where λ_k is the mean number of cells arriving in a slot given that there were k sources in the H-state in the previous slot, i.e. $\lambda_k = E[b(n+1) \mid a(n) = k]$. We can further establish that:

$$\lambda_k = \sum_{l=0}^{N} \sum_{m=l}^{N} mA(k,l)B(l,m) \quad \text{and } k = 0, 1, ..., N \qquad (7)$$

We note that the service time for a cell is unity, hence the traffic intensity $\rho = \Sigma_{k=0}^{N} x_k \lambda_k \times 1$ and we obtain

$$P_{loss} = \sum_{k=0}^{N} \frac{x_k \lambda_k P_{loss}(k)}{\rho} = \frac{\rho - (1 - \pi_0 e)}{\rho} \qquad (8)$$

The term $\pi_0 e = \Sigma_{i=0}^{N} \pi_{0i}\, P[q(k) = 0]$ represents the probability that the system is empty and can be obtained by solving the Markov chain shown in eqn. 4. Once $\pi_0 = (\pi_{00}, \pi_{01}, ..., \pi_{0N})$ is obtained, the cell loss probability can be derived from eqn. 8.

Numerical results: To cater for different types of traffic with different burstiness, two levels of burstiness are considered for system loads at 70, 80 and 95%, respectively. The cell loss performance computed using the above-mentioned method for a 4 × 4 switching block is shown in Figs. 2 and 3. From the graph shown, at a system load of 0.7, the cell-loss probability is smaller than that recommended of 10^{-6} for buffer sizes > 11 cells. At a system load of 0.8, the cell-loss probability can be extrapolated to be < 10^{-6} for a buffer size of > 20 cells. The size of a frame for most LAN applications is < 1000 bytes, or approximately 20 cells, so a buffer size of > 20 cells should be sufficient for our implementation.

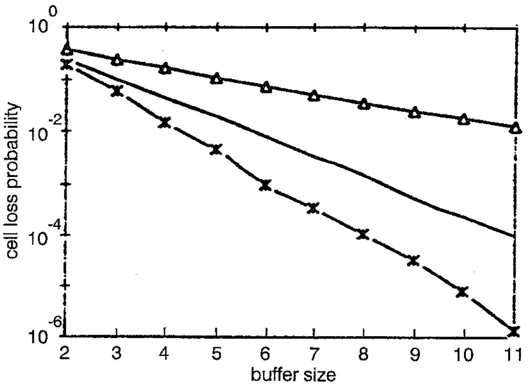

Fig. 2 *Cell-loss probability against buffer size (burstiness = 1.14)*

—△— load = 0.95
—×— load = 0.70
——— load = 0.80

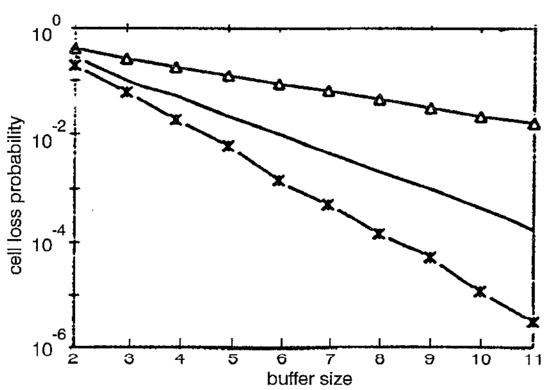

Fig. 3 *Cell-loss probability against buffer size (burstiness = 1.30)*

—△— load = 0.95
—×— load = 0.70
——— load = 0.80

Conclusion: In this Letter, we have presented a method for buffer dimensioning of an ATM25 switch. It has been shown that a buffer size of > 20 cells is sufficient for the design. Furthermore, the model and solution we have developed in this Letter for buffer dimensioning can be used, with some modification, as the basis for call admission control.

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References

1 NEWMAN, P.: 'ATM local area networks', *IEEE Comm. Mag.*, 1994, pp. 86–98
2 'Physical interface specification for 25.6 mbps over twisted pair cable'. ATM Forum, Paper af-saa-0049.00
3 CHENG, T.H., NG, C.H., and SHEN, Y.: 'Design and implementation of an ATM25 switch using FPGAs'. Tech. Rep., Nanyang Technological University, 1996

Capacity of a synchronous BFSK frequency-hopped multiple access system with RTT erasure decision

Ing-Jiunn Su and Jingshown Wu

The capacity of binary frequency shift keying (BFSK) frequency-hopped multiple access (FHMA) systems with side information is derived. The use of erasure decision with ratio-threshold test (RTT) for generating side information in fading and non-fading channels is investigated. The authors demonstrate the benefits of using erasure decision, achievable maximum capacity, optimum threshold setting and the optimal number of users in order to maximise the capacity.

Introduction: The capacity of a frequency-hopped multiple access (FHMA) system can be significantly improved by side information, which indicates the reliability of the received signal and can be used to circumvent the transmission of unreliable symbols caused by multiple access interference. However, the accuracy of the side information depends on the implemented mechanism [1] and may affect the accuracy of correction and error decision. Some researchers [2, 3] have investigated the problem without suggesting implementations. In this Letter, the side information is generated based on erasure decision with the ratio-threshold test (RTT) principle. The theoretical capacity of a synchronous FHMA system with binary frequency shift keying (BFSK) signalling is evaluated for both fading and non-fading channels. Also the optimum threshold setting and optimal number of users which maximise the capacity are found in the numerical examples.

Analysis: After removing the frequency hopping in a synchronous BFSK FHMA receiver, the decision variables generated by two envelope detectors, denoted by band 0 and band 1, can be expressed as

$$r_{i,k_i} = \left| C_{i0}\alpha_0 e^{j\theta_0} + \sum_{m=1}^{k_i} \alpha_m e^{j\theta_m} + Z_i \right| \quad i = 0, 1 \qquad (1)$$

where C_{i0} is equal to 1 or 0 to indicate whether the desired signal is present in the ith band, k_i is the number of interference signals in the ith band, α_m and θ_m are the amplitude and phase of the desired or corresponding interference signal, and Z_i is the complex Gaussian noise with zero mean and variance $\sigma^2 \equiv N_0/2$. θ_m is uniformly distributed over $[0, 2\pi)$ and $\{\alpha_m\}$ are modelled as independent and identically distributed (i.i.d.) Rician random variables with mean a and variance $2\sigma_f^2$. Here we assume that all received signals have the same average power regulated by the power control scheme. Since a^2 and $2\sigma_f^2$ represent the average power of the direct and diffused components in each signal, it is convenient to define Γ as the power ratio of the direct component to the diffused component [4]. Therefore, $\Gamma = 0$ and $\Gamma = \infty$ can represent the Rayleigh and non-fading channels, respectively. If data bits have equal likelihood of transmission, we can assume that the desired signal is present in band 0 in order to evaluate the performance of the system without loss of generality.

For the RTT decision scheme, an error decision occurs when the ratio of the desired band to the non-desired band signals is less than a predefined threshold setting, t ($0 < t \le 1$). Therefore, the error probability, conditioned on k_i interference signals in the ith band, can be expressed as [4]

$$P_e(k_0,k_1) = -\int_0^\infty \phi_{0,k_0}(\rho)\frac{d\phi_{1,k_1}(t\rho)}{d\rho}d\rho \qquad (2)$$

where

$$\phi_{i,k_i}(\rho) = e^{-\frac{[(\delta_{i0}+k_i)\sigma_f^2+\sigma^2]\rho^2}{2}} J_0^{\delta_{i0}+k_i}(a\rho)$$

is the characteristic function of r_{i,k_i}, δ_{i0} is the Kronecker delta and $J_0(.)$ is the zero-th order Bessel function. After manipulation, we found:

$$P_e(k_0,k_1) = \int_0^\infty \left[(\sigma^2 + k_1\sigma_f^2)\rho t^2\phi(t\rho) + k_1 t a J_1(ta\rho)e^{-\frac{t^2\sigma_f^2\rho^2}{2}}\right] \times \phi^{k_0+1}(\rho)\phi^{k_1-1}(t\rho)e^{-\frac{(t^2+1)\sigma^2\rho^2}{2}}d\rho \qquad (3)$$

where $J_1(.)$ is the first order Bessel function.

Conditioned on total k interference signals presented in two bands, the error probability can be written as

$$P_{e,k} = \sum_{m=0}^{k}\binom{k}{m}\left(\frac{1}{2}\right)^k P_e(m,k-m)$$
$$= \int_0^\infty \left\{t^2\sigma^2\rho + \frac{k}{2}\left[\sigma_f^2\phi(t\rho) + taJ_1(ta\rho)e^{-\frac{t^2\sigma_f^2\rho^2}{2}}\right]\right\} \times \phi(\rho)\Phi^{k-1}(\rho)e^{-\frac{(t^2+1)\sigma^2\rho^2}{2}}d\rho \qquad (4)$$

where $\Phi(\rho) = [\phi(\rho) + \phi(t\rho)]/2$. If K users have signals in the system with q available frequency slots, the average bit error probability can be found by averaging $P_{e,k}$ and it can be expressed as

$$P_e(K) = \int_0^\infty \left\{t\sigma^2\rho X(\rho) + \frac{K-1}{2}P_h\left[t\sigma_f^2\rho\phi(t\rho) + aJ_1(ta\rho)e^{-\frac{t^2\sigma_f^2\rho^2}{2}}\right]\right\} \times t\phi(\rho)X^{k-2}(\rho)e^{-\frac{(t^2+1)\sigma^2\rho^2}{2}}d\rho \qquad (5)$$

where $X(\rho) = 1 - P_h + P_h\Phi(\rho)$ and $P_h = 1/q$ is the probability of a hit, where the desired frequency slot is corrupted by another signal, in a synchronous FHMA system with random frequency hopping scheme.

Similarly, a correction decision occurs when the ratio of the non-desired band to desired band signals is less than t and its average probability can be deduced as

$$P_c(K) = \int_0^\infty \left\{X(\rho)\left[t\sigma^2\rho\phi(\rho) + aJ_1(ta\rho)e^{-\frac{t^2\sigma_f^2\rho^2}{2}}\right] + P_h\frac{K-1}{2}\left[t\sigma_f^2\rho\phi(\rho)\phi(t\rho) + 2aJ_1(ta\rho)\Phi(\rho)e^{-\frac{t^2\sigma_f^2\rho^2}{2}}\right]\right\} \times tX^{K-2}(\rho)e^{-\frac{(t^2+1)\sigma^2\rho^2}{2}}d\rho \qquad (6)$$

Given $P_e(K)$ and $P_c(K)$, the average erasure probability $P_{er}(K)$ can be easily obtained by $P_{er}(K) = 1 - P_c(K) - P_e(K)$, and the total capacity, in terms of bits per channel use, can be computed by summing the individual channel capacities:

$$C(K) = K\left\{P_c(K)\log_2 P_c(K) + P_e(K)\log_2 P_e(K) + [P_c(K) + P_e(K)]\log_2\frac{2}{P_c(K) + P_e(K)}\right\} \qquad (7)$$

Numerical results and discussion: In general, a lower threshold setting will reduce the probabilities of correction and error decision, and increase the erasure decision probability. That is suitable in generating the correct side information for a multiple access interference system. However, an increased rate of erasure decision will reduce the useful information and may result in a reduction in capacity. Therefore, there should exist an optimum t that maximises the capacity and will depend on the amount of interference. Fig. 1 shows this trend for different channel conditions (different Γ) and different numbers of users with $q = 50$. For $E_b/N_0 = 10$dB (E_b is the bit energy), the optimum t decreases as the number of interference users increases from 0 ($K = 1$) to 30 ($K = 31$). This optimum t can be lowered and the maximum capacity can be improved slightly if E_b/N_0 increases to 13dB. The reason is that the lower t can improve correct side information without decreasing the correction decision significantly in the higher E_b/N_0 for the same number of interference signals. Even in a single-user system,

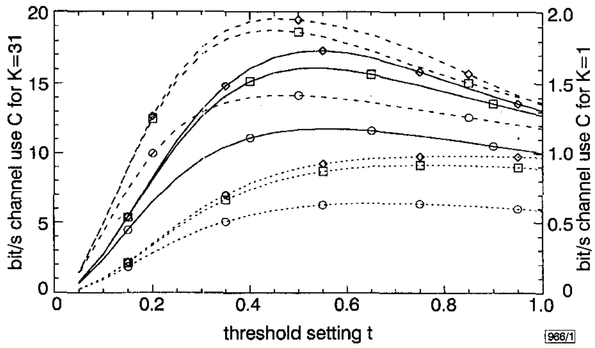

Fig. 1 *Capacity for q = 50*

○ $\Gamma = 0$ ‒ ‒ ‒ ‒ $K = 31$, $E_b/N_0 = 13$dB
□ $\Gamma = 10$ ——— $K = 31$, $E_b/N_0 = 10$dB
◇ $\Gamma = \infty$ · · · · · $K = 1$, $E_b/N_0 = 10$dB

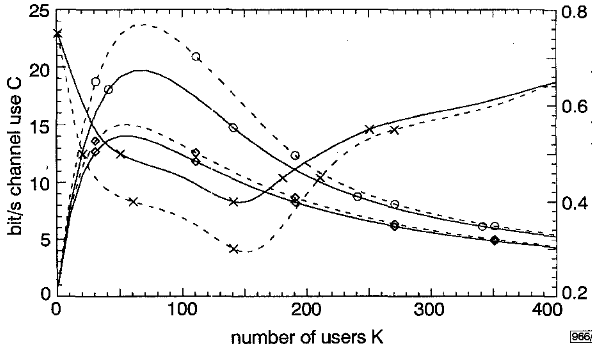

Fig. 2 *Optimum threshold setting and capacity comparison for q = 50 and Γ = 10*

‒ ‒ ‒ ‒ $E_b/N_0 = 13$dB
——— $E_b/N_0 = 10$dB
○ capacity with optimum threshold setting
◇ capacity with hard decision
× optimum threshold setting

the benefit of applying an erasure decision system over that of a hard decision ($t = 1$) system in lower Γ environments is also explicitly shown in this Figure. Through the numerical search, we find the optimum t and maximum capacity for each K. Fig. 2 shows this result and compares this capacity with that obtained using a hard decision system. The advantage in capacity obtained by using an RTT erasure decision system is explicit. From this Figure, the optimal number of users to maximise achievable capacity can also be found.

Conclusion: The theoretical capacity of a synchronous BFSK FHMA system with RTT erasure decision is investigated. The behaviour of threshold setting and advantages in capacity for erasure decision are demonstrated. The optimum threshold setting, achievable maximum capacity and optimal number of users are also obtained.

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References

1 PURSLEY, M.B.: 'The derivation and use of side information in frequency-hop spread spectrum communications', *IEICE Trans. Commun.*, 1993, **E76-B**, pp. 814–824
2 CHEUN, K., and CHOI, K.: 'Performance of FHSS multiple-access networks using MFSK modulation', *IEEE Trans.*, 1996, **COM-44**, pp. 1514–1526
3 KIM, S.W., LEE, Y.H., and KIM, S.: 'Bandwidth tradeoffs among coding, processing gain and modulation in frequency-hopped multiple access communications', *IEE Proc. Commun.*, 1994, **141**, pp. 63–69
4 HUNG, C.P., and SU, Y.T.: 'Diversity combining considerations for incoherent frequency hopping multiple access systems', *IEEE J. Sel. Areas Commun.*, 1995, **SAC-13**, pp. 333–344

Channel holding time distribution in cellular telephony

F. Barceló and J. Jordán

A simple procedure is presented for characterising the probability distribution function of the dwell time in a cellular telephony system. Other statistical figures related to mobility are also provided.

Introduction: The duration of the holding time (dwell time) in cellular telephony systems is a fraction of the total call duration due to the hand-off mechanism. Factors such as mobility and cell shape and size will cause the dwell time to have a different probability distribution (pd) function from that of the whole call duration, a greater difference being expected for higher mobility and smaller cell sizes. Several authors have attempted to model the channel holding time by making use of analytical tools and/or simulations [1–4]. All of these studies assume an exponential distribution for the whole call duration, mainly because analytical results for other distributions become highly complex or impossible to attain. From this same starting point, and by different means, all of the above mentioned studies reach the conclusion that the channel holding time is also distributed exponentially, although with a lower average.

In this Letter, the main results and conclusions of a field study of the channel occupancy in a cellular telephone system in Barcelona are presented. Further details can be found in the extended version of this Letter [5], where other interesting statistical results are also shown. This study was carried out by registering the duration of channel occupancies through a scanning receiver. The main difference with respect to other field studies [6] is that here a mixture of lognormals is the best fit. This conclusion is specially relevant because the call duration in fixed telephony has been proved to have a much better fit with a mixture of lognormal distributions than with the exponential [7]; this result can easily be extended to mobile telephone calls. Hence, we prove that the dwell-time distribution belongs to the same family of distributions as the whole call duration. The average channel holding time found in our study (40.6s) fully agrees with that found in [6] for a high mobility. When compared with the whole call duration, this average implies a high hand-off rate, so the agreement between the dwell time and whole call duration distributions does not rely on a low hand-off rate.

Data set and exponential distribution: The sample used as an example in this Letter was obtained at a frequency of 941.7125MHz during busy hours. The size of the sample displayed in Fig. 1 is $n = 2445$ occupancies and the average is $m_1 = 40.6$s. The squared coefficient of variation of this sample is $cv^2 = 1.7$. The statistical test used to check the goodness of fit was the well-known Kolmogorov-Smirnov (K-S) test, which leads to the significance level α of the candidate distribution, with respect to the empirical data. Further details on the hardware and statistical tools used in this work can be found in [5, 8].

When the exponential distribution is fitted to the empirical data (see Fig. 1) it can be observed that the probability of very short occupancies is overestimated, while the area with the highest probability in the empirical histogram is underestimated. The spikes of the empirical data set contribute to the hindrence of parameter estimation and the fit. These spikes are due to the time that the system requires from the mobile-stations before retrying the hand-off, as observed in [6].

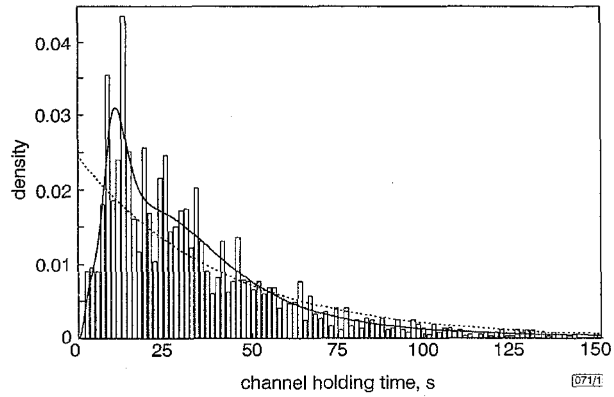

Fig. 1 *Empirical against exponential and lognormal-3 channel holding time distribution*

– – – – exponential
——— lognormal-3

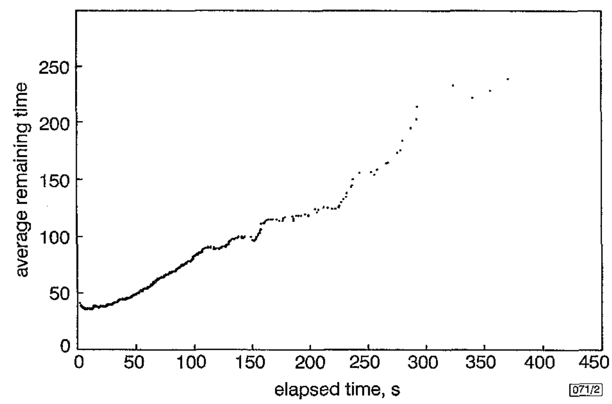

Fig. 2 *Average remaining against elapsed channel holding time*

When the remaining occupancy time is investigated as a function of the elapsed occupancy time, the average remaining time is far from being independent of the elapsed time, as should happen if the empirical data followed an exponential distribution because of the memoryless property of the latter. Fig. 2 shows that the longer the average remaining time, the longer the elapsed time. This behaviour is similar to that observed in [7] for the call duration in conventional telephony and in [8] for the transmission duration in private mobile radio (PMR) systems. The discontinuous shape for high elapsed times is due to the fact that there are fewer values to average for higher elapsed times, leading to fewer and more dispersed values.

Table 1: Moments and fitting for channel holding time

Fitting		Moments: $m_1 = 40.6$	$cv^2 = 1.7$	
Exponential	D: 5.541	β: 40.60		
Erlang-2,17	D: 2.272	β: 15.38	p: 0.957	
Erlang-2-4	D: 2.594	β_1: 15.07	β_2: 64.42	p_1: 0.894
Lognormal	α: 0.016	μ: 3.287	σ: 0.891	
Lognormal-3	α: 0.097	μ_1: 3.327	σ_1: 1.043	p_1: 0.527
		μ_2: 3.55	σ_2: 0.50	p_2: 0.339
		μ_3: 2.44	σ_3: 0.286	

Numerical results: Table 1 shows the numerical results of the K-S test: the significance α and the distance D are tabulated for several candidate probability distributions (the latter only when the significance α is negligible). Only distributions which have statistical meaning in our environment have been tested. The first family of candidates can be represented as a combination of memoryless