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Delay-dependent robust H_∞ filtering for uncertain 2-D state-delayed systems

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Abstract

For uncertain 2-D state-delayed systems in the Fornasini–Marchesini second model, this paper discusses the robust H_∞ filtering problem. Both filter analysis and synthesis problems are considered. Firstly, a stability condition and an H_∞-norm performance condition are derived. Then a set of delay-dependent sufficient conditions for the existence of desired robust H_∞ filters is expressed in terms of linear matrix inequalities. A numerical example is given in the last to show the application of the proposed filter design method.

Keywords: Robust H_∞ filter; 2-D systems; Time-delay systems; Delay-dependence; Linear matrix inequality (LMI)

1. Introduction

The filtering problem is one of the fundamental problems in various system engineering applications, especially in fields of signal processing and automatic control. In these applications, it is usually necessary to estimate the state variables from the system measurement data. One of the most popular filter design approaches is the H_∞ filtering method, which can guarantee a prescribed noise attenuation over the entire frequency range for the estimation error. In many practical physical systems, however, parameter uncertainties may appear in system models. To handle problems with modeling uncertainties, the robust H_∞ filtering methods for one-dimensional (1-D) systems have been proposed in the literature [1–3].

Recently, the discrete two-dimensional (2-D) systems, which are physical systems with dynamics depending on two independent integer variables *i* and *j*, have attracted increasing attentions due to its theoretical as well as application importance in the fields such as multi-dimensional digital filtering, linear image processing, signal processing, process control, and so on [4–6]. In recent years, the linear matrix inequality (LMI)-based methods [7,8] for 2-D systems have been widely adopted and many results have been obtained [4,9–12]. Among these results, the H_∞ filter is proposed in [4,9], stabilization and H_∞ control problems are discussed in [4,10,12], and the mixed H_2/H_∞ filtering for 2-D systems with polytopic uncertainties is reported in [11]. It is worth noting that most of the researches

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regarding this topic only deal with 2-D systems without delays.

In practical 2-D systems, there are many examples containing inherent delays, such as discretization time in discrete models describing delayed lattice differential equation [13] and partial difference equations [14,15]. The delay effects are often adverse and need to be treated properly. For 1-D state-delayed systems, there have been many works on various problems. See, e.g., [16–21], and the references cited therein. Recently, research results about the stability and control problems of uncertain 2-D discrete state-delayed systems is reported in [22], and results about the mixed H_2/H_∞ filter design problem by using a parameter-dependent Lyapunov function approach is first proposed in [14]. These works are, however, based on the delay-independent approach. In general, the delay-dependent results are less conservative than the delay-independent counterparts, especially when it is known beforehand that the delays involved are small. Therefore, it is natural to try to derive similar results on the same problems of 2-D systems with state delays.

In this paper, a delay-dependent approach to robust H_∞ filtering will be proposed for polytopic 2-D state-delayed systems described by the Fornasini–Marchesini second model. In the considered systems, it is assumed that delays appear in both the horizontal and vertical directions. The purpose of the problem under investigation is to design a 2-D filter such that, for all admissible uncertainties, the filtering error dynamics is asymptotically stable, and a prescribed H_∞-norm performance level is achieved, within specific delay ranges in both horizontal and vertical directions. Effective methods to solve the robust H_∞ filtering problem by using a parameter-dependent Lyapunov function [23,24] will be derived. Different from the quadratic stability framework [21], the use of a parameter-dependent Lyapunov function allows different Lyapunov matrices to be set for different parts of the entire polytope domain, and produces less conservative design results.

The notation used throughout the paper is quite standard. $\mathbb{Z}$ is the set of nonnegative integers, $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. Π^{T} stands for the transpose of a matrix Π, and $P>0$ (<0) means that the symmetric matrix P is positive (negative) definite. The boldface characters represent matrix variables, and $\otimes$ is the Kronecker product. In symmetric block matrices, $\star$ is used as an ellipsis for the terms that are implied by symmetry, and $\mathrm{diag}\{\cdots\}$ for block-diagonal matrices. The ℓ_2 norm of a 2-D signal $w(i,j)$ is defined and denoted by $\|w\|_2=[\sum_{i,j=0}^{\infty}\|w(i,j)\|^2]^{1/2}$, where $\|\cdot\|$ is the Euclidean vector norm. A 2-D signal $w\in\ell_2$ if it has a bounded ℓ_2 norm. Finally, for any given $\mathbf{M}>0$ $\|w(i,j)\|^2_{\mathbf{M}}$ means $w(i,j)^{\mathrm{T}}\mathbf{M}w(i,j)$.

2. Problem formulation

Consider the 2-D state-delayed polytopic system described by the Fornasini–Marchesini second model [5,25]

$$\begin{aligned} x(i+1,j+1) &= A(\alpha)\begin{bmatrix} x(i+1,j) \\ x(i,j+1)\end{bmatrix} \\ &\quad + A_d(\alpha)\begin{bmatrix} x(i+1,j-d_1) \\ x(i-d_2,j+1)\end{bmatrix} \\ &\quad + B(\alpha)\begin{bmatrix} w(i+1,j) \\ w(i,j+1)\end{bmatrix}, \end{aligned}$$

$$\begin{aligned} y(i,j) &= C(\alpha)x(i,j)+D(\alpha)w(i,j), \\ z(i,j) &= L(\alpha)x(i,j), \end{aligned} \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector, $w\in\mathbb{R}^m$ is the disturbance input vector, $y\in\mathbb{R}^p$ is the measured output vector, $z\in\mathbb{R}^q$ is the signal vector to be estimated, and d_1 and d_2 are positive integers denoting delays along vertical and horizontal directions, respectively. The matrices

$$\begin{aligned} A(\alpha) &= [A_1(\alpha)A_2(\alpha)], \\ A_d(\alpha) &= [A_{d1}(\alpha)A_{d2}(\alpha)], \\ B(\alpha) &= [B_1(\alpha)B_2(\alpha)], \end{aligned} \tag{2}$$

$C(\alpha)$, $D(\alpha)$ and $L(\alpha)$ are assumed to be constant and unknown (uncertain), but belonging to a convex compact set of polytopic type, namely

$$\begin{bmatrix} A_1(\alpha) & B_1(\alpha) & A_{d1}(\alpha) \\ A_2(\alpha) & B_2(\alpha) & A_{d2}(\alpha) \\ C(\alpha) & D(\alpha) & 0 \\ L(\alpha) & 0 & 0 \end{bmatrix} = \sum_{j=1}^{\tau}\alpha_j \begin{bmatrix} A_1^{(j)} & B_1^{(j)} & A_{d1}^{(j)} \\ A_2^{(j)} & B_2^{(j)} & A_{d2}^{(j)} \\ C^{(j)} & D^{(j)} & 0 \\ L^{(j)} & 0 & 0 \end{bmatrix}, \tag{3}$$

where $\alpha=[\alpha_1\cdots\alpha_\tau]^{\mathrm{T}}$ is unknown in the unit simplex

$$\left\{[\alpha_1\cdots\alpha_\tau]^{\mathrm{T}} : \sum_{j=1}^{\tau}\alpha_j=1, \alpha_j\geqslant 0\right\}. \tag{4}$$

The boundary conditions are defined by

$$\{x(i,j) = s_{ij},\ \forall\ i \in \mathbb{Z} \text{ and } j = -d_1, -d_1+1, \ldots, 0\},$$
$$\{x(i,j) = t_{ij},\ \forall\ j \in \mathbb{Z} \text{ and } i = -d_2, -d_2+1, \ldots, 0\},$$
$$s_{00} = t_{00}, \tag{5}$$

where s_{ij} and t_{ij} are given vectors.

In this paper, the basic objective is to find a filter of the form

$$\begin{aligned} x_f(i+1,j+1) &= \mathbf{A_{f1}} x_f(i+1,j) + \mathbf{A_{f2}} x_f(i,j+1) \\ &\quad + \mathbf{B_{f1}} y(i+1,j) + \mathbf{B_{f2}} y(i,j+1), \end{aligned}$$

$$z_f(i,j) = \mathbf{L_f} x_f(i,j) \tag{6}$$

to estimate the signal z from the measurement history of y, where $x_f(i,j) \in \mathbb{R}^n$ is the state vector of the filter, $z_f(i,j) \in \mathbb{R}^q$ is the estimation of $z(i,j)$, and $\mathbf{A_{f1}}$, $\mathbf{A_{f2}}$, $\mathbf{B_{f1}}$, $\mathbf{B_{f2}}$ and $\mathbf{L_f}$ are filter parameter matrices to be determined. Define the augmented state vector $\hat{x}(i,j) = [x^{\mathrm{T}}(i,j)\ x_f^{\mathrm{T}}(i,j)]^{\mathrm{T}}$ and the filtering error output signal $\hat{z}(i,j) = z(i,j) - z_f(i,j)$. Then the error dynamics equations are

$$\begin{aligned} \hat{x}(i+1,j+1) &= \hat{A}(\alpha) \begin{bmatrix} \hat{x}(i+1,j) \\ \hat{x}(i,j+1) \end{bmatrix} \\ &\quad + \hat{A}_d(\alpha)\tilde{J} \begin{bmatrix} \hat{x}(i+1,j-d_1) \\ \hat{x}(i-d_2,j+1) \end{bmatrix} \\ &\quad + \hat{B}(\alpha) \begin{bmatrix} w(i+1,j) \\ w(i,j+1) \end{bmatrix}, \end{aligned}$$

$$\hat{z}(i,j) = \hat{L}(\alpha)\hat{x}(i,j), \tag{7}$$

where

$$\hat{A}(\alpha) = \begin{bmatrix} A_1(\alpha) & 0 & A_2(\alpha) & 0 \\ \mathbf{B_{f1}}C(\alpha) & \mathbf{A_{f1}} & \mathbf{B_{f2}}C(\alpha) & \mathbf{A_{f2}} \end{bmatrix},$$

$$\hat{A}_d(\alpha) = \begin{bmatrix} A_{d1}(\alpha) & A_{d2}(\alpha) \\ 0 & 0 \end{bmatrix},$$

$$\hat{B}(\alpha) = \begin{bmatrix} B_1(\alpha) & B_2(\alpha) \\ \mathbf{B_{f1}}D(\alpha) & \mathbf{B_{f2}}D(\alpha) \end{bmatrix}, \quad \hat{L}(\alpha) = [L(\alpha)\ \ -\mathbf{L_f}],$$

$$\tilde{J} = [J_1^{\mathrm{T}}\ J_2^{\mathrm{T}}]^{\mathrm{T}}, \quad J_1 = [J\ \ 0], \quad J_2 = [0\ \ J],$$
$$J = [I_n\ \ 0]. \tag{8}$$

The boundary conditions of the error dynamics equations are defined by

$$\{\hat{x}(i,j) = \hat{s}_{ij},\ \forall\ i \in \mathbb{Z} \text{ and } j = -d_1, -d_1+1, \ldots, 0\},$$
$$\{\hat{x}(i,j) = \hat{t}_{ij},\ \forall\ j \in \mathbb{Z} \text{ and } i = -d_2, -d_2+1, \ldots, 0\},$$
$$\hat{s}_{00} = \hat{t}_{00}, \tag{9}$$

where $\hat{s}_{ij}$ and $\hat{t}_{ij}$ are vectors determined by (5) and boundary conditions set in the filter (6).

Throughout this paper, the following definitions apply.

Definition 1. The 2-D state-delayed system (7) is asymptotically stable if $\lim_{r\to\infty}\hat{\chi}_r = 0$ for $w = 0$ and all bounded boundary conditions in (9), where

$$\hat{\chi}_r = \sup\{\|\hat{x}(i,j)\| : i+j = r, i,j \geq 1\}. \tag{10}$$

Definition 2. The H_∞-norm of the 2-D state-delayed system (7) is defined as

$$\sup_w \left\{ \frac{\|\hat{z}\|_2}{\|w\|_2} :\ w \in \ell_2, \|w\|_2 \neq 0, \hat{s}_{ij} = \hat{t}_{ij} = 0\ \forall\ i,j \text{ in } (9) \right\}. \tag{11}$$

By the above definition, the H_∞-norm of the 2-D delay system (7) is less than or equal to γ if and only if the H_∞-norm constraint $\|\hat{z}\|^2 \leq \gamma^2\|w\|^2$ is satisfied for all $w \in \ell_2$, and $\hat{s}_{ij} = \hat{t}_{ij} = 0$ with i, j in (9).

The robust H_∞ filtering problem addressed in this paper is as follows. Given positive integers $\bar{d}_1$, $\bar{d}_2$ and a real number $\gamma > 0$, find a filter (6) such that the filtering error dynamics (7) is asymptotically stable and satisfies the H_∞-norm constraint for all admissible uncertainties and delay $d_i \in [0, \bar{d}_i]$, $i = 1, 2$.

3. Stability and H_∞-norm performance analysis

In this section, the stability and H_∞-norm performance analysis for 2-D state-delayed systems will be carried out. First, a delay-dependent sufficient condition for the asymptotic stability of 2-D state-delayed systems is presented.

3.1. Stability analysis

Consider the 2-D state-delayed system described by

$$\begin{aligned} x(i+1,j+1) &= A \begin{bmatrix} x(i+1,j) \\ x(i,j+1) \end{bmatrix} \\ &\quad + A_d \begin{bmatrix} x(i+1,j-d_1) \\ x(i-d_2,j+1) \end{bmatrix}, \end{aligned} \tag{12}$$

where $A = [A_1\ \ A_2] \in \mathbb{R}^{n\times 2n}$, $A_d = [A_{d1}\ \ A_{d2}] \in \mathbb{R}^{n\times 2n}$ are system matrices, $x \in \mathbb{R}^n$ is the state vector, and d_1, d_2 are positive integers denoting delay sizes along vertical and horizontal directions, respectively. In the following theorem, a sufficient delay-dependent condition will be given that guarantees the asymptotic stability of system (12).

Theorem 3. *Given positive integers* $\bar{d}_1$ *and* $\bar{d}_2$, *if there exist matrices* $\mathbf{P} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{Q} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{M}_l \in \mathbb{R}^{n\times n}$, $\mathbf{F}_l \in \mathbb{R}^{2n\times 2n}$ *and* $\mathbf{H}_l \in \mathbb{R}^{2n\times n}$, $l = 1, 2$, *such that*

$$\begin{bmatrix} -\mathbf{\Phi} & \star & \star \\ -\mathbf{H}^{\mathrm{T}} & -\mathbf{Q} & \star \\ \mathbf{\Lambda}\hat{A} & \mathbf{\Lambda}\hat{A}_d & -\mathbf{\Lambda} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \mathbf{F}_l & \star \\ \mathbf{H}_l^{\mathrm{T}} & \mathbf{M}_l \end{bmatrix} > 0, \quad l = 1, 2, \tag{14}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \star \\ \mathbf{P}_3^{\mathrm{T}} & \mathbf{P}_2 \end{bmatrix} > 0, \quad \mathbf{P}_3 = \mathbf{P}_3^{\mathrm{T}} \geqslant 0, \tag{15}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \star \\ \mathbf{Q}_3^{\mathrm{T}} & \mathbf{Q}_2 \end{bmatrix} > 0, \quad \mathbf{Q}_3 = \mathbf{Q}_3^{\mathrm{T}} \geqslant 0, \tag{16}$$

where

$$\begin{aligned} &\mathbf{\Phi} = \mathbf{P} - \bar{d}_1\mathbf{F}_1 - \bar{d}_2\mathbf{F}_2 - \mathbf{H} - \mathbf{H}^{\mathrm{T}}, \\ &\mathbf{\Lambda} = \mathrm{diag}\{\mathbf{R}, \bar{d}_1\mathbf{M}_1, \bar{d}_2\mathbf{M}_2\}, \\ &\mathbf{R} = \mathbf{P}_1 + 2\mathbf{P}_3 + \mathbf{P}_2 + \mathbf{Q}_1 + 2\mathbf{Q}_3 + \mathbf{Q}_2, \\ &\hat{A} = \begin{bmatrix} A \\ \begin{bmatrix} A \\ A \end{bmatrix} - I_{2n} \end{bmatrix}, \quad \hat{A}_d = \begin{bmatrix} A_d \\ A_d \\ A_d \end{bmatrix}, \\ &\mathbf{H} = [\mathbf{H}_1 \ \ \mathbf{H}_2], \end{aligned} \tag{17}$$

then system (12) *is asymptotically stable for any delay* $d_i \in [0, \ \bar{d}_i]$, $i = 1, 2$.

Proof. For notational simplicity, let

$$\tilde{Z} = \begin{bmatrix} z_2 I_n \\ z_1 I_n \end{bmatrix}, \quad \tilde{Z}_d = \begin{bmatrix} z_2^{d_1+1} I_n \\ z_1^{d_2+1} I_n \end{bmatrix}. \tag{18}$$

Suppose the condition (13)–(16) is satisfied but (12) is unstable. Then by [5,14]

$$\det(I_n - z_2A_1 - z_1A_2 - z_2^{d_1+1}A_{d1} - z_1^{d_2+1}A_{d2}) = 0 \tag{19}$$

for some $(z_1, z_2) \in \mathscr{U} = \{(z_1, z_2) | |z_1| \leqslant 1, |z_2| \leqslant 1\}$ and there exists a nonzero vector v such that

$$v = \tilde{A}\begin{bmatrix} \tilde{Z} \\ \tilde{Z}_d \end{bmatrix} v, \tag{20}$$

where $\tilde{A} = [A \ \ A_d]$. The inequalities in (14) imply that

$$v^* \begin{bmatrix} \sqrt{\bar{d}_1}\tilde{Z} \\ \frac{1}{\sqrt{\bar{d}_1}}(z_2 - z_2^{d_1+1})I_n \end{bmatrix}^* \begin{bmatrix} \mathbf{F}_1 & \star \\ \mathbf{H}_1^{\mathrm{T}} & \mathbf{M}_1 \end{bmatrix} \times \begin{bmatrix} \sqrt{\bar{d}_1}\tilde{Z} \\ \frac{1}{\sqrt{\bar{d}_1}}(z_2 - z_2^{d_1+1})I_n \end{bmatrix} v \geqslant 0, \tag{21}$$

$$v^* \begin{bmatrix} \sqrt{\bar{d}_2}\tilde{Z} \\ \frac{1}{\sqrt{\bar{d}_2}}(z_1 - z_1^{d_2+1})I_n \end{bmatrix}^* \begin{bmatrix} \mathbf{F}_2 & \star \\ \mathbf{H}_2^{\mathrm{T}} & \mathbf{M}_2 \end{bmatrix} \times \begin{bmatrix} \sqrt{\bar{d}_2}\tilde{Z} \\ \frac{1}{\sqrt{\bar{d}_2}}(z_1 - z_1^{d_2+1})I_n \end{bmatrix} v \geqslant 0, \tag{22}$$

where * denotes the complex conjugate transpose. From (21)–(22), one gets

$$v^*\left\{\bar{d}_1\tilde{Z}^*\mathbf{F}_1\tilde{Z} + \frac{1}{\bar{d}_1}(z_2 - z_2^{d_1+1})^*\mathbf{M}_1(z_2 - z_2^{d_1+1}) + 2\tilde{Z}^*\mathbf{H}_1(z_2 - z_2^{d_1+1})\right\}v \geqslant 0, \tag{23}$$

$$v^*\left\{\bar{d}_2\tilde{Z}^*\mathbf{F}_2\tilde{Z} + \frac{1}{\bar{d}_2}(z_1 - z_1^{d_2+1})^*\mathbf{M}_2(z_1 - z_1^{d_2+1}) + 2\tilde{Z}^*\mathbf{H}_2(z_1 - z_1^{d_2+1})\right\}v \geqslant 0. \tag{24}$$

Also, the inequality (13) implies that

$$v^*\begin{bmatrix} \tilde{Z} \\ \tilde{Z}_d \end{bmatrix}^* \left\{ \begin{bmatrix} -\mathbf{\Phi} & \star \\ -\mathbf{H}^{\mathrm{T}} & -\mathbf{Q} \end{bmatrix} + \tilde{A}^{\mathrm{T}}\mathbf{R}\tilde{A} + \bar{d}_1\tilde{A}_1^{\mathrm{T}}\mathbf{M}_1\tilde{A}_1 + \bar{d}_2\tilde{A}_2^{\mathrm{T}}\mathbf{M}_2\tilde{A}_2 \right\} \begin{bmatrix} \tilde{Z} \\ \tilde{Z}_d \end{bmatrix} v \leqslant 0, \tag{25}$$

where $\tilde{A}_1 = [A_1 - I_n \ \ A_2 \ \ A_{d1} \ \ A_{d2}]$ and $\tilde{A}_2 = [A_1 \ \ A_2 - I_n \ \ A_{d1} \ \ A_{d2}]$. From (25) and (12), the following inequality:

$$\begin{aligned} &v^*[-\tilde{Z}^*\mathbf{P}\tilde{Z} - \tilde{Z}_d^*\mathbf{Q}\tilde{Z}_d + \bar{d}_1\tilde{Z}^*\mathbf{F}_1\tilde{Z} + \bar{d}_2\tilde{Z}^*\mathbf{F}_2\tilde{Z} \\ &\quad + \tilde{Z}^*\mathbf{H}\tilde{Z} + \tilde{Z}^*\mathbf{H}^{\mathrm{T}}\tilde{Z} - 2\tilde{Z}^*\mathbf{H}\tilde{Z}_d]v \\ &\quad + v^*[\mathbf{R} + \bar{d}_1\mathbf{M}_1|(1 - z_2)|^2 \\ &\quad + \bar{d}_2\mathbf{M}_2|(1 - z_1)|^2]v \leqslant 0 \end{aligned} \tag{26}$$

can be deduced, which, together with (23)–(24), give

$$v^*\left[-\tilde{Z}^*\mathbf{P}\tilde{Z}-\tilde{Z}_d^*\mathbf{Q}\tilde{Z}_d-\frac{1}{\bar{d}_1}\mathbf{M}_1|(z_2-z_2^{d_1+1})|^2 -\frac{1}{\bar{d}_2}\mathbf{M}_2|(z_1-z_1^{d_2+1})|^2\right]v+v^*[\mathbf{R}+\bar{d}_1\mathbf{M}_1|(1-z_2)|^2 +\bar{d}_2\mathbf{M}_2|(1-z_1)|^2]v\leqslant 0. \quad (27)$$

It follows from (15) and (16) that

$$(z_1-z_2)^*\mathbf{P}_3(z_1-z_2)\geqslant 0,$$
$$(z_1^{d_2+1}-z_2^{d_1+1})^*\mathbf{Q}_3(z_1^{d_2+1}-z_2^{d_1+1})\geqslant 0, \quad (28)$$

which imply, respectively,

$$-z_2^*\mathbf{P}_3z_1-z_1^*\mathbf{P}_3z_2\geqslant -\mathbf{P}_3|z_2|^2-\mathbf{P}_3|z_1|^2,$$
$$-(z_2^{d_1+1})^*\mathbf{Q}_3(z_1^{d_2+1})-(z_1^{d_2+1})^*\mathbf{Q}_3(z_2^{d_1+1}) \geqslant -\mathbf{Q}_3|z_2^{d_1+1}|^2-\mathbf{Q}_3|z_1^{d_2+1}|^2, \quad (29)$$

and one has

$$-\tilde{Z}^*\mathbf{P}\tilde{Z}=-\mathbf{P}_3|z_2|^2-\mathbf{P}_3|z_1|^2-z_2^*\mathbf{P}_3z_1-z_1^*\mathbf{P}_3z_2 \geqslant -(\mathbf{P}_1+\mathbf{P}_3)|z_2|^2-(\mathbf{P}_2+\mathbf{P}_3)|z_1|^2,$$

$$-\tilde{Z}_d^*\mathbf{Q}\tilde{Z}_d=-\mathbf{Q}_1|z_2^{d_1+1}|^2-\mathbf{Q}_2|z_1^{d_2+1}|^2 -(z_2^{d_1+1})^*\mathbf{Q}_3(z_1^{d_2+1})-(z_1^{d_2+1})^*\mathbf{Q}_3(z_2^{d_1+1}) \geqslant -(\mathbf{Q}_1+\mathbf{Q}_3)|z_2^{d_1+1}|^2-(\mathbf{Q}_2+\mathbf{Q}_3)|z_1^{d_2+1}|^2. \quad (30)$$

Hence, (27) implies

$$v^*\Big\{(\mathbf{P}_1+\mathbf{P}_3)(1-|z_2|^2)+(\mathbf{P}_2+\mathbf{P}_3)(1-|z_1|^2) +(\mathbf{Q}_1+\mathbf{Q}_3)(1-|z_2^{d_1+1}|^2)+(\mathbf{Q}_2+\mathbf{Q}_3)(1-|z_1^{d_2+1}|^2) +\bar{d}_1\mathbf{M}_1\left[|(1-z_2)|^2-\frac{|z_2|^2}{\bar{d}_1^2}|(1-z_2^{d_1})|^2\right] +\bar{d}_2\mathbf{M}_2\left[|(1-z_1)|^2-\frac{|z_1|^2}{\bar{d}_2^2}|(1-z_1^{d_2})|^2\right]\Big\}v\leqslant 0. \quad (31)$$

Since $\mathbf{P}_3\geqslant 0$, $\mathbf{Q}_3\geqslant 0$, $\mathbf{P}_\kappa>0$, $\mathbf{Q}_\kappa>0$, $\mathbf{M}_\kappa>0$, $\kappa=1,2$, and $|z_2|^2/\bar{d}_1^2\leqslant 1$, $|z_1|^2/\bar{d}_2^2\leqslant 1$ for $(z_1,z_2)\in\mathscr{U}$, the left-hand side of (31) are positive except when $v=0$. This leads to a contradiction and system (12) must be asymptotically stable. This completes the proof of Theorem 3. □

Remark 4. Theorem 3 offers a new delay-dependent stability condition for 2-D state-delayed systems. By setting

$$\mathbf{F}_1=\frac{vI_{2n}}{\bar{d}_1},\quad \mathbf{M}_1=\frac{vI_n}{\bar{d}_1},$$
$$\mathbf{F}_2=\frac{vI_{2n}}{\bar{d}_2},\quad \mathbf{M}_2=\frac{vI_n}{\bar{d}_2},$$
$$\mathbf{H}=0,\quad \mathbf{P}_3=\mathbf{Q}_3=0, \quad (32)$$

in Theorem 3, the delay-dependent conditions (13)–(16) reduce to the delay-independent conditions in [14] when $v\geqslant 0$ approaches zero.

3.2. Robust H_∞-norm performance analysis

In the following theorem, a delay-dependent H_∞-norm performance criterion is established.

Theorem 5. *Given positive integers $\bar{d}_1$, $\bar{d}_2$ and a real number $\gamma>0$, system* (7) *is asymptotically stable and its H_∞-norm is less than or equal to γ if $d_i\in[0,\ \bar{d}_i]$, $i=1,2$, and there exist matrices $0<\mathbf{P}(\alpha)=\begin{bmatrix}\mathbf{P}_1(\alpha) & \star\\ \mathbf{P}_3^{\mathrm{T}}(\alpha) & \mathbf{P}_2(\alpha)\end{bmatrix}\in\mathbb{R}^{4n\times 4n}$, $\mathbf{Q}(\alpha)=\begin{bmatrix}\mathbf{Q}_1(\alpha) & \star\\ \mathbf{Q}_3^{\mathrm{T}}(\alpha) & \mathbf{Q}_2(\alpha)\end{bmatrix}\in\mathbb{R}^{2n\times 2n}$, $\mathbf{M}_l(\alpha)\in\mathbb{R}^{n\times n}$, $\mathbf{N}_l(\alpha)\in\mathbb{R}^{n\times n}$, $\mathbf{F}_l(\alpha)\in\mathbb{R}^{4n\times 4n}$, $\mathbf{G}_l(\alpha)\in\mathbb{R}^{2m\times n}$, $\mathbf{H}_l(\alpha)\in\mathbb{R}^{4n\times n}$ and $\mathbf{K}_l(\alpha)\in\mathbb{R}^{2m\times 2m}$, $l=1,2$, such that*

$$\mathbf{Q}_3(\alpha)=\mathbf{Q}_3^{\mathrm{T}}(\alpha)\geqslant 0, \quad (33)$$

$$\begin{bmatrix}\mathbf{K}_l(\alpha) & \star\\ \mathbf{G}_l^{\mathrm{T}}(\alpha) & \mathbf{N}_l(\alpha)\end{bmatrix}>0,\quad l=1,2, \quad (34)$$

$$\mathbf{P}_3(\alpha)=\mathbf{P}_3^{\mathrm{T}}(\alpha)\geqslant 0, \quad (35)$$

$$\begin{bmatrix}\mathbf{F}_l(\alpha) & \star\\ \mathbf{H}_l^{\mathrm{T}}(\alpha) & \mathbf{M}_l(\alpha)\end{bmatrix}>0,\quad l=1,2, \quad (36)$$

$$\begin{bmatrix}-\mathbf{\Phi}(\alpha) & \star & \star & \star & \star\\ -\mathbf{H}^{\mathrm{T}}(\alpha) & -\mathbf{Q}(\alpha) & \star & \star & \star\\ \mathbf{G}(\alpha)\tilde{J} & -\mathbf{G}(\alpha) & -\mathbf{\Psi}(\alpha) & \star & \star\\ \mathscr{A}(\alpha) & \mathscr{A}_d(\alpha) & \mathscr{B}(\alpha) & -\mathbf{\Omega}^{-1}(\alpha) & \star\\ \hat{L}(\alpha)\otimes I_2 & 0 & 0 & 0 & -\gamma I_{2q}\end{bmatrix}<0, \quad (37)$$

for all α in the unit simplex (4), *where*

$$\mathbf{\Phi}(\alpha)=\mathbf{P}(\alpha)-\bar{d}_1\mathbf{F}_1(\alpha)-\bar{d}_2\mathbf{F}_2(\alpha)-\mathbf{H}(\alpha)\tilde{J}-\tilde{J}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\alpha),$$
$$\mathbf{\Psi}(\alpha)=\gamma I_{2m}-\bar{d}_1\mathbf{K}_1(\alpha)-\bar{d}_2\mathbf{K}_2(\alpha),$$
$$\mathbf{\Omega}(\alpha)=\mathrm{diag}\{\mathbf{R}(\alpha),\bar{d}_1[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)], \bar{d}_2[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)]\},$$

$$\begin{aligned}\mathbf{R}(\alpha) &= \mathbf{P}_1(\alpha) + 2\mathbf{P}_3(\alpha) + \mathbf{P}_2(\alpha)\\ &\quad + J^{\mathrm{T}}[\mathbf{Q}_1(\alpha) + 2\mathbf{Q}_3(\alpha) + \mathbf{Q}_2(\alpha)]J,\\ \mathbf{H}(\alpha) &= [\mathbf{H}_1(\alpha)\ \ \mathbf{H}_2(\alpha)],\\ \mathbf{G}(\alpha) &= [\mathbf{G}_1(\alpha)\ \ \mathbf{G}_2(\alpha)],\\ \mathscr{A}(\alpha) &= \begin{bmatrix}\hat{A}(\alpha)\\ J\hat{A}(\alpha) - J_1\\ J\hat{A}(\alpha) - J_2\end{bmatrix}, \quad \mathscr{A}_d(\alpha) = \begin{bmatrix}\hat{A}_d(\alpha)\\ J\hat{A}_d(\alpha)\\ J\hat{A}_d(\alpha)\end{bmatrix},\\ \mathscr{B}(\alpha) &= \begin{bmatrix}\hat{B}(\alpha)\\ J\hat{B}(\alpha)\\ J\hat{B}(\alpha)\end{bmatrix}. \end{aligned} \tag{38}$$

Proof. First, the asymptotic stability of system (7) is established. For all α in the unit simplex (4), (37) implies (13). It follows from Theorem 3 that system (7) is asymptotically stable.

Next, the H_∞-norm performance criterion is considered. For convenience, the following definitions:

$$\tilde{x}(i,j) = \begin{bmatrix}\hat{x}(i+1,j)\\ \hat{x}(i,j+1)\end{bmatrix}, \quad \tilde{x}_d(i,j) = \begin{bmatrix}\hat{x}(i+1,j-d_1)\\ \hat{x}(i-d_2,j+1)\end{bmatrix},$$
$$\tilde{w}(i,j) = \begin{bmatrix}w^{\mathrm{T}}(i+1,j)\\ w^{\mathrm{T}}(i,j+1)\end{bmatrix},$$

and

$$\begin{aligned}\delta(i+1,t) &= x(i+1,t+1) - x(i+1,t)\\ &= (J\hat{A}(\alpha) - J_1)\tilde{x}(i,t)\\ &\quad + J\hat{A}_d(\alpha)\tilde{J}\tilde{x}_d(i,t) + J\hat{B}(\alpha)\tilde{w}(i,t),\end{aligned} \tag{39}$$

$$\begin{aligned}\delta(t,j+1) &= x(t+1,j+1) - x(t,j+1)\\ &= (J\hat{A}(\alpha) - J_2)\tilde{x}(t,j)\\ &\quad + J\hat{A}_d(\alpha)\tilde{J}\tilde{x}_d(t,j) + J\hat{B}(\alpha)\tilde{w}(t,j)\end{aligned} \tag{40}$$

are introduced. Then,

$$x(i+1,j) - x(i+1,j-d_1) = \sum_{t=j-d_1}^{j-1} \delta(i+1,t), \tag{41}$$

$$x(i,j+1) - x(i-d_2,j+1) = \sum_{t=i-d_2}^{i-1} \delta(t,j+1) \tag{42}$$

for $i \geqslant d_2$ and $j \geqslant d_1$. The inequalities (34) and (36) imply that

$$\sum_{t=j-d_1}^{j-1} \begin{bmatrix}\tilde{x}(i,j)\\ \delta(i+1,t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{F}_1(\alpha) & \star\\ \mathbf{H}_1^{\mathrm{T}}(\alpha) & \mathbf{M}_1(\alpha)\end{bmatrix} \begin{bmatrix}\tilde{x}(i,j)\\ \delta(i+1,t)\end{bmatrix} \geqslant 0, \tag{43}$$

$$\sum_{t=i-d_2}^{i-1} \begin{bmatrix}\tilde{x}(i,j)\\ \delta(t,j+1)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{F}_2(\alpha) & \star\\ \mathbf{H}_2^{\mathrm{T}}(\alpha) & \mathbf{M}_2(\alpha)\end{bmatrix} \begin{bmatrix}\tilde{x}(i,j)\\ \delta(t,j+1)\end{bmatrix} \geqslant 0, \tag{44}$$

$$\sum_{t=j-d_1}^{j-1} \begin{bmatrix}\tilde{w}(i,j)\\ \delta(i+1,t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{K}_1(\alpha) & \star\\ \mathbf{G}_1^{\mathrm{T}}(\alpha) & \mathbf{N}_1(\alpha)\end{bmatrix} \begin{bmatrix}\tilde{w}(i,j)\\ \delta(i+1,t)\end{bmatrix} \geqslant 0, \tag{45}$$

$$\sum_{t=i-d_2}^{i-1} \begin{bmatrix}\tilde{w}(i,j)\\ \delta(t,j+1)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{K}_2(\alpha) & \star\\ \mathbf{G}_2^{\mathrm{T}}(\alpha) & \mathbf{N}_2(\alpha)\end{bmatrix} \begin{bmatrix}\tilde{w}(i,j)\\ \delta(t,j+1)\end{bmatrix} \geqslant 0. \tag{46}$$

From (41)–(46), one has

$$\begin{aligned}&d_1\|\tilde{x}(i,j)\|^2_{\mathbf{F}_1(\alpha)} + \sum_{t=j-d_1}^{j-1} \|\delta(i+1,t)\|^2_{\mathbf{M}_1(\alpha)} + 2[x(i+1,j)\\ &\quad - x(i+1,j-d_1)]^{\mathrm{T}}\mathbf{H}_1^{\mathrm{T}}(\alpha)\tilde{x}(i,j) \geqslant 0,\end{aligned} \tag{47}$$

$$\begin{aligned}&d_2\|\tilde{x}(i,j)\|^2_{\mathbf{F}_2(\alpha)} + \sum_{t=i-d_2}^{i-1} \|\delta(t,j+1)\|^2_{\mathbf{M}_2(\alpha)} + 2[x(i,j+1)\\ &\quad - x(i-d_2,j+1)]^{\mathrm{T}}\mathbf{H}_2^{\mathrm{T}}(\alpha)\tilde{x}(i,j) \geqslant 0,\end{aligned} \tag{48}$$

$$\begin{aligned}&d_1\|\tilde{w}(i,j)\|^2_{\mathbf{K}_1(\alpha)} + \sum_{t=j-d_1}^{j-1} \|\delta(i+1,t)\|^2_{\mathbf{N}_1(\alpha)} + 2[x(i+1,j)\\ &\quad - x(i+1,j-d_1)]^{\mathrm{T}}\mathbf{G}_1^{\mathrm{T}}(\alpha)\tilde{w}(i,j) \geqslant 0,\end{aligned} \tag{49}$$

$$\begin{aligned}&d_2\|\tilde{w}(i,j)\|^2_{\mathbf{K}_2(\alpha)} + \sum_{t=i-d_2}^{i-1} \|\delta(t,j+1)\|^2_{\mathbf{N}_2(\alpha)} + 2[x(i,j+1)\\ &\quad - x(i-d_2,j+1)]^{\mathrm{T}}\mathbf{G}_2^{\mathrm{T}}(\alpha)\tilde{w}(i,j) \geqslant 0.\end{aligned} \tag{50}$$

By Schur's complement, (37) implies that

$$\begin{aligned}\xi^{\mathrm{T}}(i,j)\Bigg\{&\begin{bmatrix}-\mathbf{\Phi}(\alpha) & \star & \star\\ -\mathbf{H}^{\mathrm{T}}(\alpha) & -\mathbf{Q}(\alpha) & \star\\ \mathbf{G}(\alpha)\tilde{J} & -\mathbf{G}(\alpha) & -\mathbf{\Psi}(\alpha)\end{bmatrix}\\ &+ \begin{bmatrix}\mathscr{A}^{\mathrm{T}}(\alpha)\\ \mathscr{A}_d^{\mathrm{T}}(\alpha)\\ \mathscr{B}^{\mathrm{T}}(\alpha)\end{bmatrix} \mathbf{\Omega}(\alpha)[\mathscr{A}(\alpha)\ \ \mathscr{A}_d(\alpha)\ \ \mathscr{B}(\alpha)]\\ &+ \frac{1}{\gamma}\begin{bmatrix}\hat{L}^{\mathrm{T}}(\alpha)\otimes I_2\\ 0\\ 0\end{bmatrix}[\hat{L}(\alpha)\otimes I_2\ \ 0\ \ 0]\Bigg\}\xi(i,j) \leqslant 0,\end{aligned} \tag{51}$$

where $\xi^{\mathrm{T}}(i,j) = [\tilde{x}^{\mathrm{T}}(i,j)\ \ \tilde{J}\tilde{x}_d^{\mathrm{T}}(i,j)\ \ \tilde{w}^{\mathrm{T}}(i,j)]$. Also it follows from (7) and (39)–(40) that

$$\begin{aligned}&\|\hat{x}(i+1,j+1)\|^2_{\mathbf{R}(\alpha)} + d_1\|\delta(i+1,j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)}\\ &\quad + d_2\|\delta(i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} - \|\tilde{x}(i,j)\|^2_{\mathbf{P}(\alpha)}\end{aligned}$$

$$
\begin{aligned}
&+ d_1\|\tilde{x}(i,j)\|^2_{\mathbf{F}_1(\alpha)} + d_2\|\tilde{x}(i,j)\|^2_{\mathbf{F}_2(\alpha)} \\
&+ \|\tilde{x}(i,j)\|^2_{\mathbf{H}(\alpha)\tilde{J}} + \|\tilde{x}(i,j)\|^2_{\tilde{J}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\alpha)} - \|\tilde{J}\tilde{x}_d(i,j)\|^2_{\mathbf{Q}(\alpha)} \\
&- \tilde{x}^{\mathrm{T}}(i,j)\mathbf{H}(\alpha)\tilde{J}\tilde{x}_d(i,j) - \tilde{x}_d^{\mathrm{T}}(i,j)\tilde{J}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\alpha)\tilde{x}(i,j) \\
&+ \tilde{w}^{\mathrm{T}}(i,j)\mathbf{G}(\alpha)\tilde{J}\tilde{x}(i,j) + \tilde{x}^{\mathrm{T}}(i,j)\tilde{J}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}(\alpha)\tilde{w}(i,j) \\
&- \tilde{w}^{\mathrm{T}}(i,j)\mathbf{G}(\alpha)\tilde{J}\tilde{x}_d(i,j) - \tilde{x}_d^{\mathrm{T}}(i,j)\tilde{J}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}(\alpha)\tilde{w}(i,j) \\
&+ d_1\|\tilde{w}(i,j)\|^2_{\mathbf{K}_1(\alpha)} + d_2\|\tilde{w}(i,j)\|^2_{\mathbf{K}_2(\alpha)} \\
&+ \gamma^{-1}\left\|\begin{bmatrix}\hat{z}(i+1,j)\\ \hat{z}(i,j+1)\end{bmatrix}\right\|^2 - \gamma\left\|\begin{bmatrix}w(i+1,j)\\ w(i,j+1)\end{bmatrix}\right\|^2 \leqslant 0,
\end{aligned} \tag{52}
$$

which together with (47)–(50) imply

$$
\begin{aligned}
&\|\hat{x}(i+1,j+1)\|^2_{\mathbf{R}(\alpha)} + d_1\|\delta(i+1,j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&+ d_2\|\delta(i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} - \|\tilde{x}(i,j)\|^2_{\mathbf{P}(\alpha)} \\
&- \|\tilde{J}\tilde{x}_d(i,j)\|^2_{\mathbf{Q}(\alpha)} - \sum_{t=j-d_1}^{j-1}\|\delta(i+1,t)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&- \sum_{t=i-d_2}^{i-1}\|\delta(t,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} \\
&+ \gamma^{-1}\left\|\begin{bmatrix}\hat{z}(i+1,j)\\ \hat{z}(i,j+1)\end{bmatrix}\right\|^2 - \gamma\left\|\begin{bmatrix}w(i+1,j)\\ w(i,j+1)\end{bmatrix}\right\|^2 \leqslant 0.
\end{aligned} \tag{53}
$$

Since for all $\mathbf{P}_3(\alpha)\geqslant 0$ and $\mathbf{Q}_3(\alpha)\geqslant 0$,

$$
\begin{aligned}
-\|\tilde{x}(i,j)\|^2_{\mathbf{P}(\alpha)} \geqslant &- \|\hat{x}(i+1,j)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)} \\
&- \|\hat{x}(i,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)},
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
-\|\tilde{J}\tilde{x}_d(i,j)\|^2_{\mathbf{Q}(\alpha)} \geqslant &- \|x(i+1,j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)} \\
&- \|x(i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)}.
\end{aligned} \tag{55}
$$

Eq. (53) implies

$$
\begin{aligned}
&\|\hat{x}(i+1,j+1)\|^2_{\mathbf{R}(\alpha)} - \|\hat{x}(i+1,j)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)} \\
&- \|\hat{x}(i,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)} \\
&- \|x(i+1,j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)} \\
&- \|x(i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)} \\
&+ d_1\|\delta(i+1,j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&+ d_2\|\delta(i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} \\
&- \sum_{t=j-d_1}^{j-1}\|\delta(i+1,t)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&- \sum_{t=i-d_2}^{i-1}\|\delta(t,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} \\
&+ \gamma^{-1}\left\|\begin{bmatrix}\hat{z}(i+1,j)\\ \hat{z}(i,j+1)\end{bmatrix}\right\|^2 - \gamma\left\|\begin{bmatrix}w(i+1,j)\\ w(i,j+1)\end{bmatrix}\right\|^2 \leqslant 0.
\end{aligned} \tag{56}
$$

Thus, for any positive integers λ_1 and λ_2,

$$
\begin{aligned}
\sum_{i=0}^{\lambda_2-1}\sum_{j=0}^{\lambda_1-1}\Bigg\{ &\|\hat{x}(i+1,j+1)\|^2_{\mathbf{R}(\alpha)} \\
&- \|\hat{x}(i+1,j)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)} - \|\hat{x}(i,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)} \\
&- \|x(i+1,j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)} \\
&- \|x(i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)} \\
&+ d_1\|\delta(i+1,j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&+ d_2\|\delta(i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} \\
&- \sum_{t=j-d_1}^{j-1}\|\delta(i+1,t)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&- \sum_{t=i-d_2}^{i-1}\|\delta(t,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)} \\
&+\gamma^{-1}\left\|\begin{bmatrix}\hat{z}(i+1,j)\\ \hat{z}(i,j+1)\end{bmatrix}\right\|^2 - \gamma\left\|\begin{bmatrix}w(i+1,j)\\ w(i,j+1)\end{bmatrix}\right\|^2 \Bigg\} \leqslant 0,
\end{aligned} \tag{57}
$$

and

$$
\begin{aligned}
\sum_{i=0}^{\lambda_2-1}\Bigg\{ &\|\hat{x}^{\mathrm{T}}(i+1,\lambda_1)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)} \\
&+ \sum_{j=0}^{d_1}\|x(i+1,\lambda_1+j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)} \\
&+ \sum_{j=0}^{d_1-1}(d_1-j)\|\delta(i+1,\lambda_1-1-j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \\
&- \|\hat{x}^{\mathrm{T}}(i+1,0)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)} \\
&- \sum_{j=0}^{d_1}\|x(i+1,j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)} \\
&- \sum_{j=0}^{d_1-1}(d_1-j)\|\delta(i+1,-1-j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)} \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=0}^{\lambda_1-1}\left\{\|\hat{x}^{\mathrm{T}}(\lambda_2,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)}\right.\\
&+\sum_{i=0}^{d_2}\|x(\lambda_2+i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)}\\
&+\sum_{i=0}^{d_2-1}(d_2-i)\|\delta(\lambda_2-1-i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)}\\
&-\|\hat{x}^{\mathrm{T}}(0,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)}\\
&-\sum_{i=0}^{d_2}\|x(i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)}\\
&\left.-\sum_{i=0}^{d_2-1}(d_2-i)\|\delta(-1-i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)}\right\}\\
&+\gamma^{-1}\left\{\sum_{i=0}^{\lambda_2}\sum_{j=0}^{\lambda_1-1}\|\hat{z}(i,j)\|^2-\sum_{j=0}^{\lambda_1-1}\|\hat{z}(0,j)\|^2\right.\\
&\left.+\sum_{i=0}^{\lambda_2-1}\sum_{j=0}^{\lambda_1}\|\hat{z}(i,j)\|^2-\sum_{i=0}^{\lambda_2-1}\|\hat{z}(i,0)\|^2\right\}\\
&\leqslant\gamma\left\{\sum_{i=0}^{\lambda_2}\sum_{j=0}^{\lambda_1-1}\|w(i,j)\|^2-\sum_{j=0}^{\lambda_1-1}\|w(0,j)\|^2\right.\\
&\qquad\left.+\sum_{i=0}^{\lambda_2-1}\sum_{j=0}^{\lambda_1}\|w(i,j)\|^2-\sum_{i=0}^{\lambda_2-1}\|w(i,0)\|^2\right\}.
\end{aligned}\tag{58}
$$

When the boundary condition is $\hat{s}_{ij}=\hat{t}_{ij}=0$ with i,j in (9), the inequality (58) gives

$$
\begin{aligned}
&\sum_{i=0}^{\lambda_2-1}\left\{\|\hat{x}^{\mathrm{T}}(i+1,\lambda_1)\|^2_{\mathbf{P}_1(\alpha)+\mathbf{P}_3(\alpha)}\right.\\
&+\sum_{j=0}^{d_1}\|x(i+1,\lambda_1+j-d_1)\|^2_{\mathbf{Q}_1(\alpha)+\mathbf{Q}_3(\alpha)}\\
&\left.+\sum_{j=0}^{d_1-1}(d_1-j)\|\delta(i+1,\lambda_1-1-j)\|^2_{\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)}\right\}\\
&+\sum_{j=0}^{\lambda_1-1}\left\{\|\hat{x}^{\mathrm{T}}(\lambda_2,j+1)\|^2_{\mathbf{P}_2(\alpha)+\mathbf{P}_3(\alpha)}\right.\\
&+\sum_{i=0}^{d_2}\|x(\lambda_2+i-d_2,j+1)\|^2_{\mathbf{Q}_2(\alpha)+\mathbf{Q}_3(\alpha)}\\
&\left.+\sum_{i=0}^{d_2-1}(d_2-i)\|\delta(\lambda_2-1-i,j+1)\|^2_{\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)}\right\}\\
&+\gamma^{-1}\left\{\sum_{i=0}^{\lambda_2}\sum_{j=0}^{\lambda_1-1}\|\hat{z}(i,j)\|^2+\sum_{i=0}^{\lambda_2-1}\sum_{j=0}^{\lambda_1}\|\hat{z}(i,j)\|^2\right\}\\
&\leqslant\gamma\left\{\sum_{i=0}^{\lambda_2}\sum_{j=0}^{\lambda_1-1}\|w(i,j)\|^2-\sum_{j=0}^{\lambda_1-1}\|w(0,j)\|^2\right.\\
&\qquad\left.+\sum_{i=0}^{\lambda_2-1}\sum_{j=0}^{\lambda_1}\|w(i,j)\|^2-\sum_{i=0}^{\lambda_2-1}\|w(i,0)\|^2\right\}.
\end{aligned}\tag{59}
$$

Since $\mathbf{P}_3(\alpha)\geqslant 0$, $\mathbf{Q}_3(\alpha)\geqslant 0$, $\mathbf{P}_l(\alpha)>0$, $\mathbf{Q}_l(\alpha)>0$, $\mathbf{M}_l(\alpha)>0$ and $\mathbf{N}_l(\alpha)>0$, $l=1,2$, the inequality (59) implies

$$
2\gamma^{-1}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|\hat{z}(i,j)\|^2\leqslant 2\gamma\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|w(i,j)\|^2 \tag{60}
$$

when $\lambda_1\to\infty$ and $\lambda_2\to\infty$. Hence, the H_∞-norm of system (7) is no greater than γ. This completes the proof. □

Remark 6. Theorem 5 provides a new delay-dependent bounded real lemma for 2-D state-delayed systems. Similar to the case of Remark 4, when $v\geqslant 0$ approaches zero Theorem 5 with

$$
\begin{aligned}
&\mathbf{F}_1=\frac{vI_{4n}}{\bar{d}_1},\quad \mathbf{M}_1=\frac{vI_n}{\bar{d}_1},\quad \mathbf{K}_1=\frac{vI_{2m}}{\bar{d}_1},\quad \mathbf{N}_1=\frac{vI_n}{\bar{d}_1},\\
&\mathbf{F}_2=\frac{vI_{4n}}{\bar{d}_2},\quad \mathbf{M}_2=\frac{vI_n}{\bar{d}_2},\quad \mathbf{K}_2=\frac{vI_{2m}}{\bar{d}_2},\quad \mathbf{N}_2=\frac{vI_n}{\bar{d}_2},\\
&\mathbf{H}=0,\quad \mathbf{P}_3=\mathbf{Q}_3=0,
\end{aligned}
$$

reduces to the delay-independent conditions in Theorem 2 of [14] with no uncertainty. In another case, when the 2-D system has no state-delays, i.e., $A_{d1}=A_{d2}=0$, Theorem 5 reduces to Theorem 1 of [11].

4. Synthesis of robust filters

The system matrices (8) of the filtering error dynamics (7) can be expressed as

$$
\begin{aligned}
&\hat{A}(\alpha)=\sum_{j=1}^{\tau}\alpha_j\hat{A}^{(j)},\quad \hat{A}_d(\alpha)=\sum_{j=1}^{\tau}\alpha_j\hat{A}_d^{(j)},\\
&\hat{B}(\alpha)=\sum_{j=1}^{\tau}\alpha_j\hat{B}^{(j)},\quad \hat{L}(\alpha)=\sum_{j=1}^{\tau}\alpha_j\hat{L}^{(j)},
\end{aligned}\tag{61}
$$

where

$$
\begin{aligned}
&\hat{A}^{(j)}=J^{\mathrm{T}}A^{(j)}\tilde{J}+\mathscr{I}^{\mathrm{T}}\mathbf{\Xi}(I_2\otimes\mathscr{C}^{(j)}),\\
&\hat{A}_d^{(j)}=J^{\mathrm{T}}A_d^{(j)},\\
&\hat{B}^{(j)}=J^{\mathrm{T}}B^{(j)}+\mathscr{I}^{\mathrm{T}}\mathbf{\Xi}(I_2\otimes\mathscr{D}^{(j)}),\\
&\hat{L}^{(j)}=L^{(j)}J-\mathbf{L_f}\mathscr{I},\\
&A^{(j)}=[A_1^{(j)}\ \ A_2^{(j)}],\quad A_d^{(j)}=[A_{d1}^{(j)}\ \ A_{d2}^{(j)}],
\end{aligned}
$$

$$B^{(j)} = [B_1^{(j)}\ \ B_2^{(j)}],\quad \mathscr{I} = [0\ \ I_n],$$
$$\mathscr{C}^{(j)} = \begin{bmatrix} C^{(j)} & 0 \\ 0 & I_n \end{bmatrix},\quad \mathscr{D}^{(j)} = \begin{bmatrix} D^{(j)} \\ 0 \end{bmatrix},$$
$$\mathbf{\Xi} = [\mathbf{B}_{f1}\ \ \mathbf{A}_{f1}\ \ \mathbf{B}_{f2}\ \ \mathbf{A}_{f2}]. \tag{62}$$

$$\begin{bmatrix} -\mathbf{\Phi}^{(j)} & \star & \star & \star & \star & \star & \star \\ -\mathbf{H}^{(j)\mathrm{T}} & -\mathbf{Q}^{(j)} & \star & \star & \star & \star & \star \\ \mathbf{G}^{(j)}\tilde{J} & -\mathbf{G}^{(j)} & -\mathbf{\Psi}^{(j)} & \star & \star & \star & \star \\ \mathbf{S}\hat{A}^{(j)} & \mathbf{S}\hat{A}_d^{(j)} & \mathbf{S}\hat{B}^{(j)} & -\mathbf{\Lambda}_1^{(j)} & \star & \star & \star \\ \mathbf{V}_1(J\hat{A}^{(j)} - J_1) & \mathbf{V}_1 J\hat{A}_d^{(j)} & \mathbf{V}_1 J\hat{B}^{(j)} & 0 & -\mathbf{\Lambda}_2^{(j)} & \star & \star \\ \mathbf{V}_2(J\hat{A}^{(j)} - J_2) & \mathbf{V}_2 J\hat{A}_d^{(j)} & \mathbf{V}_2 J\hat{B}^{(j)} & 0 & 0 & -\mathbf{\Lambda}_3^{(j)} & \star \\ \hat{L}^{(j)} \otimes I_2 & 0 & 0 & 0 & 0 & 0 & -\gamma I_{2q} \end{bmatrix} < 0 \tag{67}$$

Therefore $\hat{A}^{(j)}$, $\hat{B}^{(j)}$ and $\hat{L}^{(j)}$ are affine functions of the filter system matrix variables $\mathbf{\Xi}$ and $\mathbf{L}_f$. This fact is useful in the subsequent development. To handle the polytopic uncertain system with less conservative design results, the use of a parameter-dependent Lyapunov function allows different Lyapunov matrices to be set for different parts of the entire polytopic domain. The following lemma is presented as a preparation step.

Lemma 7. *Given positive integers* $\bar{d}_1$, $\bar{d}_2$ *and a real number* $\gamma > 0$, *if there exist matrices* $0 < \mathbf{P}^{(j)} = \begin{bmatrix} \mathbf{P}_1^{(j)} & \star \\ \mathbf{P}_3^{(j)\mathrm{T}} & \mathbf{P}_2^{(j)} \end{bmatrix} \in \mathbb{R}^{4n\times 4n}$, $\mathbf{Q}^{(j)} = \begin{bmatrix} \mathbf{Q}_1^{(j)} & \star \\ \mathbf{Q}_3^{(j)\mathrm{T}} & \mathbf{Q}_2^{(j)} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{S} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{M}_l^{(j)} \in \mathbb{R}^{n\times n}$, $\mathbf{N}_l^{(j)} \in \mathbb{R}^{n\times n}$, $\mathbf{F}_l^{(j)} \in \mathbb{R}^{4n\times 4n}$, $\mathbf{G}_l^{(j)} \in \mathbb{R}^{2m\times n}$, $\mathbf{H}_l^{(j)} \in \mathbb{R}^{4n\times n}$, $\mathbf{K}_l^{(j)} \in \mathbb{R}^{2m\times 2m}$ *and* $\mathbf{V}_l \in \mathbb{R}^{n\times n}$, $l = 1, 2$, *such that*

$$\mathbf{Q}_3^{(j)} = \mathbf{Q}_3^{(j)\mathrm{T}} \geqslant 0, \tag{63}$$

$$\begin{bmatrix} \mathbf{K}_l^{(j)} & \star \\ \mathbf{G}_l^{(j)\mathrm{T}} & \mathbf{N}_l^{(j)} \end{bmatrix} > 0,\quad l = 1, 2, \tag{64}$$

$$\mathbf{P}_3^{(j)} = \mathbf{P}_3^{(j)\mathrm{T}} \geqslant 0, \tag{65}$$

$$\begin{bmatrix} \mathbf{F}_l^{(j)} & \star \\ \mathbf{H}_l^{(j)\mathrm{T}} & \mathbf{M}_l^{(j)} \end{bmatrix} > 0,\quad l = 1, 2, \tag{66}$$

hold for $j = 1, 2, \ldots, \tau$, *where*

$$\mathbf{\Phi}^{(j)} = \mathbf{P}^{(j)} - \bar{d}_1\mathbf{F}_1^{(j)} - \bar{d}_2\mathbf{F}_2^{(j)} - \mathbf{H}^{(j)}\tilde{J} - \tilde{J}^{\mathrm{T}}\mathbf{H}^{(j)\mathrm{T}},$$
$$\mathbf{\Psi}^{(j)} = \gamma I_{2m} - \bar{d}_1\mathbf{K}_1^{(j)} - \bar{d}_2\mathbf{K}_2^{(j)},$$
$$\mathbf{H}^{(j)} = [\mathbf{H}_1^{(j)}\ \ \mathbf{H}_2^{(j)}],\quad \mathbf{G}^{(j)} = [\mathbf{G}_1^{(j)}\ \ \mathbf{G}_2^{(j)}],$$
$$\mathbf{\Lambda}_1^{(j)} = (\mathbf{S} + \mathbf{S}^{\mathrm{T}}) - \mathbf{R}^{(j)},$$
$$\mathbf{R}^{(j)} = \mathbf{P}_1^{(j)} + 2\mathbf{P}_3^{(j)} + \mathbf{P}_2^{(j)} + J^{\mathrm{T}}[\mathbf{Q}_1^{(j)} + 2\mathbf{Q}_3^{(j)} + \mathbf{Q}_2^{(j)}]J,$$
$$\mathbf{\Lambda}_2^{(j)} = (\mathbf{V}_1 + \mathbf{V}_1^{\mathrm{T}}) - \bar{d}_1(\mathbf{M}_1^{(j)} + \mathbf{N}_1^{(j)}),$$
$$\mathbf{\Lambda}_3^{(j)} = (\mathbf{V}_2 + \mathbf{V}_2^{\mathrm{T}}) - \bar{d}_2(\mathbf{M}_2^{(j)} + \mathbf{N}_2^{(j)}), \tag{68}$$

then $\mathbf{P}(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{P}^{(j)}$, $\mathbf{Q}(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{Q}^{(j)}$, $\mathbf{F}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{F}_l^{(j)}$, $\mathbf{G}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{G}_l^{(j)}$, $\mathbf{H}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{H}_l^{(j)}$, $\mathbf{K}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{K}_l^{(j)}$, $\mathbf{M}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{M}_l^{(j)}$ and $\mathbf{N}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{N}_l^{(j)}$, $l = 1, 2$, *satisfy* (33)–(37) *of Theorem* 5 *for all* α *in the unit simplex* (4).

Proof. Clearly, (63)–(66) imply (33)–(36), respectively, for all α in the unit simplex (4). Then, for $\mathbf{\Lambda}_l(\alpha) = \sum_{j=1}^{\tau}\alpha_j\mathbf{\Lambda}_l^{(j)}$, $l = 1, 2, 3$, and all α in the unit simplex (4), (67) guarantees
Notice that (67) also ensures the nonsingularity of

$$\begin{bmatrix} -\mathbf{\Phi}(\alpha) & \star & \star & \star & \star & \star & \star \\ -\mathbf{H}^{\mathrm{T}}(\alpha) & -\mathbf{Q}(\alpha) & \star & \star & \star & \star & \star \\ \mathbf{G}(\alpha)\tilde{J} & -\mathbf{G}(\alpha) & -\mathbf{\Psi}(\alpha) & \star & \star & \star & \star \\ \mathbf{S}\hat{A}(\alpha) & \mathbf{S}\hat{A}_d(\alpha) & \mathbf{S}\hat{B}(\alpha) & -\mathbf{\Lambda}_1(\alpha) & \star & \star & \star \\ \mathbf{V}_1[J\hat{A}(\alpha) - J_1] & \mathbf{V}_1 J\hat{A}_d(\alpha) & \mathbf{V}_1 J\hat{B}(\alpha) & 0 & -\mathbf{\Lambda}_2(\alpha) & \star & \star \\ \mathbf{V}_2[J\hat{A}(\alpha) - J_2] & \mathbf{V}_2 J\hat{A}_d(\alpha) & \mathbf{V}_2 J\hat{B}(\alpha) & 0 & 0 & -\mathbf{\Lambda}_3(\alpha) & \star \\ \hat{L}(\alpha) \otimes I_2 & 0 & 0 & 0 & 0 & 0 & -\gamma I_{2q} \end{bmatrix} < 0. \tag{69}$$

$\mathbf{S}$, $\mathbf{V}_1$ and $\mathbf{V}_2$. Thus, performing the congruence transformation $\text{diag}\{I_{4n}, I_{2n}, I_{2m}, \mathbf{S}^{-\text{T}}, \mathbf{V}_1^{-\text{T}}, \mathbf{V}_2^{-\text{T}}, I_{2q}\}$ to (69) results in

$$\begin{bmatrix} -\mathbf{\Phi}(\alpha) & \star & \star & \star & \star & \star & \star \\ -\mathbf{H}^{\text{T}}(\alpha) & -\mathbf{Q}(\alpha) & \star & \star & \star & \star & \star \\ \mathbf{G}(\alpha)\tilde{J} & -\mathbf{G}(\alpha) & -\mathbf{\Psi}(\alpha) & \star & \star & \star & \star \\ \hat{A}(\alpha) & \hat{A}_d(\alpha) & \hat{B}(\alpha) & -\hat{\mathbf{\Lambda}}_1(\alpha) & \star & \star & \star \\ J\hat{A}(\alpha)-J_1 & J\hat{A}_d(\alpha) & J\hat{B}(\alpha) & 0 & -\hat{\mathbf{\Lambda}}_2(\alpha) & \star & \star \\ J\hat{A}(\alpha)-J_2 & J\hat{A}_d(\alpha) & J\hat{B}(\alpha) & 0 & 0 & -\hat{\mathbf{\Lambda}}_3(\alpha) & \star \\ \hat{L}(\alpha)\otimes I_2 & 0 & 0 & 0 & 0 & 0 & -\gamma I_{2q} \end{bmatrix} < 0, \tag{70}$$

where $\hat{\mathbf{\Lambda}}_1(\alpha) = \mathbf{S}^{-1}\mathbf{\Lambda}_1(\alpha)\mathbf{S}^{-\text{T}}$, $\hat{\mathbf{\Lambda}}_2(\alpha) = \mathbf{V}_1^{-1}\mathbf{\Lambda}_2(\alpha)\mathbf{V}_1^{-\text{T}}$ and $\hat{\mathbf{\Lambda}}_3(\alpha) = \mathbf{V}_2^{-1}\mathbf{\Lambda}_3(\alpha)\mathbf{V}_2^{-\text{T}}$. Since

$$[\mathbf{R}^{-1}(\alpha) - \mathbf{S}^{-1}]\mathbf{R}(\alpha)[\mathbf{R}^{-1}(\alpha) - \mathbf{S}^{-1}]^{\text{T}} \geqslant 0,$$
$$\{\bar{d}_1^{-1}[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)]^{-1} - \mathbf{V}_1^{-1}\}\bar{d}_1[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)] \times\{\bar{d}_1^{-1}[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)]^{-1} - \mathbf{V}_1^{-1}\}^{\text{T}} \geqslant 0,$$
$$\{\bar{d}_2^{-1}[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)]^{-1} - \mathbf{V}_2^{-1}\}\bar{d}_2[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)] \times\{\bar{d}_2^{-1}[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)]^{-1} - \mathbf{V}_2^{-1}\}^{\text{T}} \geqslant 0,$$

one has

$$-\mathbf{R}^{-1}(\alpha) \leqslant \mathbf{S}^{-1}\mathbf{R}(\alpha)\mathbf{S}^{-\text{T}} - (\mathbf{S}^{-1}+\mathbf{S}^{-\text{T}}),$$
$$-\bar{d}_1^{-1}[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)]^{-1} \leqslant \bar{d}_1\mathbf{V}_1^{-1}[\mathbf{M}_1(\alpha)+\mathbf{N}_1(\alpha)]\mathbf{V}_1^{-\text{T}} - (\mathbf{V}_1^{-1}+\mathbf{V}_1^{-\text{T}}),$$
$$-\bar{d}_2^{-1}[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)]^{-1} \leqslant \bar{d}_2\mathbf{V}_2^{-1}[\mathbf{M}_2(\alpha)+\mathbf{N}_2(\alpha)]\mathbf{V}_2^{-\text{T}} - (\mathbf{V}_2^{-1}+\mathbf{V}_2^{-\text{T}}).$$

Thus (67) implies (37) for all α in the unit simplex (4). □

It should be noted that (67) is not an LMI in the matrix variables $\mathbf{S}$, $\mathbf{A_{f1}}$, $\mathbf{A_{f2}}$, $\mathbf{B_{f1}}$ and $\mathbf{B_{f2}}$, but can be converted into one via proper transformations. This will be done in the next theorem, and an LMI based method will be developed for designing a filter (6) such that the H_∞-norm constraint is satisfied.

Theorem 8. *Consider system* (1). *Given positive integers* $\bar{d}_1$, $\bar{d}_2$ *and a real number* $\gamma > 0$, *if there exist matrices* $\hat{\mathbf{S}} = \begin{bmatrix} \hat{\mathbf{S}}_1 & \hat{\mathbf{S}}_3 \\ \hat{\mathbf{S}}_2 & \hat{\mathbf{S}}_3 \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, $0 < \hat{\mathbf{P}}^{(j)} = \begin{bmatrix} \hat{\mathbf{P}}_1^{(j)} & \hat{\mathbf{P}}_3^{(j)\text{T}} \\ \star & \hat{\mathbf{P}}_2^{(j)} \end{bmatrix} \in \mathbb{R}^{4n\times 4n}$, $\mathbf{Q}^{(j)} = \begin{bmatrix} \mathbf{Q}_1^{(j)} & \star \\ \mathbf{Q}_3^{(j)\text{T}} & \mathbf{Q}_2^{(j)} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, $\hat{\mathbf{\Xi}} \in \mathbb{R}^{n\times(2n+2m)}$, $\hat{\mathbf{L}}_\mathbf{f} \in \mathbb{R}^{q\times n}$, $\mathbf{M}_l^{(j)} \in \mathbb{R}^{n\times n}$, $\mathbf{N}_l^{(j)} \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{F}}_l^{(j)} \in \mathbb{R}^{4n\times 4n}$, $\mathbf{G}_l^{(j)} \in \mathbb{R}^{2m\times n}$, $\hat{\mathbf{H}}_l^{(j)} \in \mathbb{R}^{4n\times n}$, $\mathbf{K}_l^{(j)} \in \mathbb{R}^{2m\times 2m}$ *and* $\mathbf{V}_l \in \mathbb{R}^{n\times n}$, $l = 1, 2$, *such that* (63)–(64) *and the following LMIs*:

$$\hat{\mathbf{P}}_3^{(j)} = \hat{\mathbf{P}}_3^{(j)\text{T}} \geqslant 0, \tag{71}$$

$$\begin{bmatrix} \hat{\mathbf{F}}_l^{(j)} & \star \\ \hat{\mathbf{H}}_l^{(j)\text{T}} & \mathbf{M}_l^{(j)} \end{bmatrix} > 0, \tag{72}$$

$$\begin{bmatrix} -\mathbf{\Theta}_{11} & \star & \star & \star & \star & \star & \star \\ -\hat{\mathbf{H}}^{(j)\text{T}} & -\mathbf{Q}^{(j)} & \star & \star & \star & \star & \star \\ \mathbf{G}^{(j)}\tilde{J} & -\mathbf{G}^{(j)} & -\mathbf{\Theta}_{33} & \star & \star & \star & \star \\ \mathbf{\Theta}_{41} & \mathbf{\Theta}_{42} & \mathbf{\Theta}_{43} & -\mathbf{\Theta}_{44} & \star & \star & \star \\ \mathbf{\Theta}_{51} & \mathbf{V}_1A_d^{(j)} & \mathbf{V}_1B^{(j)} & 0 & -\mathbf{\Theta}_{55} & \star & \star \\ \mathbf{\Theta}_{61} & \mathbf{V}_2A_d^{(j)} & \mathbf{V}_2B^{(j)} & 0 & 0 & -\mathbf{\Theta}_{66} & \star \\ \mathbf{\Theta}_{71} & 0 & 0 & 0 & 0 & 0 & -\gamma I_{2q} \end{bmatrix} < 0 \tag{73}$$

hold for $j = 1, 2, \ldots, \tau$, *where*

$$\mathbf{\Theta}_{11} = \hat{\mathbf{P}}^{(j)} - \bar{d}_1\hat{\mathbf{F}}_1^{(j)} - \bar{d}_2\hat{\mathbf{F}}_2^{(j)} - \hat{\mathbf{H}}^{(j)}\tilde{J} - \tilde{J}^{\text{T}}\hat{\mathbf{H}}^{(j)\text{T}},$$

$$\mathbf{\Theta}_{33} = \gamma I_{2m} - \bar{d}_1\mathbf{K}_1^{(j)} - \bar{d}_2\mathbf{K}_2^{(j)},$$

$$\mathbf{\Theta}_{41} = \hat{\mathbf{S}}J^{\text{T}}A^{(j)}\tilde{J} + \Pi^{\text{T}}\hat{\mathbf{\Xi}}(I_2 \otimes \mathscr{C}^{(j)}),$$

$$\mathbf{\Theta}_{42} = \hat{\mathbf{S}}J^{\text{T}}A_d^{(j)},$$

$$\mathbf{\Theta}_{43} = \hat{\mathbf{S}}J^{\text{T}}B^{(j)} + \Pi^{\text{T}}\hat{\mathbf{\Xi}}(I_2 \otimes \mathscr{D}^{(j)}),$$

$$\mathbf{\Theta}_{44} = \hat{\mathbf{S}} + \hat{\mathbf{S}}^{\text{T}} - \hat{\mathbf{P}}_1^{(j)} - 2\hat{\mathbf{P}}_3^{(j)} - \hat{\mathbf{P}}_2^{(j)} - J^{\text{T}}(\mathbf{Q}_1^{(j)} + 2\mathbf{Q}_3^{(j)} + \mathbf{Q}_2^{(j)})J,$$

$$\mathbf{\Theta}_{51} = \mathbf{V}_1(A^{(j)}\tilde{J} - J_1),$$

$$\mathbf{\Theta}_{55} = \mathbf{V}_1 + \mathbf{V}_1^{\text{T}} - \bar{d}_1(\mathbf{M}_1^{(j)} + \mathbf{N}_1^{(j)}),$$

$$\mathbf{\Theta}_{61} = \mathbf{V}_2(A^{(j)}\tilde{J} - J_2),$$

$$\mathbf{\Theta}_{66} = \mathbf{V}_2 + \mathbf{V}_2^{\mathrm{T}} - \bar{d}_2(\mathbf{M}_2^{(j)} + \mathbf{N}_2^{(j)}),$$

$$\mathbf{\Theta}_{71} = I_2 \otimes (L^{(j)}J) - I_2 \otimes (\hat{\mathbf{L}}_{\mathbf{f}}\mathscr{I}),$$

$$\hat{\mathbf{H}}^{(j)} = [\hat{\mathbf{H}}_1^{(j)}\ \hat{\mathbf{H}}_2^{(j)}],$$

$$\mathbf{G}^{(j)} = [\mathbf{G}_1^{(j)}\ \mathbf{G}_2^{(j)}],$$

$$\Pi = [I_n\ I_n], \tag{74}$$

then the H_∞ filtering problem is solvable for $d_i \in [0,\ \bar{d}_i]$, $i = 1, 2$, and the filter system matrices for (6) *can be obtained from any feasible $\hat{\mathbf{S}}$ and $\hat{\mathbf{\Xi}} = [\hat{\mathbf{B}}_{\mathbf{f}1}\ \hat{\mathbf{A}}_{\mathbf{f}1}\ \hat{\mathbf{B}}_{\mathbf{f}2}\ \hat{\mathbf{A}}_{\mathbf{f}2}]$ as $\mathbf{\Xi} = [\hat{\mathbf{B}}_{\mathbf{f}1}\ \hat{\mathbf{A}}_{\mathbf{f}1}\hat{\mathbf{S}}_3^{-1}\ \hat{\mathbf{B}}_{\mathbf{f}2}\ \hat{\mathbf{A}}_{\mathbf{f}2}\hat{\mathbf{S}}_3^{-1}]$ in* (62) *and $\mathbf{L}_f = \hat{\mathbf{L}}_{\mathbf{f}}\hat{\mathbf{S}}_3^{-1}$.*

Proof. In (73), $-\mathbf{\Theta}_{44} < 0$ ensures that $\hat{\mathbf{S}}$ is nonsingular, which implies that $\hat{\mathbf{S}}_3$ is nonsingular. Define

$$\Upsilon = \begin{bmatrix} I_n & 0 \\ 0 & \hat{\mathbf{S}}_3^{\mathrm{T}} \end{bmatrix}, \tag{75}$$

$\widetilde{\Upsilon} = I_2 \otimes \Upsilon$, and change the variables as

$$\mathbf{P}^{(j)} = \widetilde{\Upsilon}^{-1}\hat{\mathbf{P}}^{(j)}\widetilde{\Upsilon}^{-T},$$

$$\mathbf{S} = \Upsilon^{-1}\hat{\mathbf{S}}\Upsilon^{-T} = \begin{bmatrix} \hat{\mathbf{S}}_1 & I_n \\ \hat{\mathbf{S}}_3^{-T}\hat{\mathbf{S}}_2 & \hat{\mathbf{S}}_3^{-T} \end{bmatrix},$$

$$\mathbf{F}_l^{(j)} = \widetilde{\Upsilon}^{-1}\hat{\mathbf{F}}_l^{(j)}\widetilde{\Upsilon}^{-T}, \quad l = 1, 2,$$

$$\mathbf{H}_l^{(j)} = \widetilde{\Upsilon}^{-1}\hat{\mathbf{H}}_l^{(j)}, \quad l = 1, 2,$$

$$\mathbf{\Xi} = \hat{\mathbf{\Xi}}\widetilde{\Upsilon}^{-T}, \quad \mathbf{L}_{\mathbf{f}} = \hat{\mathbf{L}}_{\mathbf{f}}\hat{\mathbf{S}}_3^{-1}. \tag{76}$$

Then, it is easy to check that

$$J\Upsilon = J\Upsilon^{\mathrm{T}} = J,$$

$$\Upsilon^{\mathrm{T}}\mathscr{D}^{(j)} = \mathscr{D}^{(j)},$$

$$\Upsilon^{\mathrm{T}}\mathscr{C}^{(j)} = \mathscr{C}^{(j)}\Upsilon^{\mathrm{T}},$$

$$\Upsilon\mathbf{S}\mathscr{I}^{\mathrm{T}} = \Pi^{\mathrm{T}},$$

$$\hat{\mathbf{S}}_3\mathscr{I} = \mathscr{I}\Upsilon^{-\mathrm{T}},$$

$$\Upsilon\mathbf{S}\hat{A}^{(j)}\widetilde{\Upsilon}^{\mathrm{T}} = \hat{\mathbf{S}}J^{\mathrm{T}}A^{(j)}\tilde{J} + \Pi^{\mathrm{T}}\hat{\mathbf{\Xi}}(I_2 \otimes \mathscr{C}^{(j)}),$$

$$\Upsilon\mathbf{S}\hat{A}_d^{(j)} = \hat{\mathbf{S}}J^{\mathrm{T}}A_d^{(j)},$$

$$\Upsilon\mathbf{S}\hat{B}^{(j)} = \hat{\mathbf{S}}J^{\mathrm{T}}B^{(j)} + \Pi^{\mathrm{T}}\hat{\mathbf{\Xi}}(I_2 \otimes \mathscr{D}^{(j)}),$$

$$\mathbf{V}_1(J\hat{A}^{(j)} - J_1)\widetilde{\Upsilon}^{\mathrm{T}} = \mathbf{V}_1(A^{(j)}\tilde{J} - J_1),$$

$$\mathbf{V}_2(J\hat{A}^{(j)} - J_2)\widetilde{\Upsilon}^{\mathrm{T}} = \mathbf{V}_2(A^{(j)}\tilde{J} - J_2),$$

$$\hat{L}^{(j)}\Upsilon^{\mathrm{T}} = L^{(j)}J - \hat{\mathbf{L}}_{\mathbf{f}}\mathscr{I}. \tag{77}$$

Applying the congruence transformations $\Upsilon^{-\mathrm{T}}$, $\mathrm{diag}\{\widetilde{\Upsilon}^{-\mathrm{T}}, I_n\}$ and $\mathrm{diag}\{\widetilde{\Upsilon}^{-\mathrm{T}}, I_{2n}, I_{2m}, \Upsilon^{-\mathrm{T}}, I_n, I_n, I_{2q}\}$ to (71), (72) and (73), respectively, yield (65), (66) and (67). Thus the proof is completed. □

Remark 9. With the variables $\hat{\mathbf{S}}$, $\mathbf{V}_1$ and $\mathbf{V}_2$ independent of the index j, Theorem 8 provides an effective method to solve the robust H_∞ filtering problems involving parameter-dependent system matrices. Recently, a new approach is proposed in [1], in which all Lyapunov matrix variables are set to be parameter-dependent. This new methods in [1] may offer less conservative results, especially for 1-D polytopic systems. However, the filtering synthesis method in [1] for 2-D polytopic systems will involve matrices of quite large dimensions, and the number of LMIs increases faster as the number of vertices in the polytope domain increases.

Remark 10. In Theorem 8, γ is a given real number. But the conditions in Theorem 8 are still LMIs when γ is regarded as a variable. Thus it is possible to formulate the following LMI optimization problem to find a filter with the smallest H_∞-norm:

$$\begin{aligned} &\text{Minimize:} \quad \gamma \\ &\text{subject to:} \quad (63){-}(64),\ (71){-}(73) \end{aligned} \tag{78}$$

with respect to γ and the variables stated in Theorem 8.

In the delay-independent case, Theorem 8 may be reduced to following simpler results.

Corollary 11. *Consider system* (1) *and let $\gamma > 0$ be a given real number. If there exist matrices $\hat{\mathbf{S}} = \begin{bmatrix} \hat{\mathbf{S}}_1 & \hat{\mathbf{S}}_3 \\ \hat{\mathbf{S}}_2 & \hat{\mathbf{S}}_3 \end{bmatrix}$, $\hat{\mathbf{P}}^{(j)} = \begin{bmatrix} \hat{\mathbf{P}}_1^{(j)} & \star \\ \hat{\mathbf{P}}_3^{(j)\mathrm{T}} & \hat{\mathbf{P}}_2^{(j)} \end{bmatrix}$, $\mathbf{Q}^{(j)} = \begin{bmatrix} \mathbf{Q}_1^{(j)} & \star \\ \mathbf{Q}_3^{(j)\mathrm{T}} & \mathbf{Q}_2^{(j)} \end{bmatrix}$, $\hat{\mathbf{\Xi}}$ and $\hat{\mathbf{L}}_{\mathbf{f}}$ such that the LMI*

$$\begin{bmatrix} -\hat{\mathbf{P}}^{(j)} & \star & \star & \star & \star \\ 0 & -\mathbf{Q}^{(j)} & \star & \star & \star \\ 0 & 0 & -\gamma I_{2m} & \star & \star \\ \mathbf{\Theta}_{41} & \mathbf{\Theta}_{42} & \mathbf{\Theta}_{43} & -\mathbf{\Theta}_{44} & \star \\ \mathbf{\Theta}_{71} & 0 & 0 & 0 & -\gamma I_{2q} \end{bmatrix} < 0 \tag{79}$$

hold for $j = 1, 2, \ldots, \tau$, where $\mathbf{\Theta}_{41}$, $\mathbf{\Theta}_{42}$, $\mathbf{\Theta}_{43}$, $\mathbf{\Theta}_{44}$ and $\mathbf{\Theta}_{71}$ are defined in Theorem 8, *then the H_∞ filtering problem is solvable for all delay sizes, and the filter system matrices in* (6) *can be obtained from any feasible $\hat{\mathbf{S}}$ and $\hat{\mathbf{\Xi}} = [\hat{\mathbf{B}}_{\mathbf{f}1}\ \hat{\mathbf{A}}_{\mathbf{f}1}\ \hat{\mathbf{B}}_{\mathbf{f}2}\ \hat{\mathbf{A}}_{\mathbf{f}2}]$ as $\mathbf{\Xi} = [\hat{\mathbf{B}}_{\mathbf{f}1}\ \hat{\mathbf{A}}_{\mathbf{f}1}\hat{\mathbf{S}}_3^{-1}\ \hat{\mathbf{B}}_{\mathbf{f}2}\ \hat{\mathbf{A}}_{\mathbf{f}2}\hat{\mathbf{S}}_3^{-1}]$ in* (62) *and $\mathbf{L}_f = \hat{\mathbf{L}}_{\mathbf{f}}\hat{\mathbf{S}}_3^{-1}$.*

Remark 12. In Corollary 11, an upper bound of H_∞-norm can be found from the following LMI optimization problem:

$$\begin{aligned}&\text{Minimize:} \quad \gamma\\&\text{subject to:} \quad (79)\end{aligned} \tag{80}$$

with respect to γ and the variables stated in Corollary 11.

Remark 13. When Theorem 8 and Corollary 11 are compared, it is seen that matrices of the same dimensions are involved in (73) and (79), but the number of matrix variables to be determined in Corollary 11 is less than that in Theorem 8.

5. Numerical example

Consider a heat diffusion system along a line described by the partial differential equation

$$\frac{\partial^2}{\partial \xi^2} u(\xi, t) = c(\alpha)\frac{\partial}{\partial t} u(\xi, t) + f(\xi, t) + w(\xi, t), \tag{81}$$

which has been studied in [14]. In (81), $\xi \in [0,\ \bar{\xi}]$ is the spatial variable, $t \in [0,\ \infty)$ is the time variable, $u(\xi, t)$ is the temperature of the line at ξ and t, $c(\alpha)$ is the thermal diffusivity depending on an uncertain parameter vector $\alpha = [\alpha_1\ \ \alpha_2]^T$, $f(\xi, t)$ is the control input, and $w(\xi, t)$ is the noise input. Suppose $c(\alpha) = 0.8 - 0.4\alpha_1 + 0.32\alpha_2$ and the system is controlled by a "mixed" state feedback law $f(\xi, t) = -200u(\xi, t) + 102u(\xi - \xi_0, t)$, where $\xi_0 = 0.1$. By the central and back difference approximations as in [14], (81) can be converted into the Fornasini–Marchesini second model of the form (1) with

$$\begin{aligned}
&x(i,j) = [u(i,j)\ \ 0.5c(\alpha)u(i,j-1) + 0.5u(i-1,j)]^T,\\
&y(i,j) = c(\alpha)u(i,j-1) + u(i-1,j),\\
&z(i,j) = u(i,j),\\
&A_1(\alpha) = \begin{bmatrix} 0 & 0 \\ 0.5c(\alpha) & 0 \end{bmatrix}, \quad A_2(\alpha) = \begin{bmatrix} 0.1c(\alpha) & -0.2 \\ 0.5 & 0 \end{bmatrix},\\
&A_{d1} = 0, \quad A_{d2} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0 \end{bmatrix},\\
&B_1 = 0, \quad B_2 = [0.01\ \ 0]^T,\\
&C = [0\ \ 2], \quad D = 0, \quad L = [1\ \ 0],\\
&d_1 = 0, \quad d_2 = 1.
\end{aligned}$$

The matrices $A_1(\alpha)$ and $A_2(\alpha)$ can be, respectively, represented by

$$A_1(\alpha) = \Lambda_0 + \alpha_1\Lambda_1 + \alpha_2\Lambda_2, \tag{82}$$

$$A_2(\alpha) = \Pi_0 + \alpha_1\Pi_1 + \alpha_2\Pi_2, \tag{83}$$

where

$$\begin{aligned}
&\Lambda_0 = \begin{bmatrix} 0 & 0 \\ 0.4 & 0 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 0 & 0 \\ -0.2 & 0 \end{bmatrix},\\
&\Lambda_2 = \begin{bmatrix} 0 & 0 \\ 0.16 & 0 \end{bmatrix},\\
&\Pi_0 = \begin{bmatrix} 0.08 & -0.2 \\ 0.5 & 0 \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} -0.04 & 0 \\ 0 & 0 \end{bmatrix},\\
&\Pi_2 = \begin{bmatrix} 0.032 & 0 \\ 0 & 0 \end{bmatrix}.
\end{aligned}$$

By solving the optimization problem (78) from Theorem 8 and the optimization problem (80) from Corollary 11, the optimal H_∞-norm bounds of 0.0184 and ∞, respectively, are obtained. The bound of ∞ means no feasible solutions exist for the delay-independent case. For the delay-dependent case the corresponding filter matrices are

$$\begin{aligned}
&\mathbf{A_{f1}} = \begin{bmatrix} -0.0107 & -0.5503 \\ -0.0063 & -0.9748 \end{bmatrix} \times 10^{-3},\\
&\mathbf{A_{f2}} = \begin{bmatrix} 0.0998 & -1.7315 \\ -0.0003 & -0.0078 \end{bmatrix},\\
&\mathbf{B_{f1}} = \begin{bmatrix} 0.2696 \\ -0.9646 \end{bmatrix} \times 10^{-4}, \quad \mathbf{B_{f2}} = \begin{bmatrix} 13.9253 \\ -0.1860 \end{bmatrix},\\
&\mathbf{L_f} = [-0.0083\ \ -0.0159].
\end{aligned}$$

Fig. 1 shows the magnitude plot of the filtering error dynamics over grid frequencies in the range of $[-\pi, \pi]$ for $\alpha_1 = 1$ and $\alpha_2 = 0$. It can be seen that the maximum magnitude is below the guaranteed H_∞-norm bound. This is also true for other checked uncertainties $\alpha = [0\ \ 1]^T$, $[0.1\ \ 0.9]^T, \ldots, [0.9\ \ 0.1]^T$.

6. Conclusion

For 2-D state-delayed systems with polytopic uncertainties described by the Fornasini–Marchesini second model, this paper proposes filter synthesis methods under the LMI framework. Based on the delay-dependent H_∞-norm performance criteria, effective methods for solving the robust H_∞ filtering problems with a parameter-dependent Lyapunov function approach are derived. An example is given to illustrate the usage of the proposed methods.

A natural extension of current results would be the consideration of feedback control problems for the 2-D state-delayed systems. However, when

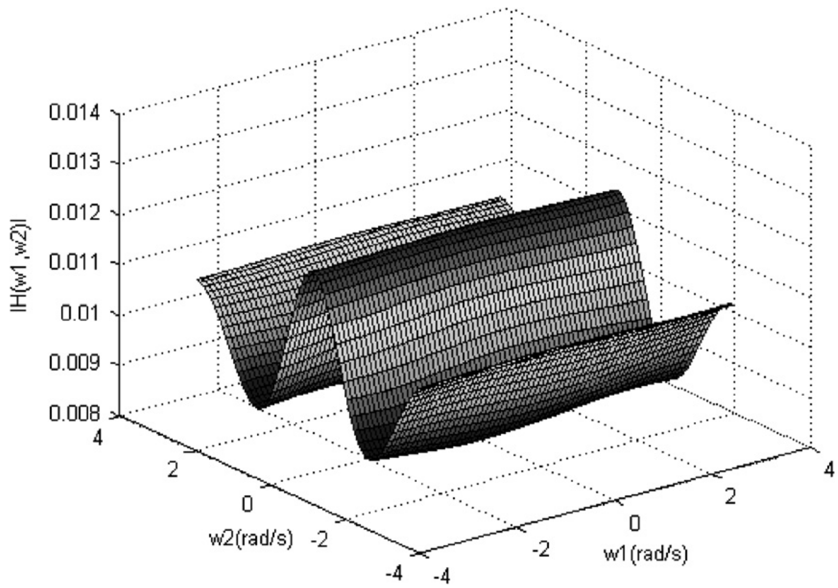

Fig. 1. Magnitudes of the filtering error transfer dynamics at different frequencies for $\alpha_1 = 1$ and $\alpha_2 = 0$.

applied to the control problems the conditions in Theorems 3 and 5 will become non-convex bilinear matrix inequalities. To develop a set of new LMI-based delay-dependent conditions for the design of feedback controllers is a research topic yet to be studied.

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