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H_∞ Filter Design for Uncertain Discrete-Time Singular Systems via Normal Transformation*

Ching-Min Lee[1] *and I-Kong Fong*[1,2]

Abstract. This paper concerns the robust H_∞ filtering problem for discrete-time singular systems with norm-bounded uncertainties. Based on the admissibility assumption of singular systems, a set of necessary and sufficient conditions for the existence of the desired filters is established, and a normal filter design method under the linear matrix inequality framework is developed. A numerical example is given to illustrate the application of the proposed method.
Key words: Singular system, restricted system equivalence, admissibility, robust filter, LMI.

1. Introduction

In the past decades, the H_∞ filtering problem for singular systems has been an important research topic. This is due not only to the theoretical interests but also to the relevance of the topic in various engineering applications. Many works [10], [17], [23], [26] consider robust filters for continuous-time singular systems, in which the filter design criteria are mainly based on the generalized Lyapunov theorem [12], [18] for singular systems, and the formulations are under the linear matrix inequality (LMI) framework for easier applications. Unlike the discrete-time singular system stabilization problem [20]–[22], [27], in the filtering problem for discrete-time singular systems, applications of the approaches parallel to those for the continuous-time systems are not often adopted. One possible reason is the difficulty in managing the resultant constraints related to the singular matrix in

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[1] Department of Electrical Engineering, National Taiwan University, Taipei, 10617, Taiwan. E-mail for Lee: cmlee61@ntu.edu.tw; E-mail for Fong: ikfong@cc.ee.ntu.edu.tw

[2] Present address: Room 211, Electrical Engineering Building II, 1 Roosevelt Rd., Sec. 4, Taipei, 10617, Taiwan.

the difference term of the state-space model, especially when the constraints need to be represented as LMIs.

In this paper, the robust H_∞ filtering problem is discussed for discrete-time singular systems with norm-bounded uncertainties. The goal of the filter is to satisfy the H_∞ performance level requirement on the filtering error dynamics. The proposed filter design method is formulated under the LMI framework. Unlike [10], [23], [26], which directly handle singular systems by using the generalized Lyapunov theorem, here a "normal transformation" to obtain normal system models (i.e., those with the system matrix for the difference term being the identity matrix) [3] from singular system models is applied first, and normal filters are found directly. Then, instead of using criteria such those in [20]–[22], [27], an easier-to-use criterion based on the direct Lyapunov theorem for normal systems is applied. It is believed that the consideration of normal filters is beneficial, because sometimes the physical realizations of singular filters are not easy [3], [4]. In order to realize a singular system, one often needs special algorithms [15] to convert a singular system model into a normal state-space form.

Some of the notation to be used subsequently is introduced here. The inequality $\mathbf{X} \geq \mathbf{0}$ means that the matrix $\mathbf{X}$ is symmetric and positive semi-definite, and $\mathbf{X} \geq \mathbf{Y}$ means $\mathbf{X}-\mathbf{Y} \geq \mathbf{0}$. Similar definitions apply to symmetric positive/negative definite matrices. For a matrix $\mathbf{M}$, $\|\mathbf{M}\|$ denotes its spectral norm, and for a stable discrete-time transfer function matrix $\mathbf{G}(z)$, $\|\mathbf{G}\|_\infty = \sup_{\omega\in[0,2\pi)} \|\mathbf{G}(e^{j\omega})\|$ is its H_∞ norm. $\mathbf{I}_r$ is the identity matrix with dimension r, the superscript $^{\mathrm{T}}$ represents the transpose of a matrix, and $\mathrm{diag}(\mathbf{X},\ \mathbf{Y}, \ldots, \mathbf{Z})$ is the block diagonal matrix with diagonal elements $\mathbf{X}, \mathbf{Y}, \ldots, \mathbf{Z}$. Finally, $*$ is used to simplify the presentation of symmetric matrices.

2. Preliminaries and problem formulation

2.1. Preliminaries

First, consider the following nominal singular system:

$$\Sigma_0 : \begin{cases} \mathbf{E}_0\mathbf{x}(k+1) = \mathbf{A}_0\mathbf{x}(k) + \mathbf{B}_0\mathbf{u}(k) \\ \mathbf{z}(k) = \mathbf{L}_0\mathbf{x}(k), \end{cases} \tag{1}$$

where $\mathbf{x}(k) \in \mathcal{R}^n$ and $\mathrm{rank}\,\mathbf{E}_0 = r < n$. The unforced singular system pair $(\mathbf{E}_0, \mathbf{A}_0)$ of (1) with $\mathbf{u}(k) \equiv \mathbf{0}$ is *regular*, if $\det(z\mathbf{E}_0 - \mathbf{A}_0)$ is not identically zero. If $\deg(\det(z\mathbf{E}_0 - \mathbf{A}_0)) = \mathrm{rank}\,\mathbf{E}_0$, then $(\mathbf{E}_0, \mathbf{A}_0)$ is said to be *causal*. The pair $(\mathbf{E}_0, \mathbf{A}_0)$ is stable if all the roots of $\det(z\mathbf{E}_0 - \mathbf{A}_0) = 0$ have magnitudes less than unity. Finally, $(\mathbf{E}_0, \mathbf{A}_0)$ is *admissible* if it is regular, causal, and stable. For Σ_0, its transfer function matrix from $\mathbf{u}(k)$ to $\mathbf{z}(k)$ is $\mathbf{G}(z) = \mathbf{L}_0(z\mathbf{E}_0 - \mathbf{A}_0)^{-1}\mathbf{B}_0$.

Definition 1 [3]**.** Suppose Σ_0 in (1) is regular. Let $\mathbf{P}_0$ and $\mathbf{Q}_0$ be two $n \times n$ non-singular matrices, and $\mathbf{E}_{0r} = \mathbf{P}_0\mathbf{E}_0\mathbf{Q}_0,\ \mathbf{A}_{0r} = \mathbf{P}_0\mathbf{A}_0\mathbf{Q}_0,\ \mathbf{B}_{0r} = \mathbf{P}_0\mathbf{B}_0,\ \mathbf{L}_{0r} =$

$\mathbf{L}_0\mathbf{Q}_0$. The system

$$\Sigma_{0r}:\begin{cases}\mathbf{E}_{0r}\mathbf{x}_r(k+1)=\mathbf{A}_{0r}\mathbf{x}_r(k)+\mathbf{B}_{0r}\mathbf{u}(k)\\ \mathbf{z}(k)=\mathbf{L}_{0r}\mathbf{x}_r(k),\end{cases} \tag{2}$$

with $\mathbf{x}_r(k)=\mathbf{Q}_0^{-1}\mathbf{x}(k)$ is *restricted system equivalent* (r.s.e.) to Σ_0.

For any given regular Σ_0, there exist [3] nonsingular matrices $\mathbf{P}_0$ and $\mathbf{Q}_0$ such that

$$\mathbf{E}_{0r}=\begin{bmatrix}\mathbf{I}_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix},\qquad \mathbf{A}_{0r}=\begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\\ \mathbf{A}_3 & \mathbf{A}_4\end{bmatrix},$$
$$\mathbf{B}_{0r}=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix},\qquad \mathbf{L}_{0r}=\begin{bmatrix}\mathbf{L}_1 & \mathbf{L}_2\end{bmatrix}. \tag{3}$$

Lemma 1 [24]**.** *Suppose Σ_{0r} in (2) is regular and has the system matrices in (3). Then the pair $(\mathbf{E}_{0r},\mathbf{A}_{0r})$ is causal and stable if and only if $\mathbf{A}_4\in\mathcal{R}^{(n-r)\times(n-r)}$ is invertible, and all the roots of* $\det(z\mathbf{E}_{0r}-\mathbf{A}_{0r})=0$ *have magnitudes less than unity.*

Lemma 1 is the discrete-time version of the corresponding Lemma in [24], and can be proved similarly [3].

Lemma 2. *Suppose Σ_{0r} in (2) is r.s.e. to Σ_0 in (1). The pair $(\mathbf{E}_0,\mathbf{A}_0)$ in (1) is admissible if and only if the pair $(\mathbf{E}_{0r},\mathbf{A}_{0r})$ in (2) is admissible.*

Proof. The pair $(\mathbf{E}_0,\mathbf{A}_0)$ is admissible if and only if [20] there exists a nonsingular matrix $\mathbf{X}$ such that

$$\mathbf{E}_0^{\mathrm{T}}\mathbf{X}\mathbf{E}_0\geq\mathbf{0},\qquad \mathbf{A}_0^{\mathrm{T}}\mathbf{X}\mathbf{A}_0-\mathbf{E}_0^{\mathrm{T}}\mathbf{X}\mathbf{E}_0<\mathbf{0}. \tag{4}$$

Since Σ_0 and Σ_{0r} are r.s.e., there exist nonsingular matrices $\mathbf{P}_0$ and $\mathbf{Q}_0$ such that $\mathbf{E}_0=\mathbf{P}_0^{-1}\mathbf{E}_{0r}\mathbf{Q}_0^{-1}$ and $\mathbf{A}_0=\mathbf{P}_0^{-1}\mathbf{A}_{0r}\mathbf{Q}_0^{-1}$. Thus (4) is equivalent to

$$\mathbf{E}_{0r}^{\mathrm{T}}\mathbf{X}_r\mathbf{E}_{0r}\geq\mathbf{0},\qquad \mathbf{A}_{0r}^{\mathrm{T}}\mathbf{X}_r\mathbf{A}_{0r}-\mathbf{E}_{0r}^{\mathrm{T}}\mathbf{X}_r\mathbf{E}_{0r}<\mathbf{0}, \tag{5}$$

with $\mathbf{X}_r=\mathbf{P}_0^{-\mathrm{T}}\mathbf{X}\mathbf{P}_0^{-1}$, which means exactly that $(\mathbf{E}_{0r},\mathbf{A}_{0r})$ is admissible. □

2.2. *System transformation*

The uncertain singular system to be discussed is

$$\Sigma:\begin{cases}\mathbf{E}\mathbf{x}(k+1)=(\mathbf{A}+\delta\mathbf{A})\mathbf{x}(k)+(\mathbf{B}+\delta\mathbf{B})\mathbf{u}(k)\\ \mathbf{y}(k)=(\mathbf{C}+\delta\mathbf{C})\mathbf{x}(k)+(\mathbf{D}+\delta\mathbf{D})\mathbf{u}(k)\\ \mathbf{z}(k)=(\mathbf{L}+\delta\mathbf{L})\mathbf{x}(k)+(\mathbf{J}+\delta\mathbf{J})\mathbf{u}(k),\end{cases} \tag{6}$$

where $\mathbf{x}(k)\in\mathcal{R}^n$ is the state vector, $\mathbf{y}(k)\in\mathcal{R}^p$ is the measured output vector, $\mathbf{z}(k)\in\mathcal{R}^q$ is the vector to be estimated, and $\mathbf{u}(k)\in\mathcal{R}^m$ is the disturbance input vector. The matrix $\mathbf{E}\in\mathcal{R}^{n\times n}$ is singular with rank $\mathbf{E}=r<n$, and the matrices $\mathbf{A}$,

B, **C**, **D**, **L**, and **J** are known real constant matrices with appropriate dimensions. The constant uncertainty matrices satisfy

$$\begin{bmatrix} \delta\mathbf{A} & \delta\mathbf{B} \\ \delta\mathbf{C} & \delta\mathbf{D} \\ \delta\mathbf{L} & \delta\mathbf{J} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \\ \mathbf{H}_z \end{bmatrix} \boldsymbol{\Delta} \begin{bmatrix} \mathbf{E}_x & \mathbf{E}_u \end{bmatrix} \tag{7}$$

with $\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta} \leq \mathbf{I}$ and $\boldsymbol{\Delta} \in \mathcal{R}^{d_1 \times d_2}$. Assume that the pair $(\mathbf{E}, \mathbf{A} + \delta\mathbf{A})$ is admissible, so there exist [3] nonsingular matrices **P** and **Q** such that Σ in (6) is r.s.e. to the system

$$\Sigma_{\mathrm{r}} : \begin{cases} \mathbf{E}_r \tilde{\mathbf{x}}(k+1) = (\mathbf{A}_r + \delta\mathbf{A}_r)\tilde{\mathbf{x}}(k) + (\mathbf{B}_r + \delta\mathbf{B}_r)\mathbf{u}(k) \\ \mathbf{y}(k) = (\mathbf{C}_r + \delta\mathbf{C}_r)\tilde{\mathbf{x}}(k) + (\mathbf{D} + \delta\mathbf{D})\mathbf{u}(k) \\ \mathbf{z}(k) = (\mathbf{L}_r + \delta\mathbf{L}_r)\tilde{\mathbf{x}}(k) + (\mathbf{J} + \delta\mathbf{J})\mathbf{u}(k), \end{cases} \tag{8}$$

where $\tilde{\mathbf{x}}(k) = \mathbf{Q}^{-1}\mathbf{x}(k) = \begin{bmatrix} \tilde{\mathbf{x}}_1^{\mathrm{T}}(k) & \tilde{\mathbf{x}}_2^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$, $\tilde{\mathbf{x}}_1(k) \in \mathcal{R}^r$, $\tilde{\mathbf{x}}_2(k) \in \mathcal{R}^{n-r}$, and the constant uncertainty matrices satisfy

$$\begin{bmatrix} \delta\mathbf{A}_r & \delta\mathbf{B}_r \\ \delta\mathbf{C}_r & \delta\mathbf{D} \\ \delta\mathbf{L}_r & \delta\mathbf{J} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{xr} \\ \mathbf{H}_y \\ \mathbf{H}_z \end{bmatrix} \boldsymbol{\Delta} \begin{bmatrix} \mathbf{E}_{xr} & \mathbf{E}_u \end{bmatrix} \tag{9}$$

with $\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta} \leq \mathbf{I}$. The matrices

$$\begin{aligned} &\mathbf{E}_r = \mathbf{PEQ} = \begin{bmatrix} \mathbf{I}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \mathbf{A}_r = \mathbf{PAQ} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}, \\ &\mathbf{B}_r = \mathbf{PB} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}, \qquad \mathbf{C}_r = \mathbf{CQ} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix}, \\ &\mathbf{L}_r = \mathbf{LQ} = \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 \end{bmatrix}, \qquad \mathbf{H}_{xr} = \mathbf{PH}_x = \begin{bmatrix} \mathbf{H}_{x1} \\ \mathbf{H}_{x2} \end{bmatrix}, \\ &\mathbf{E}_{xr} = \mathbf{E}_x\mathbf{Q} = \begin{bmatrix} \mathbf{E}_{x1} & \mathbf{E}_{x2} \end{bmatrix}. \end{aligned} \tag{10}$$

The r.s.e. system Σ_{r} in (8) may be more explicitly written as

$$\begin{aligned} \tilde{\mathbf{x}}_1(k+1) &= (\mathbf{A}_{11} + \mathbf{H}_{x1}\boldsymbol{\Delta}\mathbf{E}_{x1})\tilde{\mathbf{x}}_1(k) + (\mathbf{A}_{12} + \mathbf{H}_{x1}\boldsymbol{\Delta}\mathbf{E}_{x2})\tilde{\mathbf{x}}_2(k) \\ &\quad + (\mathbf{B}_1 + \mathbf{H}_{x1}\boldsymbol{\Delta}\mathbf{E}_u)\mathbf{u}(k), && (11) \\ \mathbf{0} &= (\mathbf{A}_{21} + \mathbf{H}_{x2}\Delta\mathbf{E}_{x1})\tilde{\mathbf{x}}_1(k) + (\mathbf{A}_{22} + \mathbf{H}_{x2}\boldsymbol{\Delta}\mathbf{E}_{x2})\tilde{\mathbf{x}}_2(k) \\ &\quad + (\mathbf{B}_2 + \mathbf{H}_{x2}\boldsymbol{\Delta}\mathbf{E}_u)\mathbf{u}(k), && (12) \\ \mathbf{y}(k) &= (\mathbf{C}_1 + \mathbf{H}_y\boldsymbol{\Delta}\mathbf{E}_{x1})\tilde{\mathbf{x}}_1(k) + (\mathbf{C}_2 + \mathbf{H}_y\boldsymbol{\Delta}\mathbf{E}_{x2})\tilde{\mathbf{x}}_2(k) \\ &\quad + (\mathbf{D} + \mathbf{H}_y\boldsymbol{\Delta}\mathbf{E}_u)\mathbf{u}(k), && (13) \\ \mathbf{z}(k) &= (\mathbf{L}_1 + \mathbf{H}_z\boldsymbol{\Delta}\mathbf{E}_{x1})\tilde{\mathbf{x}}_1(k) + (\mathbf{L}_2 + \mathbf{H}_z\Delta\mathbf{E}_{x2})\tilde{\mathbf{x}}_2(k) \\ &\quad + (\mathbf{J} + \mathbf{H}_z\boldsymbol{\Delta}\mathbf{E}_u)\mathbf{u}(k). && (14) \end{aligned}$$

By Lemma 2, the pair $(\mathbf{E}_r, \mathbf{A}_r + \delta\mathbf{A}_r)$ of Σ_r with parameter matrices in (9) and (10) is admissible. In addition, by Lemma 1, the term $(\mathbf{A}_{22} + \mathbf{H}_{x2}\boldsymbol{\Delta}\mathbf{E}_{x2})$ in (12) is nonsingular for all $\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta} \leq \mathbf{I}$, including $\boldsymbol{\Delta} = \mathbf{0}$, which implies that $\mathbf{A}_{22}$ is nonsingular. Let the nonsingular matrices $\bar{\mathbf{P}} = \text{diag}(\mathbf{I}_r, \mathbf{A}_{22}^{-1})$ and $\bar{\mathbf{Q}} = \mathbf{I}_n$. Then Σ_r in (11)–(14) is, via $\bar{\mathbf{P}}$ and $\bar{\mathbf{Q}}$, r.s.e. to

$$\tilde{\Sigma}_\mathsf{r} : \begin{cases} \mathbf{E}_r\tilde{\mathbf{x}}(k+1) = \bar{\mathbf{P}}(\mathbf{A}_r + \delta\mathbf{A}_r)\tilde{\mathbf{x}}(k) + \bar{\mathbf{P}}(\mathbf{B}_r + \delta\mathbf{B}_r)\mathbf{u}(k) \\ \mathbf{y}(k) = (\mathbf{C}_r + \delta\mathbf{C}_r)\tilde{\mathbf{x}}(k) + (\mathbf{D} + \delta\mathbf{D})\mathbf{u}(k) \\ \mathbf{z}(k) = (\mathbf{L}_r + \delta\mathbf{L}_r)\tilde{\mathbf{x}}(k) + (\mathbf{J} + \delta\mathbf{J})\mathbf{u}(k), \end{cases} \tag{15}$$

which can be represented more explicitly by (11), (13), (14), and

$$\begin{aligned} \mathbf{0} = {} & (\bar{\mathbf{A}}_{21} + \bar{\mathbf{H}}_{x2}\Delta\mathbf{E}_{x1})\tilde{\mathbf{x}}_1(k) + (\mathbf{I}_{n-r} + \bar{\mathbf{H}}_{x2}\Delta\mathbf{E}_{x2})\tilde{\mathbf{x}}_2(k) \\ & + (\bar{\mathbf{B}}_2 + \bar{\mathbf{H}}_{x2}\mathbf{\Delta}\mathbf{E}_u)\mathbf{u}(k) \end{aligned} \tag{16}$$

with $\bar{\mathbf{A}}_{21} = \mathbf{A}_{22}^{-1}\mathbf{A}_{21}$, $\bar{\mathbf{B}}_2 = \mathbf{A}_{22}^{-1}\mathbf{B}_2$, and $\bar{\mathbf{H}}_{x2} = \mathbf{A}_{22}^{-1}\mathbf{H}_{x2}$.

By Lemma 1, the term $(\mathbf{I}_{n-r} + \bar{\mathbf{H}}_{x2}\mathbf{\Delta}\mathbf{E}_{x2})$ in (16) is also nonsingular, because of the admissibility of $\tilde{\Sigma}_\mathsf{r}$ maintained by Lemma 2. Using the identity

$$(\mathbf{I} + \mathbf{MN})^{-1} = \mathbf{I} - \mathbf{M}(\mathbf{I} + \mathbf{NM})^{-1}\mathbf{N} \tag{17}$$

for any real matrices $\mathbf{M}$ and $\mathbf{N}$ with appropriate dimensions, one has

$$(\mathbf{I}_{n-r} + \bar{\mathbf{H}}_{x2}\mathbf{\Delta}\mathbf{E}_{x2})^{-1} = \mathbf{I}_{n-r} - \bar{\mathbf{H}}_{x2}\hat{\mathbf{\Delta}}\mathbf{E}_{x2}, \tag{18}$$

where $\hat{\mathbf{\Delta}} = \mathbf{\Delta}(\mathbf{I}_{d_2} + \mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\mathbf{\Delta})^{-1}$. Therefore, (16) may be rearranged as

$$\tilde{\mathbf{x}}_2(k) = -(\bar{\mathbf{A}}_{21} + \bar{\mathbf{H}}_{x2}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1})\tilde{\mathbf{x}}_1(k) - (\bar{\mathbf{B}}_2 + \bar{\mathbf{H}}_{x2}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u)\mathbf{u}(k), \tag{19}$$

where $\bar{\mathbf{E}}_{x1} = \mathbf{E}_{x1} - \mathbf{E}_{x2}\bar{\mathbf{A}}_{21}$ and $\bar{\mathbf{E}}_u = \mathbf{E}_u - \mathbf{E}_{x2}\bar{\mathbf{B}}_2$. By substituting (19) into (11), (13), and (14), the system $\tilde{\Sigma}_\mathsf{r}$ is reduced to

$$\tilde{\Sigma}_\mathsf{r} : \begin{cases} \tilde{\mathbf{x}}_1(k+1) = (\bar{\mathbf{A}}_{11} + \bar{\mathbf{H}}_{x1}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1})\tilde{\mathbf{x}}_1(k) + (\bar{\mathbf{B}}_1 + \bar{\mathbf{H}}_{x1}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u)\mathbf{u}(k) \\ \mathbf{y}(k) = (\bar{\mathbf{C}}_1 + \bar{\mathbf{H}}_y\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1})\tilde{\mathbf{x}}_1(k) + (\bar{\mathbf{D}} + \bar{\mathbf{H}}_y\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u)\mathbf{u}(k) \\ \mathbf{z}(k) = (\bar{\mathbf{L}}_1 + \bar{\mathbf{H}}_z\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1})\tilde{\mathbf{x}}_1(k) + (\bar{\mathbf{J}} + \bar{\mathbf{H}}_z\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u)\mathbf{u}(k), \end{cases} \tag{20}$$

where

$$\begin{aligned} &\bar{\mathbf{A}}_{11} = \mathbf{A}_{11} - \mathbf{A}_{12}\bar{\mathbf{A}}_{21}, && \bar{\mathbf{B}}_1 = \mathbf{B}_1 - \mathbf{A}_{12}\bar{\mathbf{B}}_2, && \bar{\mathbf{C}}_1 = \mathbf{C}_1 - \mathbf{C}_2\bar{\mathbf{A}}_{21}, \\ &\bar{\mathbf{D}} = \mathbf{D} - \mathbf{C}_2\bar{\mathbf{B}}_2, && \bar{\mathbf{L}}_1 = \mathbf{L}_1 - \mathbf{L}_2\bar{\mathbf{A}}_{21}, && \bar{\mathbf{J}} = \mathbf{J} - \mathbf{L}_2\bar{\mathbf{B}}_2, \\ &\bar{\mathbf{H}}_{x1} = \mathbf{H}_{x1} - \mathbf{A}_{12}\bar{\mathbf{H}}_{x2}, && \bar{\mathbf{H}}_y = \mathbf{H}_y - \mathbf{C}_2\bar{\mathbf{H}}_{x2}, && \bar{\mathbf{H}}_z = \mathbf{H}_z - \mathbf{L}_2\bar{\mathbf{H}}_{x2}, \\ &\bar{\mathbf{E}}_{x1} = \mathbf{E}_{x1} - \mathbf{E}_{x2}\bar{\mathbf{A}}_{21}, && \bar{\mathbf{E}}_u = \mathbf{E}_u - \mathbf{E}_{x2}\bar{\mathbf{B}}_2. \end{aligned} \tag{21}$$

Note that $\tilde{\Sigma}_\mathsf{r}$ in (20) is a normal system [3], and its stability is guaranteed by Lemma 2 with the r.s.e. relationship.

The transformation from singular to normal system models enables one to handle the robust filtering problem for uncertain singular systems more easily, because many existing filter design methods for normal systems can be applied. Besides, filters designed this way have fewer states than singular filters designed directly from the singular system models. Finally, sometimes the physical realizations of singular filters are not easy [3], [4]. In order to realize singular filters, one often needs special algorithms [15] to convert to a normal state-space form.

However, it must be pointed out that in general the transformation is not unique, and for Σ in (6), there may be more than one pair of nonsingular matrices $\{\mathbf{P}, \mathbf{Q}\}$ capable of making $\mathbf{PEQ} = \text{diag}(\mathbf{I}_r, \mathbf{0})$. Among the various methods to find a feasible pair $\{\mathbf{P}, \mathbf{Q}\}$, one is stated here. Let a singular value decomposition [9] of a given $\mathbf{E}$ in (6) be $\mathbf{E} = \bar{\mathbf{U}}\,\text{diag}(\mathbf{\Sigma}, \mathbf{0})\bar{\mathbf{V}}^{\mathrm{T}}$, where $\bar{\mathbf{U}}, \bar{\mathbf{V}} \in \mathcal{R}^{n\times n}$ are unitary, $\mathbf{\Sigma} = \text{diag}(\sigma_1, \ldots, \sigma_r)$, and $\sigma_i > 0$, $i = 1, \ldots, r$, are the singular values of $\mathbf{E}$. Thus, $\text{diag}(\mathbf{\Sigma}^{-1}, \mathbf{I}_{n-r})\bar{\mathbf{U}}^{\mathrm{T}}\mathbf{E}\bar{\mathbf{V}} = \text{diag}(\mathbf{I}_r, \mathbf{0})$, and a feasible pair $\{\mathbf{P}, \mathbf{Q}\}$ is $\{\text{diag}(\mathbf{\Sigma}^{-1}, \mathbf{I}_{n-r})\bar{\mathbf{U}}^{\mathrm{T}}, \bar{\mathbf{V}}\}$.

2.3. Problem statement

Consider the normal stable system $\tilde{\Sigma}_{\mathrm{r}}$ in (20) subject to $\hat{\mathbf{\Delta}} = \mathbf{\Delta}(\mathbf{I}_{d2} + \mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\mathbf{\Delta})^{-1}$ and $\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Delta} \leq \mathbf{I}$. To estimate $\mathbf{z}(k)$, the following filter:

$$\Sigma_{\mathrm{f}} : \begin{cases} \mathbf{x}_f(k+1) = \mathbf{A}_f\mathbf{x}_f(k) + \mathbf{B}_f\mathbf{y}(k) \\ \mathbf{z}_f(k) = \mathbf{C}_f\mathbf{x}_f(k) + \mathbf{D}_f\mathbf{y}(k) \end{cases} \tag{22}$$

is adopted, where $\mathbf{x}_f(k) \in \mathcal{R}^r$ and $\mathbf{z}_f(k) \in \mathcal{R}^q$. The matrices $\mathbf{A}_f, \mathbf{B}_f$, $\mathbf{C}_f$, and $\mathbf{D}_f$ are to be determined. From $\tilde{\Sigma}_{\mathrm{r}}$ in (20) and Σ_{f} in (22), the filtering error dynamics may be written as

$$\Sigma_{\mathrm{e}} : \begin{cases} \mathbf{x}_e(k+1) = \mathbf{A}_e\mathbf{x}_e(k) + \mathbf{B}_e\mathbf{u}(k) \\ \mathbf{e}(k) = \mathbf{C}_e\mathbf{x}_e(k) + \mathbf{D}_e\mathbf{u}(k), \end{cases} \tag{23}$$

where $\mathbf{e}(k) = \mathbf{z}(k) - \mathbf{z}_f(k)$, $\mathbf{x}_e^{\mathrm{T}}(k) = [\,\tilde{\mathbf{x}}_1^{\mathrm{T}}(k) \;\; \mathbf{x}_f^{\mathrm{T}}(k)\,]$,

$$\mathbf{A}_e = \begin{bmatrix} \hat{\mathbf{A}} & \mathbf{0} \\ \mathbf{B}_f\hat{\mathbf{C}} & \mathbf{A}_f \end{bmatrix}, \qquad \mathbf{B}_e = \begin{bmatrix} \hat{\mathbf{B}} \\ \mathbf{B}_f\hat{\mathbf{D}} \end{bmatrix},$$
$$\mathbf{C}_e = \begin{bmatrix} \hat{\mathbf{L}} - \mathbf{D}_f\hat{\mathbf{C}} & -\mathbf{C}_f \end{bmatrix}, \qquad \mathbf{D}_e = \hat{\mathbf{J}} - \mathbf{D}_f\hat{\mathbf{D}}, \tag{24}$$

and

$$\begin{aligned} &\hat{\mathbf{A}} = \bar{\mathbf{A}}_{11} + \bar{\mathbf{H}}_{x1}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1}, \qquad \hat{\mathbf{B}} = \bar{\mathbf{B}}_1 + \bar{\mathbf{H}}_{x1}\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u, \qquad \hat{\mathbf{C}} = \bar{\mathbf{C}}_1 + \bar{\mathbf{H}}_y\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1}, \\ &\hat{\mathbf{D}} = \bar{\mathbf{D}} + \bar{\mathbf{H}}_y\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u, \qquad \hat{\mathbf{L}} = \bar{\mathbf{L}}_1 + \bar{\mathbf{H}}_z\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_{x1}, \qquad \hat{\mathbf{J}} = \bar{\mathbf{J}} + \bar{\mathbf{H}}_z\hat{\mathbf{\Delta}}\bar{\mathbf{E}}_u. \end{aligned} \tag{25}$$

The purpose here is to design a stable filter Σ_{f} such that

$$\sup_{\mathbf{\Delta}} \|\mathbf{C}_e(z\mathbf{I}_{2r} - \mathbf{A}_e)^{-1}\mathbf{B}_e + \mathbf{D}_e\|_\infty < \mu_e \tag{26}$$

for a prescribed H_∞-norm bound $\mu_e > 0$.

At this point an extra assumption $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\| < 1$ is added, which is solely for enabling the LMI formulation in Theorem 2 to be developed in Section 3. Though this extra assumption limits the systems that may be handled, its validity is not affected by the choice of the transformation matrices $\mathbf{P}$ and $\mathbf{Q}$, as can be proved in the following lemma.

Lemma 3. *Consider the uncertain singular system* Σ *in (6)–(7) with the admissible pair* $(\mathbf{E}, \mathbf{A}+\delta\mathbf{A})$. *The value of* $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\|$ *is independent of the transformation matrix pair* $\{\mathbf{P}, \mathbf{Q}\}$, *making* $\mathbf{PEQ} = \mathrm{diag}(\mathbf{I}_r, \mathbf{0})$ *in (10).*

Proof. Suppose the nonsingular matrix pair $\{\tilde{\mathbf{P}}, \tilde{\mathbf{Q}}\}$ makes $\tilde{\mathbf{P}}\mathbf{E}\tilde{\mathbf{Q}} = \mathrm{diag}(\mathbf{I}_r, \mathbf{0})$ and

$$\tilde{\mathbf{P}}\mathbf{A}\tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{I}_{n-r} \end{bmatrix}, \quad \tilde{\mathbf{P}}\mathbf{H}_x = \begin{bmatrix} \mathbf{H}_{x1} \\ \mathbf{A}_{22}^{-1}\mathbf{H}_{x2} \end{bmatrix}, \quad \mathbf{E}_x\tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{E}_{x1} & \mathbf{E}_{x2} \end{bmatrix}. \tag{27}$$

By (16) and (27), the concerned norm is $\|\mathbf{E}_{x2}\mathbf{A}_{22}^{-1}\mathbf{H}_{x2}\|$. Let $\hat{\mathbf{P}}, \hat{\mathbf{Q}} \in \mathcal{R}^{n\times n}$ be any two nonsingular matrices satisfying $\hat{\mathbf{P}}\tilde{\mathbf{P}}\mathbf{E}\tilde{\mathbf{Q}}\hat{\mathbf{Q}} = \mathrm{diag}(\mathbf{I}_r, \mathbf{0})$. Partition $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$ as

$$\hat{\mathbf{P}} = \begin{bmatrix} \hat{\mathbf{P}}_{11} & \hat{\mathbf{P}}_{12} \\ \hat{\mathbf{P}}_{21} & \hat{\mathbf{P}}_{22} \end{bmatrix}, \qquad \hat{\mathbf{Q}} = \begin{bmatrix} \hat{\mathbf{Q}}_{11} & \hat{\mathbf{Q}}_{12} \\ \hat{\mathbf{Q}}_{21} & \hat{\mathbf{Q}}_{22} \end{bmatrix}, \tag{28}$$

where $\hat{\mathbf{P}}_{11}, \hat{\mathbf{Q}}_{11} \in \mathcal{R}^{r\times r}$, and $\hat{\mathbf{P}}_{22}, \hat{\mathbf{Q}}_{22} \in \mathcal{R}^{(n-r)\times(n-r)}$. Then $\hat{\mathbf{P}}_{11}\hat{\mathbf{Q}}_{11} = \mathbf{I}_r$, $\hat{\mathbf{P}}_{21} = \mathbf{0}$, and $\hat{\mathbf{Q}}_{12} = \mathbf{0}$. From (27), the (2, 2) block of $\hat{\mathbf{P}}\tilde{\mathbf{P}}(\mathbf{A} + \mathbf{H}_x\boldsymbol{\Delta}\mathbf{E}_x)\tilde{\mathbf{Q}}\hat{\mathbf{Q}}$ is $\hat{\mathbf{P}}_{22}\hat{\mathbf{Q}}_{22} + \hat{\mathbf{P}}_{22}\mathbf{A}_{22}^{-1}\mathbf{H}_{x2}\boldsymbol{\Delta}\mathbf{E}_{x2}\hat{\mathbf{Q}}_{22}$. By Lemma 1, $\hat{\mathbf{P}}_{22}\hat{\mathbf{Q}}_{22}$ is nonsingular, which implies that both $\hat{\mathbf{P}}_{22}$ and $\hat{\mathbf{Q}}_{22}$ are also nonsingular. Therefore, when $\{\hat{\mathbf{P}}\tilde{\mathbf{P}}, \tilde{\mathbf{Q}}\hat{\mathbf{Q}}\}$ is regarded as another transformation matrix pair, the corresponding new $\mathbf{E}_{x2}$ and $\bar{\mathbf{H}}_{x2}$ are $\mathbf{E}_{x2}\hat{\mathbf{Q}}_{22}$ and $(\hat{\mathbf{P}}_{22}\hat{\mathbf{Q}}_{22})^{-1}\hat{\mathbf{P}}_{22}\mathbf{A}_{22}^{-1}\mathbf{H}_{x2}$, respectively, and the concerned norm is $\|\mathbf{E}_{x2}\hat{\mathbf{Q}}_{22}(\hat{\mathbf{P}}_{22}\hat{\mathbf{Q}}_{22})^{-1}\hat{\mathbf{P}}_{22}\mathbf{A}_{22}^{-1}\mathbf{H}_{x2}\| = \|\mathbf{E}_{x2}\mathbf{A}_{22}^{-1}\mathbf{H}_{x2}\|$. □

2.4. Three useful lemmas

The following is a well-known lemma extended from the Bounded Real Lemma [7] for characterizing the H_∞-norm constraint.

Lemma 4 [8], [25]**.** *The error dynamic system* Σ_e *in (23) is quadratically stable [1] and satisfies (26) for a given* $\mu_e > 0$, *if and only if there exists a* $\mathbf{P}_e > \mathbf{0}$ *such that*

$$\begin{bmatrix} -\mathbf{P}_e & \mathbf{0} & \mathbf{A}_e^{\mathrm{T}}\mathbf{P}_e & \mathbf{C}_e^{\mathrm{T}} \\ \mathbf{0} & -\mu_e^2\mathbf{I} & \mathbf{B}_e^{\mathrm{T}}\mathbf{P}_e & \mathbf{D}_e^{\mathrm{T}} \\ \mathbf{P}_e\mathbf{A}_e & \mathbf{P}_e\mathbf{B}_e & -\mathbf{P}_e & \mathbf{0} \\ \mathbf{C}_e & \mathbf{D}_e & \mathbf{0} & -\mathbf{I} \end{bmatrix} < \mathbf{0}. \tag{29}$$

It is known [1] that the quadratic stability of a system implies its asymptotic stability. Because $\tilde{\Sigma}_\mathsf{r}$ in (20) is stable, the quadratic stability of Σ_e in (23) implies that the filter Σ_f in (22) is asymptotically stable.

The next two lemmas are useful for formulating the problem within the LMI framework.

Lemma 5 [19]. *Let* $\mathbf{I}-\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}>\mathbf{0}$, *and define the set*

$$\Upsilon=\{\boldsymbol{\Delta}(\mathbf{I}-\boldsymbol{\Gamma}\boldsymbol{\Delta})^{-1},\ \boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta}\leq\mathbf{I}\}.$$

Then, $\Upsilon=\{\boldsymbol{\Gamma}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}})^{-1}+\boldsymbol{\Pi}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{T}})^{-1/2},\ \boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{\Pi}\leq(\mathbf{I}-\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma})^{-1}\}$.

Lemma 6 [11]. *Let* $\boldsymbol{\Omega}$, $\bar{\mathbf{M}}$, $\bar{\mathbf{N}}$, *and* $\mathbf{R}>\mathbf{0}$ *be real matrices with appropriate dimensions, and let the matrix* $\bar{\boldsymbol{\Pi}}$ *satisfy* $\bar{\boldsymbol{\Pi}}^{\mathrm{T}}\bar{\boldsymbol{\Pi}}\leq\mathbf{R}$. *Then for all* $\bar{\boldsymbol{\Pi}}^{\mathrm{T}}\bar{\boldsymbol{\Pi}}\leq\mathbf{R}$ *the matrix inequality*

$$\boldsymbol{\Omega}+\bar{\mathbf{M}}\bar{\boldsymbol{\Pi}}\bar{\mathbf{N}}+\bar{\mathbf{N}}^{\mathrm{T}}\bar{\boldsymbol{\Pi}}^{\mathrm{T}}\bar{\mathbf{M}}^{\mathrm{T}}<\mathbf{0}$$

holds if and only if there exists a scalar $\varepsilon>0$ *such that*

$$\begin{bmatrix}\boldsymbol{\Omega} & \bar{\mathbf{M}}\\ \bar{\mathbf{M}}^{\mathrm{T}} & \mathbf{0}\end{bmatrix}+\varepsilon\begin{bmatrix}\bar{\mathbf{N}}^{\mathrm{T}}\mathbf{R}\bar{\mathbf{N}} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix}<\mathbf{0}.$$

3. Robust filter design

In the literature, many authors [6], [13], [14], [25] have discussed normal robust filtering problems with various specifications, mainly based on Lemma 4. Here the method for proving Theorem 1 of [13] is modified to treat a different kind of uncertainty, and to derive the following preliminary theorem, which is the first step toward developing an LMI solution to the problem stated in Section 2.

Theorem 1. *The filtering error dynamics* Σ_{e} *in (23) is quadratically stable and satisfies (26) for all admissible uncertainties, if and only if there exist* $\boldsymbol{\Phi}\in\mathcal{R}^{r\times r}$, $\mathbf{X}\in\mathcal{R}^{r\times r}$, $\mathbf{Y}\in\mathcal{R}^{q\times r}$, $\mathbf{Z}\in\mathcal{R}^{r\times q}$, $\mathbf{W}\in\mathcal{R}^{r\times r}$, *and* $\mathbf{D}_f\in\mathcal{R}^{q\times p}$ *such that*

$$\begin{bmatrix}-\boldsymbol{\Phi} & * & * & * & * & *\\ -\boldsymbol{\Phi} & -\mathbf{X} & * & * & * & *\\ \mathbf{0} & \mathbf{0} & -\mu_e^2\mathbf{I} & * & * & *\\ \boldsymbol{\Phi}\hat{\mathbf{A}} & \boldsymbol{\Phi}\hat{\mathbf{A}} & \boldsymbol{\Phi}\hat{\mathbf{B}} & -\boldsymbol{\Phi} & * & *\\ \mathbf{X}\hat{\mathbf{A}}+\mathbf{Z}\hat{\mathbf{C}}+\mathbf{W} & \mathbf{X}\hat{\mathbf{A}}+\mathbf{Z}\hat{\mathbf{C}} & \mathbf{X}\hat{\mathbf{B}}+\mathbf{Z}\hat{\mathbf{D}} & -\boldsymbol{\Phi} & -\mathbf{X} & *\\ \hat{\mathbf{L}}-\mathbf{D}_f\hat{\mathbf{C}}-\mathbf{Y} & \hat{\mathbf{L}}-\mathbf{D}_f\hat{\mathbf{C}} & \hat{\mathbf{J}}-\mathbf{D}_f\hat{\mathbf{D}} & \mathbf{0} & \mathbf{0} & -\mathbf{I}\end{bmatrix}<\mathbf{0}, \quad (30)$$

$$\begin{bmatrix}\boldsymbol{\Phi} & \boldsymbol{\Phi}\\ \boldsymbol{\Phi} & \mathbf{X}\end{bmatrix}>\mathbf{0}, \quad (31)$$

where $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$, $\hat{\mathbf{D}}$, $\hat{\mathbf{L}}$, *and* $\hat{\mathbf{J}}$ *are defined in (25). When the preceeding inequalities hold, the filter* Σ_{f} *in (22) with filter gains*

$$\mathbf{A}_f=-\mathbf{U}^{-1}\mathbf{W}\mathbf{U}^{-\mathrm{T}},\qquad \mathbf{B}_f=\mathbf{U}^{-1}\mathbf{Z},\qquad \mathbf{C}_f=-\mathbf{Y}\mathbf{U}^{-\mathrm{T}},\qquad \mathbf{D}_f \quad (32)$$

is a solution to the considered robust filtering problem, where $\mathbf{U}$ *is nonsingular and satisfies* $\mathbf{U}\mathbf{U}^{\mathrm{T}}=\mathbf{X}-\boldsymbol{\Phi}$.

Proof. (Sufficiency) By the Schur complement [2] and the inequality (31), $\mathbf{\Phi} > \mathbf{0}$ and $\mathbf{X} - \mathbf{\Phi} > \mathbf{0}$. Thus, $\mathbf{I} - \mathbf{X\Phi}^{-1}$ is nonsingular and there exist nonsingular matrices $\mathbf{U}$ and $\mathbf{V}$ such that $\mathbf{I} - \mathbf{X\Phi}^{-1} = \mathbf{UV}^{\mathrm{T}}$. Let

$$\hat{\mathbf{T}} = \begin{bmatrix} \mathbf{\Phi}^{-1} & \mathbf{I} \\ \mathbf{V}^{\mathrm{T}} & \mathbf{0} \end{bmatrix}, \qquad \check{\mathbf{T}} = \begin{bmatrix} \mathbf{I} & \mathbf{X} \\ \mathbf{0} & \mathbf{U}^{\mathrm{T}} \end{bmatrix}, \tag{33}$$

where $\hat{\mathbf{T}}$ is nonsingular as $\hat{\mathbf{T}}^{-1} = \begin{bmatrix} \mathbf{0} & \mathbf{V}^{-\mathrm{T}} \\ \mathbf{I} & -\mathbf{\Phi}^{-1}\mathbf{V}^{-\mathrm{T}} \end{bmatrix}$. Define $\mathbf{P}_e = \check{\mathbf{T}}\hat{\mathbf{T}}^{-1} = \begin{bmatrix} \mathbf{X} & \mathbf{U} \\ \mathbf{U}^{\mathrm{T}} & \mathbf{I} \end{bmatrix}$ by letting $\mathbf{U} = -\mathbf{\Phi V}$. Under this arrangement $\mathbf{P}_e > \mathbf{0}$ because $\mathbf{X} - \mathbf{UU}^{\mathrm{T}} = \mathbf{X} + \mathbf{UV}^{\mathrm{T}}\mathbf{\Phi} = \mathbf{\Phi} > \mathbf{0}$. Next, pre- and post-multiply (30) by $\mathrm{diag}(\mathbf{\Phi}^{-1}, \mathbf{I}, \mathbf{I}, \mathbf{\Phi}^{-1}, \mathbf{I}, \mathbf{I})$ at the same time. Substituting (24), (32), (33), $\mathbf{U} = -\mathbf{\Phi V}$, and $\mathbf{P}_e = \check{\mathbf{T}}\hat{\mathbf{T}}^{-1}$ into the resulting inequality, as well as pre- and post-multiplying by $\mathrm{diag}(\hat{\mathbf{T}}^{-\mathrm{T}}, \mathbf{I}, \hat{\mathbf{T}}^{-\mathrm{T}}, \mathbf{I})$ and $\mathrm{diag}(\hat{\mathbf{T}}^{-1}, \mathbf{I}, \hat{\mathbf{T}}^{-1}, \mathbf{I})$, respectively, give (29). By Lemma 4, the error dynamics in (23) is quadratically stable, which implies the filter in (22) with gains in (32) is asymptotically stable, and the H_∞ performance requirement (26) is satisfied for all admissible uncertainties.

(Necessity) If the filtering error dynamics Σ_{e} is quadratically stable and has the H_∞-norm bound μ_e, then by Lemma 4 there exists a $\mathbf{P}_e > \mathbf{0}$ such that (29) is satisfied. Let $\mathbf{P}_e$ and its inverse $\mathbf{P}_e^{-1}$ be partitioned as

$$\mathbf{P}_e = \begin{bmatrix} \mathbf{X} & \mathbf{U} \\ \mathbf{U}^{\mathrm{T}} & \mathbf{\Psi} \end{bmatrix}, \qquad \mathbf{P}_e^{-1} = \begin{bmatrix} \mathbf{\Phi}^{-1} & \mathbf{V} \\ \mathbf{V}^{\mathrm{T}} & \star \end{bmatrix}, \tag{34}$$

where $\mathbf{X} > \mathbf{0}$, $\mathbf{\Phi} > \mathbf{0}$, and $\star$ denotes the submatrix which is insignificant in this proof. From $\mathbf{P}_e\mathbf{P}_e^{-1} = \mathbf{I}$, it is seen that $\mathbf{I} - \mathbf{X\Phi}^{-1} = \mathbf{UV}^{\mathrm{T}}$ with $\mathbf{U}$, $\mathbf{V}$ nonsingular [16], and $\mathbf{U} = -\mathbf{\Phi V\Psi}^{\mathrm{T}}$. Form a nonsingular matrix $\hat{\mathbf{T}}$ as in (33). Substitute $\mathbf{P}_e$ in (34) into (29), and pre- and post-multiply the resultant inequality by $\mathrm{diag}(\hat{\mathbf{T}}^{\mathrm{T}}, \mathbf{I}, \hat{\mathbf{T}}^{\mathrm{T}}, \mathbf{I})$ and $\mathrm{diag}(\hat{\mathbf{T}}, \mathbf{I}, \hat{\mathbf{T}}, \mathbf{I})$, respectively. Then (30) is obtained when

$$\mathbf{A}_f = \mathbf{U}^{-1}\mathbf{W\Phi}^{-1}\mathbf{V}^{-\mathrm{T}}\mathbf{\Psi}^{-1}, \qquad \mathbf{B}_f = \mathbf{U}^{-1}\mathbf{Z}, \qquad \mathbf{C}_f = \mathbf{Y\Phi}^{-1}\mathbf{V}^{-\mathrm{T}}\mathbf{\Psi}^{-1} \tag{35}$$

are substituted, and the resultant inequality is pre- and post-multiplied by $\mathrm{diag}(\mathbf{\Phi}, \mathbf{I}, \mathbf{I}, \mathbf{\Phi}, \mathbf{I}, \mathbf{I})$ at the same time. A similar but much simpler procedure applied to $\mathbf{P}_e$ in (34) produces the inequality in (31). □

Note that in addition to the filter gain matrices shown in the sufficiency part of Theorem 1, the following filter gains:

$$\mathbf{A}_f = (\mathbf{\Phi} - \mathbf{X})^{-1}\mathbf{W}, \qquad \mathbf{B}_f = (\mathbf{X} - \mathbf{\Phi})^{-1}\mathbf{Z}, \qquad \mathbf{C}_f = -\mathbf{Y}, \qquad \mathbf{D}_f \tag{36}$$

are also usable, because the transfer function matrix $\mathbf{G}_f(z)$ of the filter from $\mathbf{y}(k)$ to $\mathbf{z}_f(k)$ satisfies

$$\begin{aligned}\mathbf{G}_f(z) &= -\mathbf{Y}\mathbf{U}^{-\mathrm{T}}(z\mathbf{I}+\mathbf{U}^{-1}\mathbf{W}\mathbf{U}^{-\mathrm{T}})^{-1}\mathbf{U}^{-1}\mathbf{Z}+\mathbf{D}_f \\ &= -\mathbf{Y}[z\mathbf{I}+(\mathbf{U}\mathbf{U}^{\mathrm{T}})\mathbf{W}]^{-1}(\mathbf{U}\mathbf{U}^{\mathrm{T}})^{-1}\mathbf{Z}+\mathbf{D}_f \\ &= -\mathbf{Y}[z\mathbf{I}-(\boldsymbol{\Phi}-\mathbf{X})^{-1}\mathbf{W}]^{-1}(\mathbf{X}-\boldsymbol{\Phi})^{-1}\mathbf{Z}+\mathbf{D}_f.\end{aligned} \tag{37}$$

Next, in order to put the results of Theorem 1 under the LMI framework, the uncertainty $\hat{\boldsymbol{\Delta}}$ is reformulated by the equivalent description

$$\hat{\boldsymbol{\Delta}} = \boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}_{d_2}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}+\boldsymbol{\Pi}^{\mathrm{T}}(\mathbf{I}_{d_2}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1/2}, \tag{38}$$

by Lemma 5 and the assumption $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\| < 1$, where $\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{\Pi} \le (\mathbf{I}_{d1}-\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Theta})^{-1}$ and $\boldsymbol{\Theta} = -\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}$. Correspondingly, the matrices in (25) may be represented as

$$\begin{aligned}&\hat{\mathbf{A}} = \tilde{\mathbf{A}}+\bar{\mathbf{H}}_{x1}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{x1}, && \hat{\mathbf{B}} = \tilde{\mathbf{B}}+\bar{\mathbf{H}}_{x1}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{u}, && \hat{\mathbf{C}} = \tilde{\mathbf{C}}+\bar{\mathbf{H}}_{y}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{x1},\\ &\hat{\mathbf{D}} = \tilde{\mathbf{D}}+\bar{\mathbf{H}}_{y}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{u}, && \hat{\mathbf{L}} = \tilde{\mathbf{L}}+\bar{\mathbf{H}}_{z}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{x1}, && \hat{\mathbf{J}} = \tilde{\mathbf{J}}+\bar{\mathbf{H}}_{z}\boldsymbol{\Pi}^{\mathrm{T}}\tilde{\mathbf{E}}_{u},\end{aligned} \tag{39}$$

where

$$\begin{aligned}\tilde{\mathbf{A}} &= \bar{\mathbf{A}}_{11}+\bar{\mathbf{H}}_{x1}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{x1}, & \tilde{\mathbf{B}} &= \bar{\mathbf{B}}_{1}+\bar{\mathbf{H}}_{x1}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{u},\\ \tilde{\mathbf{C}} &= \bar{\mathbf{C}}_{1}+\bar{\mathbf{H}}_{y}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{x1}, & \tilde{\mathbf{D}} &= \bar{\mathbf{D}}+\bar{\mathbf{H}}_{y}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{u},\\ \tilde{\mathbf{L}} &= \bar{\mathbf{L}}_{1}+\bar{\mathbf{H}}_{z}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{x1}, & \tilde{\mathbf{J}} &= \bar{\mathbf{J}}+\bar{\mathbf{H}}_{z}\boldsymbol{\Theta}^{\mathrm{T}}(\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1}\bar{\mathbf{E}}_{u},\\ \tilde{\mathbf{E}}_{x1} &= (\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1/2}\bar{\mathbf{E}}_{x1}, & \tilde{\mathbf{E}}_{u} &= (\mathbf{I}-\boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}})^{-1/2}\bar{\mathbf{E}}_{u}.\end{aligned} \tag{40}$$

Then Theorem 2 below is an LMI version of Theorem 1.

Theorem 2. *Under the assumption of* $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\| < 1$*, the filtering error dynamics* Σ_{e} *in (23) is quadratically stable and satisfies (26) for a given* $\mu_e > 0$ *with all considered uncertainties, if and only if there exist* $\boldsymbol{\Phi} \in \mathcal{R}^{r\times r}$, $\mathbf{X} \in \mathcal{R}^{r\times r}$, $\mathbf{Y} \in \mathcal{R}^{q\times r}$, $\mathbf{Z} \in \mathcal{R}^{r\times q}$, $\mathbf{W} \in \mathcal{R}^{r\times r}$, $\mathbf{D}_f \in \mathcal{R}^{q\times p}$, *and* $\varepsilon^{-1} > 0$ *such that the LMIs in (31) and*

$$\begin{bmatrix} -\boldsymbol{\Phi} & * & * & * & * & * & * & * \\ -\boldsymbol{\Phi} & -\mathbf{X} & * & * & * & * & * & * \\ \mathbf{0} & \mathbf{0} & -\mu_e^2\mathbf{I}_m & * & * & * & * & * \\ \boldsymbol{\Phi}\tilde{\mathbf{A}} & \boldsymbol{\Phi}\tilde{\mathbf{A}} & \boldsymbol{\Phi}\tilde{\mathbf{B}} & -\boldsymbol{\Phi} & * & * & * & * \\ \mathbf{M}_{51} & \mathbf{M}_{52} & \mathbf{M}_{53} & -\boldsymbol{\Phi} & -\mathbf{X} & * & * & * \\ \mathbf{M}_{61} & \mathbf{M}_{62} & \mathbf{M}_{63} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_q & * & * \\ \varepsilon^{-1}\tilde{\mathbf{E}}_{x1} & \varepsilon^{-1}\tilde{\mathbf{E}}_{x1} & \varepsilon^{-1}\tilde{\mathbf{E}}_{u} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\varepsilon^{-1}\mathbf{I}_{d_2} & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{H}}_{x1}^{\mathrm{T}}\boldsymbol{\Phi} & \mathbf{M}_{85} & \mathbf{M}_{86} & \mathbf{0} & \mathbf{M}_{88} \end{bmatrix} < \mathbf{0}, \tag{41}$$

are satisfied, where

$$\begin{aligned}&\mathbf{M}_{51} = \mathbf{X}\tilde{\mathbf{A}}+\mathbf{Z}\tilde{\mathbf{C}}+\mathbf{W}, && \mathbf{M}_{52} = \mathbf{X}\tilde{\mathbf{A}}+\mathbf{Z}\tilde{\mathbf{C}}, && \mathbf{M}_{53} = \mathbf{X}\tilde{\mathbf{B}}+\mathbf{Z}\tilde{\mathbf{D}},\\ &\mathbf{M}_{61} = \tilde{\mathbf{L}}-\mathbf{D}_f\tilde{\mathbf{C}}-\mathbf{Y}, && \mathbf{M}_{62} = \tilde{\mathbf{L}}-\mathbf{D}_f\tilde{\mathbf{C}}, && \mathbf{M}_{63} = \tilde{\mathbf{J}}-\mathbf{D}_f\tilde{\mathbf{D}},\\ &\mathbf{M}_{85} = \bar{\mathbf{H}}_{x1}^{\mathrm{T}}\mathbf{X}+\bar{\mathbf{H}}_{y}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}, && \mathbf{M}_{86} = \bar{\mathbf{H}}_{z}^{\mathrm{T}}-\bar{\mathbf{H}}_{y}^{\mathrm{T}}\mathbf{D}_f^{\mathrm{T}}, && \mathbf{M}_{88} = -\varepsilon^{-1}(\mathbf{I}_{d_1}-\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Theta}).\end{aligned} \tag{42}$$

When the above inequalities hold, the filter $\Sigma_\mathfrak{f}$ in (22) with filter gains (32) or (36) is a solution to the considered robust filtering problem.

Proof. It is enough to establish the equivalence of (30) and (41) with an $\varepsilon^{-1} > 0$. By (38), (30) may be rewritten as

$$\tilde{\mathbf{\Omega}} + \tilde{\mathbf{M}}\mathbf{\Pi}\tilde{\mathbf{N}} + \tilde{\mathbf{N}}^{\mathrm{T}}\mathbf{\Pi}^{\mathrm{T}}\tilde{\mathbf{M}}^{\mathrm{T}} < \mathbf{0}, \tag{43}$$

with $\mathbf{\Pi}^{\mathrm{T}}\mathbf{\Pi} \le (\mathbf{I}_{d_1} - \mathbf{\Theta}^{\mathrm{T}}\mathbf{\Theta})^{-1}$, where

$$\tilde{\mathbf{\Omega}} = \begin{bmatrix} -\mathbf{\Phi} & * & * & * & * & * \\ -\mathbf{\Phi} & -\mathbf{X} & * & * & * & * \\ \mathbf{0} & \mathbf{0} & -\mu_e^2\mathbf{I}_m & * & * & * \\ \mathbf{\Phi}\tilde{\mathbf{A}} & \mathbf{\Phi}\tilde{\mathbf{A}} & \mathbf{\Phi}\tilde{\mathbf{B}} & -\mathbf{\Phi} & * & * \\ \mathbf{X}\tilde{\mathbf{A}}+\mathbf{Z}\tilde{\mathbf{C}}+\mathbf{W} & \mathbf{X}\tilde{\mathbf{A}}+\mathbf{Z}\tilde{\mathbf{C}} & \mathbf{X}\tilde{\mathbf{B}}+\mathbf{Z}\tilde{\mathbf{D}} & -\mathbf{\Phi} & -\mathbf{X} & * \\ \tilde{\mathbf{L}}-\mathbf{D}_f\tilde{\mathbf{C}}-\mathbf{Y} & \tilde{\mathbf{L}}-\mathbf{D}_f\tilde{\mathbf{C}} & \tilde{\mathbf{J}}-\mathbf{D}_f\tilde{\mathbf{D}} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_q \end{bmatrix}, \tag{44}$$

$$\tilde{\mathbf{M}}^{\mathrm{T}} = \begin{bmatrix} \tilde{\mathbf{E}}_{x1} & \tilde{\mathbf{E}}_{x1} & \tilde{\mathbf{E}}_u & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{45}$$

$$\tilde{\mathbf{N}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{H}}_{x1}^{\mathrm{T}}\mathbf{\Phi} & \bar{\mathbf{H}}_{x1}^{\mathrm{T}}\mathbf{X}+\bar{\mathbf{H}}_y^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}} & \bar{\mathbf{H}}_z^{\mathrm{T}}-\bar{\mathbf{H}}_y^{\mathrm{T}}\mathbf{D}_f^{\mathrm{T}} \end{bmatrix}. \tag{46}$$

By Lemma 6 and the Schur complement, it is seen that (43) is equivalent to (41) with an $\varepsilon^{-1} > 0$. □

Remark 1. Based on Theorem 2, the following convex optimization problem may be formulated with respect to a chosen pair $\{\mathbf{P}, \mathbf{Q}\}$ in (10) to find the H_∞ optimal filter of the form (22) such that (26) is satisfied with the minimal μ_e:

$$\min_{\mu_e^2,\ \varepsilon^{-1}, \mathbf{\Phi},\ \mathbf{W},\ \mathbf{X},\ \mathbf{Y},\ \mathbf{Z},\ \mathbf{D}_f} \mu_e^2, \tag{47}$$

subject to the LMIs (31), (41), $\varepsilon^{-1} > 0$, and $\mu_e^2 > 0$.

4. A numerical example

In this section, an example is worked out to illustrate the proposed filter design method. Suppose that the system matrices of the system Σ in (6) are as follows:

$$\mathbf{E} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0.1530 & 0.0450 & 0.0690 \\ 0.1560 & 0.2520 & 0.1560 \\ 0.1350 & -0.1710 & -0.6360 \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ 0.2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.1 & 0 & 0.5 \end{bmatrix}, \quad \mathbf{D} = -0.5, \tag{48}$$
$$\mathbf{L} = \begin{bmatrix} -1 & 0.3 & -0.5 \end{bmatrix}, \quad \mathbf{J} = 0.$$

The uncertainty matrices in (7) are assumed to be

$$\mathbf{H}_x^{\mathrm{T}} = \begin{bmatrix} 1.5 & 3 & 1.5 \end{bmatrix}, \qquad \mathbf{H}_y = -1, \qquad \mathbf{H}_z = 2,$$
$$\mathbf{E}_x = \begin{bmatrix} 0.05 & 0 & 0.1 \end{bmatrix}, \qquad \mathbf{E}_u = 1, \tag{49}$$

and $|\Delta| \leq 1$. The prescribed H_∞-norm bound μ_e in (26) is 2. It is easy to verify that $(\mathbf{E}, \mathbf{A} + \mathbf{H}_x \Delta \mathbf{E}_x)$ is an admissible pair, and rank $\mathbf{E} = 2$. By applying singular value decomposition to $\mathbf{E}$, one may choose

$$\mathbf{P} = \begin{bmatrix} 0.2283 & 0.2045 & 0.0238 \\ 0.2850 & -0.3977 & 0.6827 \\ -0.5774 & 0.5774 & 0.5774 \end{bmatrix},$$
$$\mathbf{Q} = \begin{bmatrix} 0.2521 & 0.9677 & 0 \\ 0.8655 & -0.2255 & 0.4472 \\ 0.4328 & -0.1128 & -0.8944 \end{bmatrix}. \tag{50}$$

Because $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\| = 0.5291 < 1$, the assumption of Theorem 2 is satisfied. The filter Σ_{f} in (22) is designed by solving the LMIs of Theorem 2, and the filter gains (36) are found to be

$$\mathbf{A}_f = \begin{bmatrix} 0.0236 & -0.0619 \\ -0.1601 & 0.2558 \end{bmatrix}, \qquad \mathbf{B}_f = \begin{bmatrix} -0.7226 \\ 1.1036 \end{bmatrix},$$
$$\mathbf{C}_f = \begin{bmatrix} -0.0535 & 0.9527 \end{bmatrix}, \qquad \mathbf{D}_f = -0.9518, \tag{51}$$

which is a second-order normal stable filter as desired. With respect to the chosen $\{\mathbf{P}, \mathbf{Q}\}$ in (50), the corresponding H_∞ optimal filter is also designed by solving the convex optimization problem mentioned in Remark 1, which is implemented by the *MATLAB LMI Control Toolbox* [5]. The resulting optimal μ_e is 1.1761, and the filter gains (36) are found to be

$$\mathbf{A}_f = \begin{bmatrix} 0.0251 & -0.0740 \\ -0.1564 & 0.2626 \end{bmatrix}, \qquad \mathbf{B}_f = \begin{bmatrix} -0.6156 \\ 1.0481 \end{bmatrix},$$
$$\mathbf{C}_f = \begin{bmatrix} -0.0667 & 1.0134 \end{bmatrix}, \qquad \mathbf{D}_f = -0.8753. \tag{52}$$

Of course, for the $\mathbf{E}$ in (48) there are other choices of $\{\mathbf{P}, \mathbf{Q}\}$ capable of making $\mathbf{PEQ} = \mathrm{diag}(\mathbf{I}_2, 0)$, and an example is

$$\mathbf{P} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \qquad \mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 1 \\ 0 & 0 & -2 \end{bmatrix}. \tag{53}$$

Corresponding to this choice, the concerned norm $\|\mathbf{E}_{x2}\bar{\mathbf{H}}_{x2}\|$ is still $0.5291 < 1$. When Theorem 2 is applied again with $\mu_e = 2$, the corresponding filter gains (36) are found to be

$$\mathbf{A}_f = \begin{bmatrix} 0.1874 & -0.0883 \\ -0.2544 & 0.0968 \end{bmatrix}, \qquad \mathbf{B}_f = \begin{bmatrix} 0.8676 \\ -1.9944 \end{bmatrix},$$
$$\mathbf{C}_f = \begin{bmatrix} 0.9448 & -0.1419 \end{bmatrix}, \qquad \mathbf{D}_f = -0.9202, \tag{54}$$

which are clearly different from those in (51). Similarly, resolving the convex optimization problem mentioned in Remark 1 gives a different optimal $\mu_e = 1.8531$ from the previous one.

5. Conclusion

The H_∞ filter design problem has been considered for discrete-time singular systems with norm-bounded uncertainties. The algebraic equations in the singular system model are eliminated, and a normal dynamic system model is constructed with uncertainties in the linear fractional transformation form. For the H_∞ filter design problem, the normal system model allows one to utilize many existing methods to design normal filters directly, but the question of how to utilize the degrees of freedom in the choices of normal system models is worthy of further investigations. In this paper, a set of necessary and sufficient conditions is provided in terms of LMIs for a normal filter design.

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