively. The closer θ_{i0} and ϑ_{i0} to θ_i^* and ϑ_i^* is, the smaller ρ^* becomes. This, in turn, results in better tracking.

Remark 5. Suppose that some partial knowledge about the manipulator to be controlled is known in the form of "approximation to $f(q,\dot{q})$" and "approximation to $G(q,\dot{q})$" denoted by the terms $f^{\circ}(x|$ nominal parameters of the arm) and $G^{\circ}(x|$ nominal parameters of the arm), respectively. Then a set of initial weights can be selected by using the well-known least square algorithm or etc., such that θ_{i0} will close to θ_i^* and a smaller ρ^* can be achieved.

5. Simulation

The equations of motion of the manipulator can be expressed in matrix form as follows,

$$\begin{bmatrix} (m_1+m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 c_2 + J_1 & m_2 r_2^2 + m_2 r_1 r_2 c_2 \\ m_2 r_2^2 + m_2 r_1 r_2 c_2 & m_2 r_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -m_2 r_1 r_2 s_2 \dot{q}_1(\dot{q}_1+\dot{q}_2) \\ m_2 r_1 r_2 s_2 \dot{q}_2^2 \end{bmatrix} + \begin{bmatrix} ((m_1+m_2)l_1 c_2 + m_2 l_2 c_{12})g \\ (m_2 l_2 c_{12})g \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \quad (33)$$

where $r_1 = 0.5l_1$, $r_2 = 0.5l_2$, and $c_1 \equiv \cos(q_1)$, $s_{12} \equiv \sin(q_1+q_2)$, *etc.*. The kinematics and inertial parameters of the manipulator are given in Table I. The excessive ratio between m_1 and m_2 is to emphasize the load effect. The trajectory to be followed by the ith joint is given by two decoupled linear systems as

$$\ddot{q}_{Mi} = \alpha_{i1} q_{Mi} + \alpha_{i2}\dot{q}_{Mi} + v_i, \quad i = 1,2. \quad (34)$$

The model parameters are chosen as follows: $\alpha_{11} = -1.0$, $\alpha_{12} = -1.0$, $\alpha_{21} = -4.0$, $\alpha_{22} = -2.0$ and the driving inputs to the reference model are sinusoidal functions $v_1 = \pi\sin(0.8\pi t)$, $v_2 = 1.5\pi\cos(\pi t)$. The reference model and the plant are assumed to have the same initial states as $q_1(0) = -1.5rad$, $q_2(0) = -1.2rad$, $\dot{q}_1(0) = 0rad/\sec$ and $\dot{q}_2(0) = 0rad/\sec$. The membership functions of states q_1, $\dot{q}_1$, q_2, and $\dot{q}_2$ (represented by generic variable x) for the qualitative statements are defined as {*NB, NS, ZE, PS, PB*} where *NB:* A_i(x)=exp(-4(x+1.8)^2), *NS:* A_i(x)=exp(-4(x+0.8)^2), *ZE:* A_i(x)=exp(-4(x)^2), *PS:* A_i(x)=exp(-4(x-0.8)^2), *PB:* A_i(x)=exp(-4(x-1.8)^2). In (32) and (34), the design parameters are given by $Q_1 = Q_2 = 10\,I_{2\times 2}$, R_1=Block diag $[0.01I_{256\times 256}, 32000I_{256\times 256}, 20000I_{256\times 256}]$, R_2=Block diag *diag* $[0.025I_{256\times 256}, 20000I_{256\times 256}$, $\sigma_1 = 0.002$, σ_2=0.001, and $\varepsilon = 0.005$.

For the purpose of comparison, simulations for the NFLC control are carried out for situations with and without the nominal parameters of the manipulator. In the case that the nominal parameters are known *a priori*, through the training data $x^{(k)}$, the initial parameters θ_i' and ω_{ij} are chosen based on the element-by-element minimization of the following objective function

$$\sum_k \left\| f^0(x^{(k)} \mid \text{nominal parameters of the arm}) - \hat{f}(x^{(k)}, \theta_i') \right\|^2$$

$$\sum_k \left\| G^0(x^{(k)} \mid \text{nominal parameters of the arm}) - \hat{G}(x^{(k)}, \omega_{ij}) \right\|^2$$

We chose 32 testing points as training data, $x^{(k)}$, from either along the desired trajectories or nearby of them. In the other case, no nominal parameters of the manipulator is used, and the elements in θ_i' and ω_{ij} are chosen randomly in the interval $(-10,10)$ and $(-2,2)$, respectively.

Model following control under disturbances

The combined effects of friction and external torque disturbance are

$$\begin{aligned} d_1 &= 2.0\sin(\dot{q}_1) + 2.5\sin(\dot{q}_2) + 0.5\sin(t) \\ d_2 &= 5.0\sin(\dot{q}_1) + 4.0\sin(\dot{q}_2) + 0.4\sin(t) \end{aligned} \quad (35)$$

For the case without measurement noise, the simulation results are presented in Fig. 3. Fig. 3(a) and (b) show the case of the NFLC with and without using nominal parameters of the manipulator. We see from Fig. 3 that the NFLC achieves faster convergence when the initial parameters are chosen on the basis of nominal robot parameters.

Tracking control with measurement noise

The noise sources are assumed to be white with uniform distribution [-0.056, 0.05]. We assume that the noise effects of different sensors are independent of each other. The external disturbances are set like the above subsection. Fig. 4 shows the tracking error of joint 1 and joint 2. This simulation shows that the proposed control scheme is robust against measurement noise.

6. Conclusion

A method of robust neuro-fuzzy model-following control for multi-link robot manipulators has been constructed. The NFLC is composed of an multi-input/multi-output fuzzy system with adjustable rule credit assignment and interconnections compensating network. The interconnections compensating network can decompose the robot dynamics into decoupled subsystems and the fuzzy part approximates the unknown non-linearity of the robot by learning. By using fine tuning mechanism in the consequent membership functions, the NFLC based control system can achieve some degree of robust properties. The overall robust adaptation scheme has been proved to be able to guarantee the output tracking error to converge to a residual set ultimately. Simulations of an example plant have confirmed the robustness of the design to actuator input noise and measurement noise. If the rough plant model and parameters of the robot are available, the NFLC can be trained in advance to achieve faster convergence of the weight adaptation and model

Acknowledgement

The financial support for this research from the National Science Council of Taiwan, R. O. C. under NSC87-2212-E002-024 is gratefully acknowledged.

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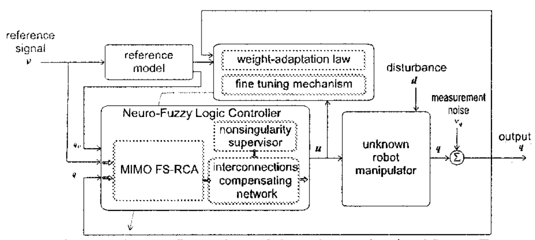


Fig. 1. The configuration of the robust adaptive Neuro-Fuzzy model-following control design.

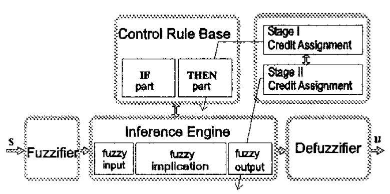
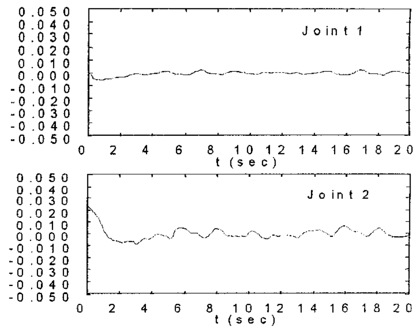


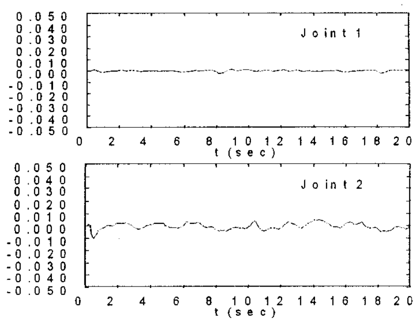
Fig. 2. Diagrammatic representation of fuzzy system with stage adjustable rule credit assignment.

TABLE I
Parameters of the robot

Parameter	Symbols	Real	Nominal
mass of link 1	$m_1\ (kg)$	0.60	0.48
mass of link2	$m_2\ (kg)$	7.02	6.3
inertia of link 1	$J_1\ (kgm^2)$	4.5	4.8
inertia of link 2	$J_2\ (kgm^2)$	4.5	5.1
length of link 1	$l_1\ (m)$	2.04	2.0
length of link 2	$l_2\ (m)$	1.66	1.6

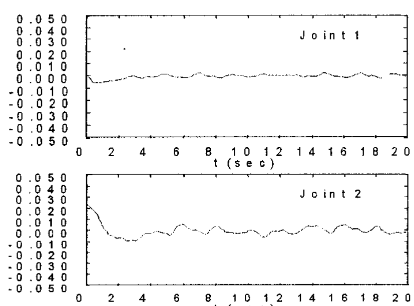


(a)

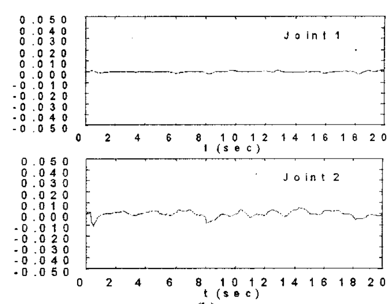


(b)

Fig. 3. Tracking error of joint 1 and 2 without measurement noise, (a) without, and (b) with rough mathematical model and nominal parameters.



(a)



(b)

Fig. 4. Tracking error of joint 1 and 1 with measurement noise, (a) without, and (b) with rough mathematical model and nominal parameters.