Decentralized Control of Interconnected Systems with Unmodelled Nonlinearity and Interaction*

CHENG-JYI MAO† and WEI-SONG LIN†‡

A proposed completely decentralized controller for interconnected systems with unmodelled nonlinearity and interaction gives each subsystem a near-optimal performance close to the decomposed, linearized optimal response.

Key Words-Decentralized control; interconnected systems; unmodelled; near-optimal control.

Abstract-This paper presents a completely decentralized control scheme for the control of interconnected systems with unmodelled nonlinearity and interaction. The interactions due to the interconnection and the intrinsic nonlinearities associated with each subsystem are represented by aggregative deviations of state derivatives from their linearized nominal values of the decomposed subsystems. Then, based on a model following technique, the aggregative deviations are tracked by on-line improvement. The solution involves the design of the decentralized control giving each subsystem a near-optimal performance close to the decomposed, linearized optimal response and the generation of corrective signals for the aggregative deviations of state derivatives. This approach is completely decentralized and all the operations are subsystem based, therefore the burden of computations is reduced significantly. Moreover, the proposed control method is robust to modelling errors and is initial state independent. By the Lyapunov's direct method, a sufficient condition for the stability of the global system even under any structural perturbations is established. Computer simulations for the decentralized control of a two-link, $\theta-r$ manipulator are conducted.

1. INTRODUCTION

THE DECENTRALIZED control schemes, different from the classical centralized information structures, have been considered with significant interests for the control of linear or nonlinear interconnected systems in recent years. When a large-scale system is concerned, the centralized pattern often fails to hold due to either lack of

† Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, 10764, R.O.C.

‡ Author to whom correspondence should be addressed.

the overall information or lack of the centralized computing capability. Although much of the early work dealing with linear interconnected systems by multilevel approaches have been made (Šiljak and Sundareshan, 1976; Sundareshan, 1977; Singh et al., 1979), these controllers still require information transfer among subsystems. With the constraint that only local information is observed by the local control station, some studies concerning this problem have been proposed. For example, the suppression of information transfer was achieved by calculating the block diagonal gain matrices using hierarchical structures (Hassan and Singh, 1978a; Hassan et al., 1979). Besides, Hassan and Singh (1978b, 1980) have shown two on-line completely decentralized control methods for linear interconnected systems: one by improving the interaction model and the other by improving the estimated interactions. For the problem of nonlinear interconnected systems, control methods by using high-gain feedback (Khalil and Saberi, 1982) and by solving a set of appropriate local optimal problems (Saberi, 1988) have also been derived.

Practically, physical systems possess unmodelled nonlinearities to some extent, and the interactions among subsystems of interconnected systems are poorly known or not known at all after the subsystems being integrated. How to deal with these problems becomes more interesting in this area. The model reference adaptive control technique has been applied to the decentralized control of interconnected systems (Bundell, 1985; Gavel and Šiljak, 1985; Ioannou and Kokotovic, 1985; Ioannou, 1986), where these methods considered the subsystems as if they were decoupled and accommodated

^{*} Received 19 August 1988; revised 13 June 1989; received in final form 24 June 1989. The original version of this paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor M. Jamshidi under the direction of Editor A. P. Sage.

with the unknown interactions. Other approaches for the decentralized control of linear unknown systems by the technique of so-called "multivariable tuning regulators" (Davison, 1978) and by nonlinear dynamic compensation (Hmamed and Radouane, 1982) have been proposed.

Especially, the two on-line approaches (Hassan and Singh, 1978b, 1980) mentioned above have shown a possible way to design the decentralized controllers for linear interconnected systems by giving the interaction a model. However, if the interaction is unmodelled, the procedure as in those for choosing the interaction model parameters from the global system matrix is not possible. In the present paper, with the advantages of simple computation and easy implementation, a completely decentralized control scheme for the control of interconnected systems with unmodelled nonlinearity and interaction is proposed. The nonlinearities and interactions associated with each subsystem are represented by aggregative deviations of state derivatives. Consequently, by using a model following technique, the deviations are tracked by the on-line improving technique. Therefore exact modelling of the deviations is not required. Based on improving the deviation model, the well-known optimal control technique is applied to obtain a completely decentralized control scheme. This eliminates the redesign procedure of subsystem controllers after the decomposed subsystems being integrated. Moreover, this method is robust to modelling errors and is initial state independent. The proposed decentralized controller yields a near-optimal performance which is close to the decomposed optimal response where no deviations are considered. Besides, since the calculation of the controller gains is subsystem based, it is simple to compute and the controllers are easy to be implemented. The rest of the paper is divided into five sections. Sections 2 and 3 give the formulation of the problem and the derivations of the new approach, respectively. In Section 4, the sufficient condition to guarantee the stability of the global system is established. It is seen that the satisfaction of this condition provides a robust design which is insensitive to any structural perturbations in the sense of Šiljak (Šiljak, 1973). In Section 5, computer simulations for the decentralized control of a two-link, θ -r manipulator are given and responses by the decomposed optimal control, local optimal control without any compensation and proposed decentralized control are compared. Section 6 is a brief conclusion.

2. PROBLEM FORMULATION

Consider a composite system which is an interconnection of s subsystems as

$$\dot{x}_i(t) = f_i(x_i, u_i) + \sum_{\substack{j=1 \ j \neq i}}^s g_{ij}(t, x_j) \quad i = 1, 2, \ldots, s$$
(1)

where $x_i(t)$ is the n_i -dimensional state vector, $u_i(t)$ is the m_i -dimensional control vector, $f_i(x_i, u_i)$ is continuous in time with $f_i(0, 0) = 0$, $g_{ij}(t, x_j)$ is the interconnection term for the *i*th subsystem from the *j*th subsystem and $g_{ij}(t, 0) =$ 0. The interconnection terms are assumed to satisfy

$$\sup_{t>0} ||g_{ij}(t, x_j)|| \le \alpha_{ij} ||x_j(t)||$$
(2)

where α_{ij} is a non-negative constant number, and the norms hereafter are all Euclidean and induced Euclidean norms for vectors and matrices, respectively. From the linearization of system (1) about the equilibrium point and the aggregation of the instant deviations of state derivatives due to the nonlinearities and interactions, equation (1) can be rewritten as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + z_i(t)$$
 $i = 1, 2, ..., s$

(3)

where A_i , B_i are constant matrices with appropriate dimensions, (A_i, B_i) is assumed to be a completely controllable pair and $z_i(t)$ is the n_i -dimensional deviation vector which is an aggregation of the nonlinearities and interactions.

With the presence of the unmodelled aggregative deviations, the goal of the decentralized controller is to give each subsystem a near-optimal performance close to the decomposed optimal response with respect to the following quadratic performance index

$$\min J_i = \frac{1}{2} \int_0^\infty \left[x_i'(t) Q_i x_i(t) + u_i'(t) R_i u_i(t) \right] dt \quad (4)$$

subject to (3) with $z_i(t)$ being neglected, where $Q_i = D_i D_i^t$ is positive semidefinite, (A_i, D_i) is completely observable, R_i is a positive definite matrix, and the superscript, t hereafter means the transpose of a vector or matrix.

3. THE DECENTRALIZED CONTROLLER

Based on a model following technique, the reference model for the *i*th subsystem can be constructed as

$$\hat{x}_{i}(t) = A_{i}\hat{x}_{i}(t) + B_{i}\{u_{i}(t) + K_{i}[x_{i}(t) - \hat{x}_{i}(t)]\} + \hat{z}_{i}(t)$$
(5)

$$\hat{z}_i(t) = A_{zi}\hat{z}_i(t) + v_i(t)$$
 (6)

where $\hat{z}_i(t)$ is the tracked aggregative deviation, A_{zi} is an arbitrary stable matrix, (A_{zi}, I_i) is a controllable pair, where I_i is the n_i -dimensional identity matrix and $v_i(t)$ is the improving signal left to be determined. The gain, K_i is chosen such that $(A_i - B_i K_i)$ is asymptotically stable. Let

$$\bar{x}_i(t) = x_i(t) - \hat{x}_i(t) \tag{7}$$

$$\tilde{z}_i(t) = z_i(t) - \hat{z}_i(t) \tag{8}$$

then we have

$$\dot{\bar{x}}_i(t) = A_i^* \bar{x}_i(t) + \bar{z}_i(t)$$
 (9)

where $A_i^* = (A_i - B_i K_i)$.

Since the aggregative deviation, $z_i(t)$ is unmeasurable and unmodelled, one way to make (5) a good model is to improve the crude interaction model (6) on-line such that the errors (7) (8), are as small as possible. This can be done by minimizing the following performance index:

$$\min J_{zi} = \frac{1}{2} \int_0^\infty \left[\tilde{x}_i^t(t) Q_{zi} \tilde{x}_i(t) + \tilde{z}_i^t(t) R_{zi} \tilde{z}_i(t) \right] dt$$
(10)

subject to the constraint (9), where $Q_{zi} = D_{zi}D_{zi}^{t}$ is positive semidefinite, (A_{i}^{*}, D_{zi}) is completely observable and R_{zi} is a positive definite matrix.

The optimal solution is

$$\tilde{z}_i(t) = -L_i \tilde{x}_i(t) \tag{11}$$

$$\bar{L}_i = R_{zi}^{-1} P_{zi} \tag{12}$$

and P_{zi} is a steady state solution of an appropriate Riccati equation.

From (8) and (11), we have

$$z_i^*(t) = \hat{z}_i(t) - \tilde{L}_i \bar{x}_i(t).$$
(13)

Consider that $z_i^*(t)$ is a good estimation of the unknown deviation and accept it as a substitution of $z_i(t)$ in the design. The substitutions of (13) into (3) and (11) into (9) give the following approximate state equations:

$$\dot{x}_{i}'(t) = A_{i}x_{i}'(t) + B_{i}u_{i}(t) + \hat{z}_{i}(t) - \bar{L}_{i}\bar{x}_{i}'(t) \quad (14)$$

$$\dot{\bar{x}}_i'(t) = \tilde{A}_i \bar{x}_i'(t) \tag{15}$$

where $x'_i(t)$, $\bar{x}'_i(t)$ are approximate states and $\tilde{A}_i = (A_i^* - \tilde{L}_i)$.

Define the extended vectors, $\bar{X}_i(t)$ and $\bar{U}_i(t)$ as

$$\bar{X}_{i}^{t}(t) = [x_{i}^{\prime\prime}(t)\hat{z}_{i}^{t}(t)\bar{x}_{i}^{\prime\prime}(t)], \quad \bar{U}_{i}^{t}(t) = [u_{i}^{t}(t)v_{i}^{t}(t)]$$
(16)

then we obtain the following extended system:

$$\bar{X}_i(t) = \bar{A}_i \bar{X}_i(t) + \bar{B}_i \bar{U}_i(t)$$
(17)

where

$$\bar{A}_{i} = \begin{bmatrix} A_{i} & I_{i} & -\tilde{L}_{i} \\ 0 & A_{zi} & 0 \\ 0 & 0 & \bar{A}_{i} \end{bmatrix}, \quad \bar{B}_{i} = \begin{bmatrix} B_{i} & 0 \\ 0 & I_{i} \\ 0 & 0 \end{bmatrix}. \quad (18)$$

The control variable $u_i(t)$ and the improving signal $v_i(t)$ can thus be determined by minimizing the following performance index:

$$\min \bar{J}_{i} = \frac{1}{2} \int_{0}^{\infty} \left[\bar{X}_{i}^{\prime}(t) \bar{Q}_{i} \bar{X}_{i}(t) + \bar{U}_{i}^{\prime}(t) \bar{R}_{i} \bar{U}_{i}(t) \right] dt$$
(19)

subject to the constraint (17).

Since the pair (\bar{A}_i, \bar{B}_i) is partially controllable and is stabilizable, we can choose

$$\bar{Q}_{i} = \begin{bmatrix} Q_{i} & 0 & 0\\ 0 & Q_{ui} & 0\\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{R}_{i} = \begin{bmatrix} R_{i} & 0\\ 0 & R_{vi} \end{bmatrix}$$
(20)

where Q_i , R_i are chosen the same as those in (4), and Q_{ui} , R_{vi} are respectively positive semidefinite and positive definite matrices such that $\bar{Q}_i = \bar{D}_i \bar{D}_i^t$ is positive semidefinite, (\bar{A}_i, \bar{D}_i) is detectable and \bar{R}_i is a positive definite matrix.

The solution of (19) is

where

$$U_i(t) = -\tilde{G}_i \tilde{X}_i(t)$$

$$\bar{G}_{i} = \bar{R}_{i}^{-1} \bar{B}_{i}^{t} \bar{P}_{i} \triangleq \begin{bmatrix} G_{i1} & G_{i2} & G_{i3} \\ H_{i1} & H_{i2} & H_{i3} \end{bmatrix}$$
(22)

and \bar{P}_i is a steady state solution of an appropriate Riccati equation.

Though (21) is only a result of approximate state equations, it can still be accepted as the desired control and improving laws. From (21) and (22), $u_i(t)$ and $v_i(t)$ can be described by

$$u_i(t) = -G_{i1}x_i(t) - G_{i2}\hat{z}_i(t) - G_{i3}\tilde{x}_i(t) \quad (23)$$

$$v_i(t) = -H_{i1}x_i(t) - H_{i2}\hat{z}_i(t) - H_{i3}\bar{x}_i(t).$$
 (24)

The block diagram of the proposed decentralized controller is depicted in Fig. 1.



FIG. 1. The decentralized controller.

(21)

4. STABILITY

For the sake of incorporating structural perturbations during the operation period, the on-off connective parameter $e_{ij}(t)$ is introduced (Šiljak, 1973). The state equation (3) can then be rewritten as

$$\dot{x}_{i}(t) = A_{i}x_{i}(t) + B_{i}u_{i}(t) + \sum_{j=1}^{s} e_{ij}(t)g_{ij}(t, x_{j})$$
$$i = 1, 2, \dots, s \quad (25)$$

where $g_{ii}(t, x_i)$ represents the nonlinearities of the *i*th subsystem and is assumed to satisfy (2) where the corresponding Lipschitz constant (Vidyasagar, 1978) is denoted by α_{ii} .

The sufficient condition for the asymptotic stability of the decentralized control system is stated by the following theorem.

Theorem 1. The nonlinear interconnected system when controlled by the decentralized controllers is asymptotically stable under any structural perturbations if the following sufficient condition is satisfied;

$$\max_{i} \lambda_{\mathcal{M}}(\beta_{i}) + 2\sqrt{2}s \max_{i} \lambda_{\mathcal{M}}(\bar{P}_{i}) \max_{i,j} \alpha_{ij} < \min_{i} \lambda_{m}(w_{i}) \quad (26)$$

where

$$\beta_i = \mathcal{A}_{i2}^t \bar{P}_i + \bar{P}_i \mathcal{A}_{i2}$$

$$\alpha_{ij} \text{ is the same as that in (2)}$$

$$w_i = -(\mathcal{A}_{i1}^t \bar{P}_i + \bar{P}_i \mathcal{A}_{i1})$$

$$\lambda_i(x_i) = \text{the minimum eigenvalue of the arguments}$$

 $\lambda_m(\cdot)$ = the minimum eigenvalue of the argument

 $\lambda_{\mathcal{M}}(\cdot)$ = the maximum eigenvalue of the argument.

Proof. When the system described by (25) is controlled by (23) and (24) and a simplified notation is introduced, we have

$$\dot{X}_i(t) = \mathscr{A}_i X_i(t) + \mathscr{B}_i \mathscr{G}_i(t)$$
(27)

where

$$\begin{aligned} X_{i}^{t}(t) &= \left[x_{i}^{t}(t) \hat{z}_{i}^{t}(t) \bar{x}_{i}^{t}(t) \right] \\ \mathscr{A}_{i} &= \begin{bmatrix} A_{i} - B_{i} G_{i1} & -B_{i} G_{i2} & -B_{i} G_{i3} \\ -H_{i1} & A_{zi} - H_{i2} & -H_{i3} \\ 0 & -I_{i} & A_{i} - B_{i} K_{i} \end{bmatrix} \\ \mathscr{B}_{i} &= \begin{bmatrix} I_{i} & 0 & I_{i} \end{bmatrix}^{t} \end{aligned}$$

and $\mathscr{G}_i(t) = \sum_{j=1}^s e_{ij}(t)g_{ij}(t, x_j).$

Choose the Lyapunov function V[X(t)] as

$$V[X(t)] = \sum_{i=1}^{s} V_i[X_i(t)] = \sum_{i=1}^{s} X_i^t(t) \bar{P}_i X_i(t) \quad (28)$$

where \bar{P}_i is the same as that in (22) and is symmetric, positive definite.

The sufficient condition for this system to be asymptotically stable is $\dot{V}[X(t)] < 0$. On using (27), we obtain

$$\dot{V}[X(t)] = \sum_{i=1}^{s} \left[\dot{X}_{i}^{t}(t) \bar{P}_{i} X_{i}(t) + X_{i}^{t}(t) \bar{P}_{i} \dot{X}_{i}(t) \right]$$

$$= \sum_{i=1}^{s} \left[X_{i}^{t}(t) (\mathcal{A}_{i}^{t} \bar{P}_{i} + \bar{P}_{i} \mathcal{A}_{i}) X_{i}(t) + 2 X_{i}^{t}(t) \bar{P}_{i} \mathcal{B}_{i} \mathcal{G}_{i}(t) \right]$$

$$< 0. \qquad (29)$$

Let $\mathcal{A}_i = \mathcal{A}_{i1} + \mathcal{A}_{i2}$, where

$$\begin{split} \mathcal{A}_{i1} &= \bar{A}_i - \bar{B}_i \bar{G}_i \\ &= \begin{bmatrix} A_i - B_i G_{i1} & I_i - B_i G_{i2} & -\bar{L}_i - B_i G_{i3} \\ -H_{i1} & A_{zi} - H_{i2} & -H_{i3} \\ 0 & 0 & \bar{A}_i \end{bmatrix} \\ \mathcal{A}_{i2} &= \begin{bmatrix} 0 & -I_i & \bar{L}_i \\ 0 & 0 & 0 \\ 0 & -I_i & \bar{L}_i \end{bmatrix}. \end{split}$$

From the substitution of \mathcal{A}_i by $\mathcal{A}_{i1} + \mathcal{A}_{i2}$, (28) becomes

$$\sum_{i=1}^{3} \left[X_{i}^{t}(t) (\mathscr{A}_{i1}^{t} \bar{P}_{i} + \bar{P}_{i} \mathscr{A}_{i1}) X_{i}(t) + X_{i}^{t}(t) (\mathscr{A}_{i2}^{t} \bar{P}_{i} + \bar{P}_{i} \mathscr{A}_{i2}) X_{i}(t) + 2 X_{i}^{t}(t) \bar{P}_{i} \mathscr{B}_{i} \mathscr{G}_{i}(t) \right] < 0.$$
(30)

Using \tilde{P}_i , the solution of the Riccati equation, we have

$$\sum_{i=1}^{s} X_{i}^{i}(t) (\mathcal{A}_{i1}^{t} \bar{P}_{i} + \bar{P}_{i} \mathcal{A}_{i1}) X_{i}(t) = \sum_{i=1}^{s} X_{i}^{i}(t) (-w_{i}) X_{i}(t)$$
(31)

where $w_i = -(\mathscr{A}_{i1}^t \bar{P}_i + \bar{P}_i \mathscr{A}_{i1})$ is a positive definite, symmetric matrix. Then

$$\sum_{i=1}^{s} \left[X_{i}^{t}(t) (\mathscr{A}_{i2}^{t} \tilde{P}_{i} + \tilde{P}_{i} \mathscr{A}_{i2}) X_{i}(t) + 2 X_{i}^{t}(t) \tilde{P}_{i} \beta_{i} \mathscr{G}_{i}(t) \right]$$
$$< \sum_{i=1}^{s} X_{i}^{t}(t) (w_{i}) X_{i}(t). \quad (32)$$

Since

$$\sum_{i=1}^{s} X_{i}^{t}(t)(w_{i})X_{i}(t) \geq \min_{i} \lambda_{m}(w_{i})\sum_{i=1}^{s} ||X_{i}(t)||^{2}$$
$$\sum_{i=1}^{s} X_{i}^{t}(t)(\mathscr{A}_{i2}^{t}\bar{P}_{i} + \bar{P}_{i}\mathscr{A}_{i2})X_{i}(t)$$
$$\leq \max_{i} \lambda_{M}(\beta_{i})\sum_{i=1}^{s} ||X_{i}(t)||^{2}$$

where $\beta_i = (\mathscr{A}_{i2}^t \tilde{P}_i + \tilde{P}_i \mathscr{A}_{i2})$, and

$$\sum_{i=1}^{s} 2X_{i}^{t}(t)\bar{P}_{i}\mathfrak{B}_{i}\mathfrak{G}_{i}(t)$$

$$\leq \sum_{i=1}^{s} 2\sqrt{2} \lambda_{M}(\bar{P}_{i}) \sum_{j=1}^{s} ||X_{i}(t)|| \alpha_{ij} ||X_{j}(t)||$$

$$\leq 2\sqrt{2} s \max_{i} \lambda_{M}(\bar{P}_{i}) \max_{i,j} \alpha_{ij} \sum_{i=1}^{s} ||X_{i}(t)||^{2}.$$



FIG. 2. Schematic representation of the model.

The result is

$$\max_{i} \lambda_{\mathcal{M}}(\beta_{i}) + 2\sqrt{2} s \max_{i} \lambda_{\mathcal{M}}(\bar{P}_{i}) \max_{i,j} \alpha_{ij}$$

$$< \min_{i} \lambda_{m}(w_{i}).$$

5. EXAMPLE

Consider a two-link, θ -r manipulator which is assumed to perform horizontally. Figure 2 shows the model schematically.

The mass of the rotary link is assumed at its center of mass, m_1 at constant distance r_1 from the center of rotation. The prismatic arm and load are modelled as a mass m_2 at distance r. By using the Lagrangian (Snyder, 1985), the model of this manipulator can be described by

$$T(t) = m_1 r_1^2 \ddot{\theta}(t) + m_2 r^2(t) \ddot{\theta}(t) + 2m_2 r(t) \dot{r}(t) \dot{\theta}(t) + B_{\theta} \dot{\theta}(t) \quad (33) F(t) = m_2 [\ddot{r}(t) - r(t) \dot{\theta}^2(t)] + B_r \dot{r}(t) \quad (34)$$

where T(t) is the input torque, F(t) is the input force, $\theta(t)$ is the rotary variable, r(t) is the linear variable, B_{θ} and B_r are the rotary and linear viscous friction coefficients, respectively. Define $x_1^t(t) = [x_{11}(t)x_{12}(t)] = [\theta(t)\dot{\theta}(t)], x_2^t(t) =$ $[x_{21}(t)x_{22}(t)] = [r(t)\dot{r}(t)]$ and $[u_1(t)u_2(t)] =$ [T(t)F(t)]. The nonlinear system (33) and (34) can be linearized about the operating point to obtain the following state equations

$$\dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t) + z_1(t)$$
(35)

$$\dot{x}_2(t) = A_2 x_2(t) + B_2 u_2(t) + z_2(t)$$
(36)

where A_i , B_i are the parameters of the resulting linearized model and $z_1(t)$, $z_2(t)$ are aggregative deviation vectors containing the interactions and nonlinearities.

Assume that the manipulator performs a return motion from an initial condition. Our problem is to determine the decentralized control laws for $u_1(t)$ and $u_2(t)$ such that the robot can perform close to the decomposed optimal response with respect to the following optimization problem:

$$\min J_i = \frac{1}{2} \int_0^\infty \left[x_i'(t) Q_i x_i(t) + u_i'(t) R_i u_i(t) \right] dt \quad (37)$$



subject to (35), (36) with $z_1(t)$, $z_2(t)$ being neglected, where $Q_i = D_i D_i^t$ is positive semidefinite, (A_i, D_i) is completely observable and R_i is positive definite.

The numerical values are set as $B_{\theta} = 2.2 \text{ Nt}$ m s rad⁻¹, $B_r = 2.8$ Nt s m⁻¹, $r_1 = 0.2$ m, $m_1 =$ 10 kg, $m_2 = 8$ kg, and the initial condition is $[x_1'(0)x_2'(0)] = [1.0, 0, 0.6, 0]$. The responses of the system are simulated on a digital computer for three different cases. The first case is the decomposed optimal control of the manipulator with single joint operation where no interactions and no nonlinearities are considered. The second one simply uses the local controllers without any compensation for the interactions and nonlinearities. The third case uses the proposed decentralized controllers for the control of the manipulator with unmodelled interactions and nonlinearities between the links. In order to compare the results, the same weighting matrices, $Q_i = 10I_2$ and $R_i = I_1$ are applied to the performance indices, respectively. All of the state trajectories are plotted in Figs 3-6.

The performance indices associated with each control method are $J_{non-deviation} = J_{n1} + J_{n2} = 40.203$, $J_{local} = J_{l1} + J_{l2} = 111.36$ and $J_{decentralized} = J_{d1} + J_{d2} = 50.937$, respectively. The results have shown the attraction of the proposed decentralized controller in the control of interconnected systems.



267

AUTO 26:2-F



FIG. 5. State trajectories of linear positions.



FIG. 6. State trajectories of linear velocities.

6. CONCLUSION

In this paper we have proposed a completely decentralized control scheme for the control of interconnected systems with unmodelled interaction and nonlinearity. The aggregative deviations are tracked by using a model following technique with on-line improvement. The sufficient condition for which the global system when controlled by the decentralized controllers is asymptotically stable even under structural perturbations in the sense of Šiljak has been established. Since the controller is completely decentralized, the subsystem autonomy results in better reliability and simplifies the design and implementation effectively. The decentralized control technique has been applied to a two-link, $\theta - r$ manipulator which is a highly nonlinear, coupled

system and responses by three different control methods are compared.

REFERENCES

- Bundell, G. A. (1985). Robust discrete decentralized model reference adaptive control. Proc. 24th Conf. on Decision and Control, pp. 1486-1488.
- Davison, E. J. (1978). Decentralized robust control of unknown systems using tuning regulators. *IEEE Trans. Aut. Control* AC-23, 276-289.
- Gavel, D. T. and D. D. Šiljak (1985). High gain adaptive decentralized control. *Proc. Am. Control. Conf.*, pp. 568-573.
- Hassan, M. F. and M. G. Singh (1978a). A hierarchical structure for computing near optimal decentralized control. *IEEE Trans. Syst. Man & Cybern.* SMC-8, 575-579.
- Hassan, M. F. and M. G. Singh (1978b). Robust decentralized controller for linear-interconnected dynamical systems. *Proc. IEE*, 125, 429-432.
 Hassan, M. F. and M. G. Singh (1980). Decentralized
- Hassan, M. F. and M. G. Singh (1980). Decentralized controller with online interaction trajectory improvement. *Proc. IEE*, **127**, 142–148.
- Hassan, M. F., M. G. Singh and A. Titli (1979). A near optimal decentralized controller with a pre-specified degree of stability. *Automatica*, **15**, 483-488.
- Hmamed, A. and L. Radouane (1982). On decentralized nonlinear feedback stabilization of interconnected systems. *Proc. 3rd IFAC Symp. on Software for Comuter Control*, pp. 345-350.
- Ioannou, P. A. (1986). Decentralized adaptive control of interconnected systems. *IEEE Trans. Aut. Control* AC-31, 291-298.
- Ioannou, P. A. and P. Kokotovic (1985). Decentralized adaptive control of interconnected systems with reducedorder models. *Automatica*, 21, 401-412.
- Khalil, H. and A. Saberi (1982). Decentralized stabilization of nonlinear interconnected systems using high-gain feedback. *IEEE Trans. Aut. Control* AC-27, 265-268.
- Saberi, A. (1988). On optimality of decentralized control for a class of nonlinear interconnected systems. *Automatica*, 24, 101-104.
- Šiljak, D. D. (1973). On stability of large scale systems under structural perturbations. *IEEE Trans. Syst. Man & Cybern.*, SMC-3, 415-417.
- Šiljak, D. D. and M. K. Sundareshan (1976). A multi-level optimization of large-scale dynamic systems. *IEEE Trans. Aut. Control* AC-21, 79-84.
- Singh, M. G., M. F. Hassan and A. Titli (1979). A feedback solution for large interconnected dynamical systems using the prediction principle. *IEEE Trans. Syst. Man & Cybern.*, SMC-6, 233-239.
- Cybern., SMC-6, 233-239. Snyder, W. E. (1985). Industrial Robots, Computer Interfacing and Control, pp. 202-205. Prentice-Hall, Englewood Cliffs, NJ.
- Sundareshan, M. K. (1977). Generation of multi-level control and estimation schemes for large scale systems: a perturbation approach. *IEEE Trans. Syst. Man & Cybern.*, SMC-7, 144-152.
- Vidyasagar, M. (1978). Nonlineor Systems Analysis, pp. 79-80. Prentice-Hall, Englewood Cliffs, New Jersey.