Robust Fuzzy Model-Following Control of Robot Manipulators

Chih-Hsin Tsai, Chi-Hsiang Wang, and Wei-Song Lin

***Abstract*—A robust fuzzy model-following control system is proposed for the control of robot manipulators. The application field to n-link robot manipulators with torque disturbance and measurement noise is addressed. The control objective is obtained by tailoring a nominal adaptation process of parameters to implement appropriate function approximation and facilitating a self-tuning mechanism on the consequent membership functions to overcome the equivalent uncertainty. The major differences comparing with previous adaptive fuzzy control approaches are that a novel fuzzy system with self-tuning mechanism provided robust property and the rulebase in the form of "IF *situation* THEN *the control input*" rather than "IF *situation* THEN *the value of some nonlinear function of the robot*" are introduced. The proposed multilayer fuzzy logic controller can improve both transient and stability margins without *a priori* knowledge about the dynamic model or parameters of the robotic system. Using the Lyapunov stability method, the uniform ultimate boundedness of tracking error has been proved. The performance is demonstrated by simulating the control of a two-link robot in various situations.**

***Index Terms*—Fuzzy control, model following control, robot.**

I. Introduction

MODEL following control of robot manipulators has been widely studied and shown to have many practical applications. In order to guarantee the state error between the reference model and the controlled plant approaches asymptotically to zero, several methods have been investigated. The variable structure model-following control (VSMFC) has the advantage of insensitivity to parameter variations and disturbances and, hence, precise system model is generally not required [1]–[3]. However, it obtains adaptive control laws that are discontinuous on an attractive hyperplane in the error state space. Besides, most of the VSMFC designs are based on the restrictive assumption that the ranges of parameter variations are known and the resulted control efforts are excessive. Adaptive model-following control (AMFC) was another choice to overcome the difficulties found in robot control [4]–[8]. But the strict positive realness is invariably required, and real-time parameter identification, which is sensitive to numerical precision and measurement noise, creates other problems [9].

In this paper, a robust multilayer fuzzy controller (RMFC) for the model-following control of robot manipulators is proposed. In the RMFC, the parameters of the controlled plant are not assumed to be linear as in standard adaptive control techniques. No prior knowledge about the parameters and system matrix including its size is required. The scheme of the RMFC has been inspired from the previous works [10]–[16] and we extend the application field to n-link robot manipulators with torque disturbance and measurement noise. The methods proposed in [13]–[16] use the fuzzy basis function expansion (FBFE) proposed by Wang [11] to represent the unknown nonlinearity of plants. These papers take the advantage of fuzzy basis function and a stable parameter adaptation scheme is then determined by Lyapunov method. Despite the success of stable adaptive fuzzy control based on fuzzy basis functions allows the inclusion of *a priori* knowledge about the plant. The fuzzy rule base considered in these papers consists of "IF *situation* THEN *the value of some nonlinear function of plant*," which are inherent of the plant dynamic rather than the familiar expressions of human expert knowledge. As an improvement, this paper first develops a novel multilayer fuzzy system with fuzzy rules in the form of "IF *situation* THEN *the control input*." This form of rules preserves the superior advantage of the conventional fuzzy systems that has direct translation from linguistic rules provided by a human expert. This architecture enhances adaptive fuzzy controller such that all adjustable parameters are meaningful and can be incorporated with and directly extracted out linguistic rules in a more reasonable way. Besides, due to a self-tuning mechanism on the consequent membership functions, the proposed scheme can specifically deal with the measurement noise and the effect of uncertain modeling error and considerably shrink the tracking error residual set. This gives the RMFC system some degree of adaptability in the sense that the self-tuning mechanism can reflect to variant disturbances and plant uncertainties. If the nominal model of the robot manipulator is available, the *a priori* model knowledge can be utilized to train the RMFC off-line to achieve faster convergence of output tracking error. Using Lyapunov method, it is shown that the proposed parameter adaptation scheme has some degree of robustness and the output tracking error can converge to a residual set ultimately.

The remainder of this paper is organized as follows. In Section II, a brief outline of the model-following control of n-link robot manipulators is presented. In Section III, the RMFC in which several fuzzy systems with embedded rule credit assignments and self-tuning mechanism on the consequent membership functions is proposed. In Section IV, a robust parameter adaptation scheme for the RMFC is proposed and stability analysis by using Lyapunov method is shown. In Section V, the performance of the RMFC systems is examined by the control of a two-link robot in variant situations. Section VI is the conclusion.

II. System Description and Control

A. Notation and Definition

An n-component column vector f is denoted as $f = [f_i]_n \equiv [f_1, \cdots, f_n]^T \in R^n$; an $m \times n$ matrix A is denoted as $A =$

Manuscript received September 29, 1999; revised January 21, 2000. This work was supported by the National Science Council of Taiwan, R.O.C. under Contract NSC87-2212-E002-024.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

Publisher Item Identifier S 1063-6706(00)06594-2.

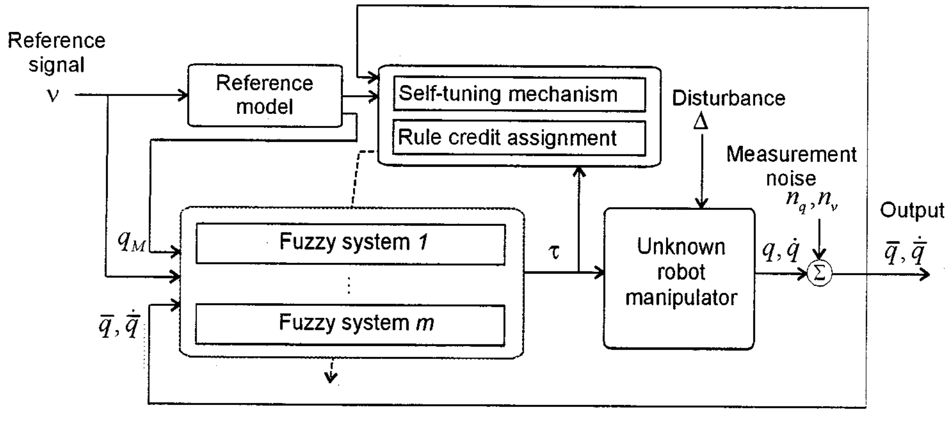

Fig. 1. Configuration of a robust multilayer fuzzy model-following control system.

$[a_{ij}]_{m\times n} \in R^{m\times n}$; and a diagonal matrix B is denoted as $B =$ Block diag $[b_i]_{n\times n} =$ Block diag $[b_1, \cdots, b_n] \in R^{n\times n}$. Given a matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the operation of taking the minimum and maximum eigenvalues of A, respectively.

B. Control Problem

The dynamics of an n-degree-of-freedom rigid manipulator can be described in general form as follows: [17]

$$\tau + \Delta(q, \dot{q}, t) = H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g'(q) \tag{1}$$

where τ is the control torque vector, Δ is the actuator input noise, $H(q)$ is an $n \times n$ inertia matrix dependent on position vector $q, \dot{q}$ and $\ddot{q}$ are the velocity and acceleration vectors, respectively, $C(q, \dot{q})\dot{q}$ is the Coriolis and centripetal force vector, and $g'(q)$ is the gravity force vector. Let $H'(q) = [h'_{ij}(q)]_{n\times n} = H^{-1}(q), d(q, \dot{q}, t) = [d_i(q, \dot{q}, t)]_n = H^{-1}(q)\Delta(q, \dot{q}, t)$, (1) can be written through some mathematical operations as

$$\ddot{q} = f(q, \dot{q}) + G(q)\tau + z(q, \dot{q}, t) \tag{2}$$

where

$$\begin{aligned} f(q, \dot{q}) &= H'(q)(-C(q, \dot{q})\dot{q} - g'(q)) \\ G(q) &= \text{Block diag } [g_i(q)]_{n\times n} \\ &= \text{Block diag } [h'_{ij}(k)]_{n\times n} \quad \text{and} \\ z(q, \dot{q}, t) &\equiv [z_i(q, \dot{q}, t)]_n = \left[\sum_{j=1, j\neq i}^{n} h'_{ij}(q)\tau_j + d_i(q, \dot{q}, t) \right]_n \end{aligned}$$

denotes the combination of unknown interaction between the subsystems and the time-varying disturbance term. Considering the measurement noise, we have $\overline{q} = q + n_p, \dot{\overline{q}} = \dot{q} + n_v$ where noise vectors n_p, n_v, and Δ are supposed to be bounded random perturbations and we assume that $n_p \in C^2$. f_i and $g_i, i = 1, \cdots, n$ are assumed to be smooth functions. The problem of control focuses on the uncertain robot with its structure being known but without the parameters or functions of the mathematical model. Let $q_M \in R^n$ and $v \in R^n$ denote the reference output and input, respectively. The control strategy is to decouple the joint dynamics and to conduct (2) asymptotically following a linear reference model of the following form:

$$\ddot{q}_M = \Lambda_1 q_M + \Lambda_2 \dot{q}_M + v \tag{3}$$

where $\Lambda_1 =$ Block diag $[\alpha_{1i}]_{n\times n}$ and $\Lambda_2 =$ Block diag $[\alpha_{2i}]_{n\times n}$, in the presence of bounded disturbance and measurement noise. The constants α_{1i} and α_{2i} are selected such that an asymptotically stable reference model with desirable properties is obtained. As shown in Fig. 1, the model-following control system adopted comprises m fuzzy systems with rule credit assignment and self-tuning mechanism on the consequent membership functions. Its basic components (fuzzy systems, rule credit assignment, and self-tuning mechanism) are explained in more detail in Section III.

III. Robust Multilayer Fuzzy Controller

Considering the request of numerical input and output of the fuzzy system, a particular class of fuzzy system with the singleton fuzzified, algebraic product T-norm, the sup-star compositional operator [11], and the local mean-of-maximum [18] method is used. Fig. 2 shows the ith fuzzy system of the RMFC.

1) Fuzzy rule base: Let

$$\begin{aligned} x &= [x_i]_{2n} = [q^T, \dot{q}^T]^T, \\ \overline{x} &= [\overline{x}_i]_{2n} = [\overline{q}^T, \dot{\overline{q}}^T]^T \in R^{2n} \end{aligned}$$

an n-degree-of-freedom rigid manipulator can be controlled by the following $N + 1$ linguistic rules

$$\begin{aligned} R^j\text{:}\quad & \text{IF } \overline{x}_1 \text{ is } A_i^j \; AND \; \cdots \; AND \; \overline{x}_{2n} \text{ is } A_{2n}^j \\ & \text{THEN } \tau_1 \; is \; B_1^j \; AND \; \cdots \; AND \; \tau_n \; is \; B_n^j \\ & \text{for} \quad j = 1, \cdots, N+1. \end{aligned}$$

The fuzzy sets A_k^j and B_i^j are linguistic terms characterized by the fuzzy membership functions

$$\mu_{A_k^j}(x_k) = \exp(-(x_k - m_k^j)^2/a_k^j) \tag{4}$$

and

$$\mu_{B_i^j}(\tau_i) = \begin{cases} (1 + ((c_i^j - \tau)/a_{Li})^2)^{-1}, & \text{if } \tau_i \le c_i^j \\ (1 + ((\tau_i - c_i^j)/a_{Ri})^2)^{-1}, & \text{if } \tau_i > c_i^j \end{cases} \tag{5}$$

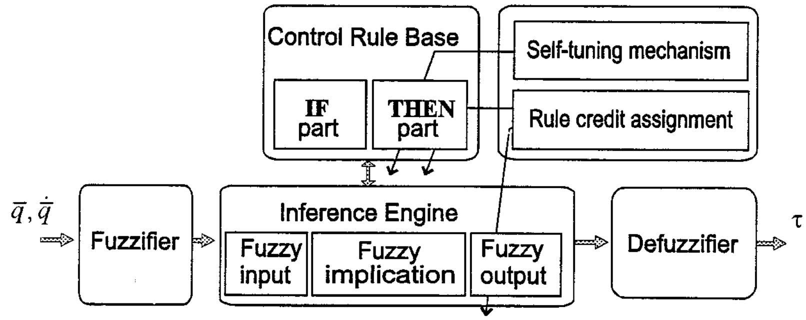

Fig. 2. Diagrammatic representation of the proposed fuzzy system with rule credit assignment of self-tuning mechanism.

where $\{a_k^j, m_k^j\}$ and $\{a_{Li}, a_{Ri}, c_i^j\}$ are referred to the premise and consequence parameters, respectively.

2) Rule credit assignment: The basic idea of the rule credit assignment is to reward good rules by increasing the confidence of the consequent fuzzy sets and the recommendation fuzzy output of this rule. Denote $\omega_i^j > 1$ (or $\omega_i^j < 1$) as a reward (or a punishment) offered to the jth rule in the ith knowledge rule base, then the consequent membership function (5) can be reshaped into

$$\mu_{\tilde{B}_i^j}(\tau_i) = \begin{cases} (1 + (\omega_i^j(c_i^j - \tau_i)/a_{Li})^2)^{-1}, & \text{if } \tau_i \le c_i^j \\ (1 + (\omega_i^j(\tau_i - c_i^j)/a_{Ri})^2)^{-1}, & \text{if } \tau_i > c_i^j \end{cases} \tag{6}$$

and the recommendation fuzzy output of each rule is determined in singleton form as follows:

$$\omega_i^j I(\mu^j, \mu_{\tilde{B}_i^j}) = \begin{cases} \omega_i^j \mu^j, & \text{for } u_i = \tilde{c}_i^j \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where "·" is the multiplication operation, $\mu^j = \mu_{A_1^j}(\bar{x}_1)\mu_{A_2^j}(\bar{x}_2)\cdots\mu_{A_{2n}^j}(\bar{x}_{2n})$ denotes the matching degree, respectively, I is the implication function and $\tilde{c}_i^j$ denotes the location of the singleton implication fuzzy set defined as [18]

$$\tilde{c}_i^j = \text{the centroid of the set } \{\tau_i\colon \mu_{\tilde{B}_i^j}(\tau_i) \ge \mu^j(\bar{x})\}. \tag{8}$$

Since no direct error measurement of the multilayer fuzzy system is possible in the case of control problem, only error information of the plant and desired trajectory can be used for the choice of rule credit ω_i^j. In this paper, our approach is to treat the entire problem in the context of Lyapunov-based adaptive systems theory as shown in the next section.

3) Self-tuning mechanism: Referring to Fig. 3 and by (6), for the matching degree μ^j at the left intersection point τ_P we have

$$\mu^j = (1 + (\omega_i^j(c_i^j - \tau_P)/a_{Li})^2)^{-1} \tag{9}$$

or

$$\tau_P = c_i^j - \frac{a_{Li}}{\omega_i^j}\sqrt{\frac{1}{\mu^j} - 1}. \tag{10}$$

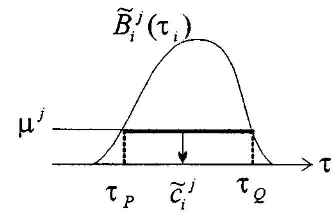

Fig. 3. The definition of the centroid of the line segment at height intercepted by the fuzzy membership function.

Similarly, at the right intersection point τ_Q, we have the following result:

$$\tau_Q = c_i^j + \frac{a_{Ri}}{\omega_i^j}\sqrt{\frac{1}{\mu^j} - 1}. \tag{11}$$

Thus, by definition (8), we have

$$\begin{aligned} \tilde{c}_i^j &= (\tau_P + \tau_Q)/2 \\ &= c_i^j - \frac{a_{LRi}}{\omega_i^j}\sqrt{\frac{1}{\mu^j} - 1} \end{aligned} \tag{12}$$

where $a_{LRi} = (a_{Li} - a_{Ri})/2$. Physically, the parameter a_{LRi} represents the difference between the left and right spreads of the consequent membership functions. In the conventional fuzzy logic control systems, a_{Li} is set to be equivalent to a_{Ri} or the consequent membership is just in singleton form [11]. In this paper, this term is employed as a robust control component and a robust adaptive law for it is proposed in the next section.

4) Analytical formulation of the multi-layer fuzzy system: Using the center average defuzzification, the output response of the fuzzy controller is

$$\tau_i(t) = \frac{\sum_{j=1}^{N+1} \omega_i^j \cdot \mu^j \cdot \tilde{c}_i^j}{\sum_{j=1}^{N+1} \omega_i^j \cdot \mu^j}. \tag{13}$$

In the rule base, let the $(N+1)$th rule be chosen to be of Takagi–Sugeno type and its consequent fuzzy set vector B_i^{N+1} be singleton with support represented as the form of the synthesis input

$$c' = [c_i']_n = \Lambda_1 \bar{q} + \Lambda_2 \dot{\bar{q}} + v. \tag{14}$$

The curvature control parameter a_k^{N+1} of its antecedent membership function is assumed to approach to infinity so that this rule will be fired whatever $\overline{x}$ is. The credit assignment takes place in rules $R^j, j = 1, \cdots, N$ but assigned to be one for R^{N+1}. Accordingly, using (12) and (14), the analytical formulation of the multilayer fuzzy system in (13) resolve into

$$\tau = \hat{G}^{-1}(-\hat{f} + c' - a_{LR}\phi) \tag{15}$$

where, $\hat{G} = \text{Block diag } [\omega_i^T \mu']_{n\times n}, \omega_i$ and μ' are $(N+1)\times 1$ column vectors composed of ω_i^j and μ^j, $\hat{f} = [\beta_i^T \mu]_n$, β_i and μ are $N \times 1$ column vectors composed of $\omega_i^j c_i^j$ and $\mu^j(k), a_{LR} \in R^n$ and

$$\mu_{LR} = \sum_{j=1}^{N} \mu^j \sqrt{\frac{1}{\mu^j} - 1} \in R.$$

In viewing (15), the control law is not well defined when $\omega_i = 0$. To avoid this problem, we project $\theta = [\beta_1^T, \omega_1^T, \cdots, \beta_n^T, \omega_n^T]^T$ inside an estimated feasible region $M_\theta = \{\theta: |\beta_i| \le \beta_{i,\max}, 0 < \omega_i \le \omega_{i,\max}, i = 1, \cdots, n\}$ by properly adapting the parameter values.

IV. Learning Algorithm and Performance

Let $\theta^* = [\beta_1^{*T}, \omega_1^{*T}, \cdots, \beta_n^{*T}, \omega_n^{*T}]^T$, where β_i^* and ω_i^* are the parameters of the best function approximation

$$\begin{aligned} \beta_i^* &= \arg \min_{\beta_i \in M_\theta} [\sup |f_i - \beta_i^T \mu|] \\ \omega_i^* &= \arg \min_{\omega_i \in M_\theta} [\sup |g_i - \omega_i^T \mu'|]. \end{aligned} \tag{16}$$

Then (2) can be rewritten in terms of the measured output $\overline{q}(k+1)$ and expressed as

$$\begin{aligned} \ddot{\overline{q}} &= f(x) + G(x)\tau + z(x,u,t) + \ddot{n}_p \\ &= \hat{f}^*(\overline{x}) + \zeta_f + (\hat{G}^*(\overline{x}) + \zeta_g)\tau + z(x,u,t) + \ddot{n}_p \end{aligned} \tag{17}$$

where

$$\begin{aligned} \hat{f}^*(\overline{x}) &= [\beta_i^{*T}\mu(\overline{x})]_n \\ \hat{G}^*(\overline{x} &= \text{Block diag } [\omega_i^{*T}\mu'(\overline{x})]_{n\times n} \end{aligned}$$

and

$$\begin{aligned} \zeta_f &= f(x) - \beta_i^{*T}\mu(x)]_n - [\beta_i^{*T}\Delta\mu(x,n_x)]_n \\ \zeta_g &= \text{Block diag } [g_i(x) - \omega_i^{*T}\mu'(x) - \omega_i^{*T}\Delta\mu'(x,n_x)]_{n\times n} \end{aligned} \tag{18}$$

with

$$\begin{aligned} \Delta\mu(x,n_x) &= \mu(\overline{x}) - \mu 9x) \\ \Delta\mu'(x,n_x) &= \mu'(\overline{x}) - \mu'(x) \end{aligned} \tag{19}$$

being measures of sensitivity of the nominal model $z(x,\tau,t) \equiv n_x \equiv n_p \equiv 0$ with respect to the measurement noise n_x. By (17), subtracting $\hat{G}\tau$ and adding $-\hat{f} + c' - a_{LR}\phi$ to the right-hand side of (19), we obtain the error equation

$$\ddot{\overline{q}} - \ddot{q}_M = -\Lambda_1 q_M - \Lambda_2 \dot{q}_M - v - \tilde{f}(\overline{x}) - \tilde{G}(\overline{x})\tau - a_{LR}\phi + \zeta \tag{20}$$

or

$$\dot{e} = Ae - BW^T\tilde{\theta} - Ba_{LR}\phi + B\zeta \tag{21}$$

where

$$\begin{aligned} \zeta &= \zeta_f + \zeta_g\tau + z(x,\tau,t) + \ddot{n}_p \in R^n \\ \tilde{f}(\overline{x}) &= [(\beta_i^T(k+1) - \beta_i^{*T}\mu(\overline{x})]_n \\ \tilde{G}(\overline{x}) &= \text{Block diag } [(\omega_i^T - \omega_i^{*T})\mu']_{n\times n} \\ e &= [e_i^T]_n, \quad e_i = [\overline{q}_i - q_{Mi}, \dot{\overline{q}} - \dot{q}_{Mi}]^T, \\ A &= \text{Block diag } [A_i]_{2n\times 2n} \\ B &= \text{Block diag } [b_i] \in R^{2n\times n}, \end{aligned}$$

and

$$\tilde{\theta} = \theta - \theta^*$$

denotes the parameter estimation error and

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & 1 \\ -\alpha_{1i} & -\alpha_{2i} \end{bmatrix}, \quad b_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ W &= \text{Block diag } [w_i] \in R^{n(2N+1)\times n} \\ w_i &= \begin{bmatrix} \mu \\ \mu' u_i \end{bmatrix} \in R^{2N+1}. \end{aligned} \tag{22}$$

In the following paragraph, a robust tuning algorithm for θ and a_{LRi} motivated by an attempt to modify the basic steepest descent technique and provide treatment to the exogenous signals, disturbance, and approximation error term ζ is proposed.

To counteract the equivalent uncertainty, the self-tuning mechanism $a_{LR}\phi$ is employed. The parameter a_{LR} is chosen as $a_{LR}(\vartheta) = \varphi\vartheta$, where ϑ is an auxiliary adjustable parameter, φ = Block diag $[\varphi_i]_{n\times n}, \varphi_i = \tanh(b_i^T P_i e_i \phi/\varepsilon)$, and ε is a small positive constant.

Assumption 1: There exists the smallest nonnegative parameter vector $\vartheta^* \ge 0$ such that for all $x \in R^n$ and $t \in R^+$

$$|\zeta| \le \vartheta^* \phi. \tag{23}$$

Use Assumption 1 and let $M_\vartheta = \{\vartheta: |\vartheta_i| < \vartheta_{i,\max}, i = 1, \cdots, n\}$ be the bound of ϑ, M_ϑ^ε be the union of M_ϑ and its boundary layer of thickness ε_ϑ and M_θ^ε be the union of M_ϑ and its boundary layer of thickness ε_θ. The following smooth robust parameter adaptation scheme is proposed:

$$\dot{\theta}(t) = \begin{cases} 0, & \text{if } e^T P b b^T P e \le d_0^2 \\ (I - d_\theta \theta_\perp \theta_\perp^T) R^{-1} [W B^T P e - \sigma_1(\theta - \theta_0)], & \text{otherwise} \end{cases} \tag{24}$$

with d_θ = Block diag $[d_{\theta_i}]_{n(2N+1)\times n(2N+1)}$

$$d_\theta = \begin{cases} 0, & \text{if } \theta_\perp^T [W B^T P e - \sigma_1(\theta - \theta_0)] \le 0 \\ \min[1, \text{ dist } (\theta, M_\theta)/\varepsilon_\theta], & \text{otherwise} \end{cases} \tag{25}$$

and

$$\dot{\vartheta}(t) = \begin{cases} 0, & \text{if } e^T P b b^T P e \le d_0^2 \\ (I - d_\vartheta) R_\vartheta^{-1} [W' B^T P e - \sigma_2(\vartheta - \vartheta_0)], & \text{otherwise} \end{cases} \tag{26}$$

with d_ϑ = Block diag $[d_{\vartheta_i}]_{n\times n}, W'$ = Block diag $[w_i']_{n\times n}$,

$$d_{\vartheta_i} = \begin{cases} 0, & \text{if } \vartheta_i [b_i^T P_i e_i w_i' - \sigma_2(\vartheta_i - \vartheta_{i0})] \le 0 \\ \min[1, \text{ dist } (\vartheta_i, \mathbf{M}_{\vartheta_i})/\varepsilon_\vartheta], & \text{otherwise} \end{cases} \tag{27}$$

$$w_i' = \phi\varphi_i \tag{28}$$

where R = Block diag $[R_1, \cdots, R_n]$, R_ϑ = Block diag $[r_{\vartheta_i}]_n$, R_i and R_ϑ are diagonal matrix with positive diagonal elements,

$P =$ Block diag $[P_1, \cdots, P_n], P_i$ is a symmetric positive-definite matrix satisfying the Lyapunov equation $A_i^T P_i + P_i A_i = -Q_i$, with the design parameters $Q_i > 0$, and σ_1 and σ_2 are chosen small but positive constant to keep θ and ϑ from growing unbounded.

Theorem 1: Consider the nonlinear robotic system (2) with unknown but bounded f_i, h'_{ij}, d_i, n_p and n_v. Let Assumption 1 hold. Use the RMFC (15) and the parameter adaptation law (24) and (26). Then in the bounded state $(q, \dot{q}) \in \Omega = \{(q, \dot{q})$: $|(q, \dot{q})| \le \gamma\}$.

1) θ, ϑ and the control input τ are uniformly ultimately bounded.
2) Given any ρ satisfying $\rho^* < \rho \le \gamma$ where (29), shown at the bottom of the page, with $\vartheta_i^M \equiv \max\{\vartheta_i^*, \vartheta_{i0}\}$ and κ being a constant that satisfies $\kappa = e^{-(\kappa+1}$, i.e., $\kappa = 0.2785$, there exists T such that for $T \le t \le \infty$ the tracking error e converges to the residual set

$$\{e\text{: } e^T Pe \le \rho \text{ or } e^T Pbb^T Pe \le d_0^2\}. \tag{30}$$

Proof: Refer to Appendix for details.

Remark 1: In the RMFC some degree of adaptability is achieved by applying the self-tuning mechanism, $a_{LR}\phi$, to deal with the disturbance and noise.

Remark 2: In view of (29), if the design constants $\varepsilon, \sigma_1, \sigma_2, R_\vartheta, R$, and Q are appropriately chosen, tracking to a small neighborhood around $e \approx 0$ is possible.

Remark 3: The initial design of parameters θ_0 and ϑ_0 in the RMFC can be considered to be initial estimates of the best parameters, θ^* and ϑ^*, respectively. The closer θ_0 and ϑ_0 to θ^* and ϑ^* are, the smaller ρ^* becomes. This, in turn, results in better tracking.

Remark 4: Suppose *a priori* knowledge about the manipulator to be controlled is available in the form of "approximation to $f(q, \dot{q})$" and "approximation to $G(q, \dot{q})$" denoted by the terms $f^0(q, \dot{q}|$ *nominal parameters of the arm*) and G^0 ($q|$ *nominal parameters of the arm*), respectively. Then the initial parameters can be selected by using the well-known least square algorithm, etc., so that θ_0 will close to θ^* and a smaller ρ^* is achieved.

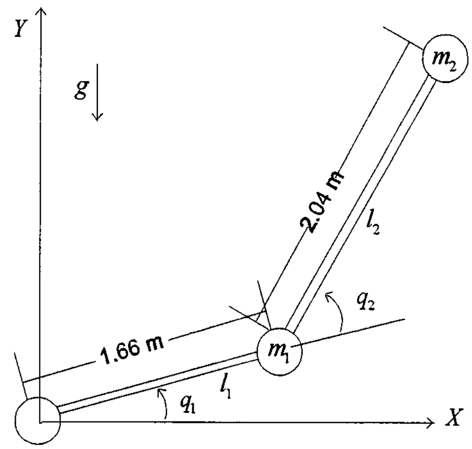

Fig. 4. A two-link planar robot.

V. SIMULATION

Consider a two joints planar manipulator as shown in Fig. 4. The equations of motion of the manipulator can be expressed in matrix form as (31), as shown at the bottom of the page where $r_1 = 0.5l_1$, $r_2 = 0.5l_2$, $c_1 \equiv \cos(q_1)$, $s_{12} \equiv \sin(q_1 + q_2)$, etc. The kinematics and inertial parameters of the manipulator are listed in Table I. The excessive ratio between m_1 and m_2 is to emphasize the load effect. The trajectory to be followed is described by (3) and the model parameters are chosen as $\Lambda_1 =$ Block diag $[-1.0, -4.0]$, and $\Lambda_2 =$ Block diag $[-1.0, -2.0]$. The driving inputs to the reference models are sinusoidal functions $v_1 = \pi \sin(0.8\pi t)$ and $v_2 = 1.5\pi \cos(\pi t)$, respectively. Fig. 5 shows the trajectories. The reference model and the plant are assumed to have the same initial states as $q_1(0) = -1.5$ rad, $q_2(0) = -1.2$ rad, $\dot{q}_1(0) = 0$ rad/sec, and $\dot{q}_2(0) = 0$ rad/s. The membership functions of states $\overline{q}_1, \dot{\overline{q}}_1, \overline{q}_2$, and $\dot{\overline{q}}_2$ (represented by generic variable x) for the qualitative statements are defined as $\{NB, NS, ZE, PS, PB\}$ where NB: $A_i(x) = \exp(-4(x+1.8)^2)$, NS: $A_i(x) = \exp(-4(x+0.8)^2)$, ZE: $A_i(x) = \exp(-4x^2)$, PS: $A_i(x) = \exp(-4(x-0.8)^2)$, PB: $A_i(x) = \exp(-4(x-1.8)^2)$. In (24) and (26), the

$$\rho^* = \frac{\sum_{i=1}^{n} [\sigma_1(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \sigma_2(\vartheta_i^* - \vartheta_{o0})^2 + 2\kappa\vartheta_i^M\varepsilon]}{\min_i \min\left\{\frac{\lambda_{\min}(Q'_i)}{\lambda_{\max}(P_i)}, \frac{\sigma_1}{\lambda_{\max}(R_i)}, \frac{\sigma_2}{\gamma_\vartheta}\right\}} \tag{29}$$

$$\begin{bmatrix} (m_1+m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2c_2 + J_1 & m_2r_2^2 + m_2r_1r_2c_2 \\ m_2r_2^2 + m_2r_1r_2c_2 & m_2r_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -m_2r_1r_2s_2\dot{q}_1(\dot{q}_1 + \dot{q}_2) \\ m_2r_1r_2s_2\dot{q}_2^2 \end{bmatrix} + \begin{bmatrix} ((m_1+m_2)l_1c_2 + m_2l_2c_{12})g \\ (m_2l_2c_{12})g \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{31}$$

TABLE I
PARAMETERS OF THE ROBOT

Parameter	Symbols	Real	Nominal
mass of link 1	m_1 (kg)	0.60	0.48
mass of link2	m_2 (kg)	7.02	6.30
inertia of link 1	J_1 (kgm^2)	4.50	4.80
inertia of link 2	J_2 (kgm^2)	4.50	5.10
length of link 1	l_1 (m)	1.66	1.60
length of link 2	l_2 (m)	2.04	2.00

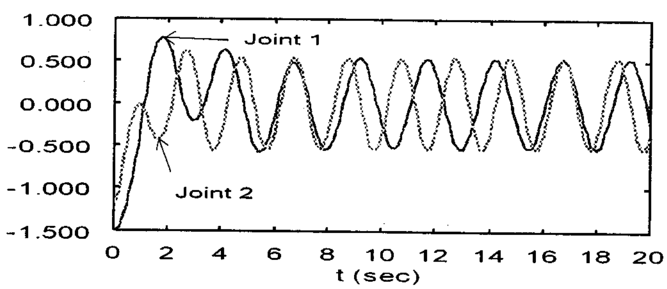

Fig. 5. Reference outputs of joint 1 and 2.

design parameters are given by $Q_1 = Q_2 = 10I_{2\times 2}, R_1 =$ Block diag $[0.01I_{625\times 625},\ 32\,000I_{625\times 625},\ 20\,000I_{625\times 625}]$, $R_2 =$ Block diag $[0.025I_{625\times 625},\ 20\,000I_{625\times 625},\ 32\,000I_{625\times 625}]$, $\sigma_1 = 0.002, \sigma_2 = 0.001$, and $\varepsilon = 0.005$.

The RMFC control is also simulated for situations with and without the nominal parameters of the manipulator. In the case that the nominal parameters are known *a priori*, the initial parameters β_i and ω_i are chosen based on the training data $x^{(k)}$ and the element-by-element minimization of the following objective function:

$$\sum_k |f^9(x^{(k)}| \text{ nominal parameters of the arm}) - \hat{f}(x^{(k)}, \beta_i)|^2$$

$$\sum_k |G^0(x^{(k)}| \text{ nominal parameters of the arm}) - \hat{G}(x^{(k)}, \omega_i)|^2$$

32 testing points from either along the desired trajectories or nearby of them are chosen as training data $x^{(k)}$. When no nominal parameters of the manipulator is available, the elements of β_i and ω_i are chosen randomly in the intervals (−10, 10) and (0, 2), respectively.

1) Model Following Control Subject to Disturbances: The combined friction and external torque disturbance are

$$\begin{aligned} \Delta_1 &= 2.0\sin(\dot{q}_1) + 2.5\sin(\dot{q}_2) + 0.5\sin(t) \\ \Delta_2 &= 5.0\sin(\dot{q}_1) + 4.0\sin(\dot{q}_2) + 0.4\sin(t). \end{aligned} \tag{32}$$

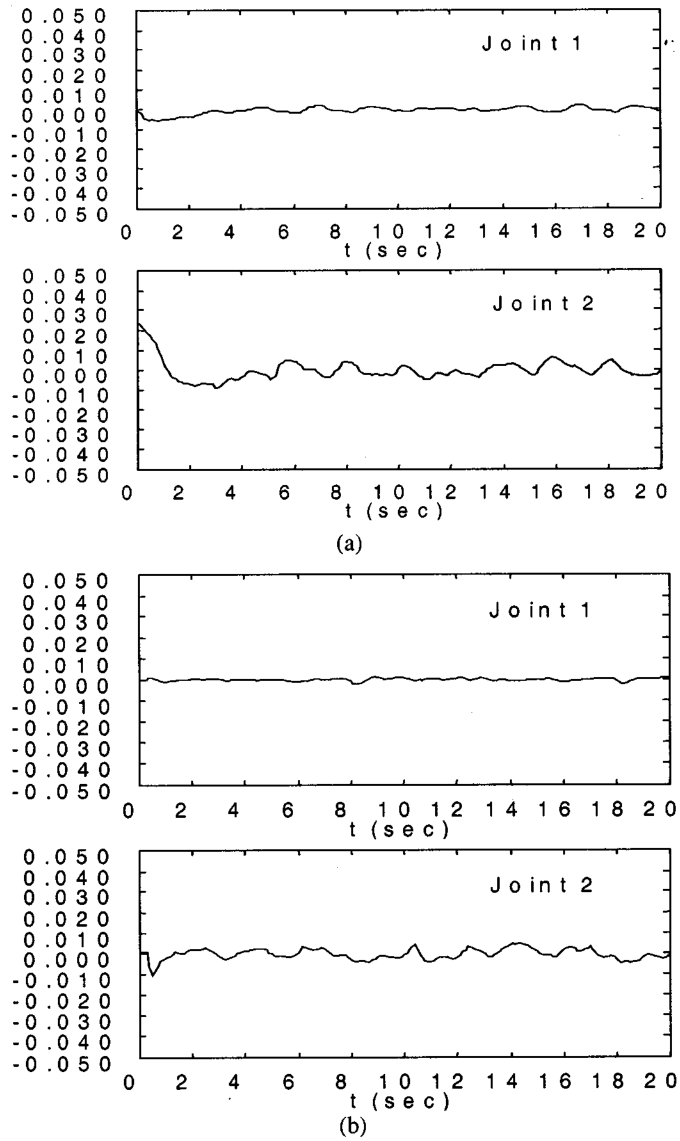

Fig. 6. Tracking error of joint 1 and 2 without measurement noise (a) without and (b) with rough mathematical model and nominal parameters.

For the case without measurement noise, the simulation results are presented in Fig. 6. Fig. 6(a) and (b) shows the cases of the RMFC with and without using nominal parameters of the manipulator. Obviously the RMFC achieves faster convergence when the initial parameters are chosen based on the nominal model.

2) Tracking control with measurement noise: The noises of different sensors are assumed to be independent of each other and white with uniform distribution within [−0.056, 0.05]. External disturbances are set the same as the above subsection. Fig. 7 shows the tracking errors of joint 1 and joint 2. The RMFC scheme appears to be robust against measurement noise by this simulation.

VI. CONCLUSION

A RMFC and its adaptation method for the model-following control of multilink robot manipulators have been constructed. The RMFC is mainly composed of a multi-input/multi-output

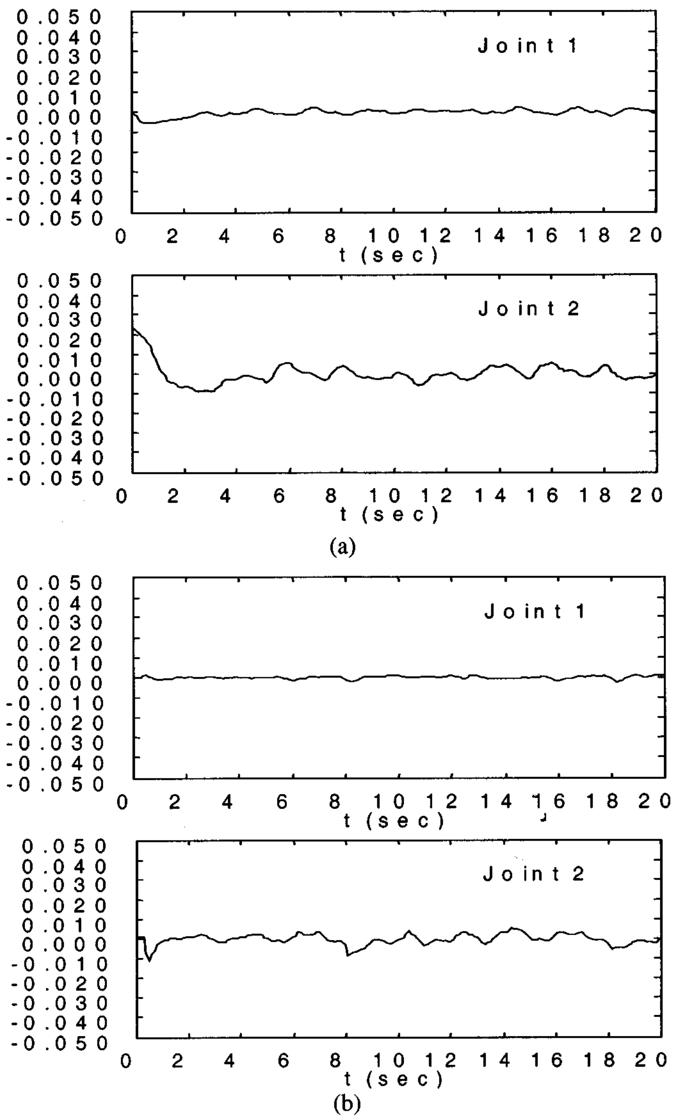

Fig. 7. Tracking error of joint 1 and 1 with measurement noise (a) without and (b) with rough mathematical model and nominal parameters.

fuzzy system with adjustable rule credit assignment and the interconnections compensating network. The interconnections compensating network can compensate the interaction among the subsystems and the fuzzy part of the RMFC approximates the unknown nonlinearity of the robot by learning. By on-line tuning the consequent membership functions, the RMFC system achieves some degree of robust properties. The overall adaptation scheme has been proved to be able to guarantee the output tracking error to converge to a residual set ultimately. Simulations of a robot control system have confirmed the robustness of the design to actuator and measurement noise. If the nominal model of the robot is available, the RMFC can be trained in advance to achieve faster convergence of the output tracking error. Applications of the RMFC to other affine nonlinear dynamic systems are straightforward.

APPENDIX

Proof of Theorem 1: Let V_θ and V_ϑ be positive-definite functions of the forms $V_\theta = \frac{1}{2}\,; \Sigma_{i=1}^n (\theta_i^T \theta_i)$ and $V_\vartheta = \frac{1}{2}\, \Sigma_{i=1}^n \vartheta^2$, respectively. Their time derivative are $\dot{V}_\theta = \Sigma_{i=1}^n \theta_i^T \dot{\theta}_i$ and $\dot{V}_\vartheta = \Sigma_{i=1}^n \vartheta_i^T \dot{\vartheta}_i$, respectively. If the first line of (25) is true then $d_{\theta_i} = 0$, and the conclusion $\dot{V}_\theta \le 0$ is trivial. If the second line of (25) is true then $d_{\theta_i} < 1$ and $\theta_i \in M_{\theta_i}^\varepsilon$. Therefore, either $\dot{V}_\theta \le 0$ or $\theta_i \in M_{\theta_i}^\varepsilon$ is obtained. Similarly we have either $\dot{V}_\vartheta < 0$ or $\vartheta_i \in M_{\vartheta_i}^\varepsilon$. Therefore the boundedness of θ_i, ϑ_i, and τ is guaranteed. To show the performance of the closed-loop system formed by (2), (15), (24), and (26), we choose the following positive-definite functions:

$$V = V_1 + \cdots + V_n \tag{A.1}$$

where

$$V_i(t) = \begin{cases} \frac{1}{2} d_0^2 + \frac{1}{2} \tilde{\theta}_i^T R_i \tilde{\theta}_i + \frac{1}{2} \gamma_{\vartheta_i} \tilde{\vartheta}_i^2, \\ \quad \text{if } e^T P b b^T P e \le d_0^2 \\ \frac{1}{2} e_i^T P e_i + \frac{1}{2} \tilde{\theta}_i^T R_i \tilde{\theta}_i + \frac{1}{2} \gamma_{\vartheta_i} \tilde{\vartheta}_i^2, \\ \quad \text{otherwise} \end{cases}$$

$\tilde{\vartheta}_i = \vartheta_i - \vartheta_i^M$ is the auxiliary adjustable parameter error and $\vartheta_i^M \equiv \max\{\vartheta_i^*, \vartheta_{i0}\}$. Taking the derivative of V_i along the trajectories of the closed-loop system and taking (21), (24), and (26) into account we obtain: $\dot{V}_i = 0$ for $e^T P b b^T P e \le d_0^2$ and

$$\begin{aligned} \dot{V}_i(t) =& e_i^T P_i (A_i e_i - b_i \tilde{\theta}_i^T w_i + b_i(\zeta_i - \alpha_{LRi} \phi)) \\ &+ \tilde{\theta}_i^T (I - d_\theta \theta_{i\perp} \theta_{i\perp}^T) \\ &\cdot [w_i b_i^T P_i e_i - \sigma_1 (\theta_i - \theta_{i0})] \\ &+ \tilde{\vartheta}_i (1 - d_\vartheta) [w_i' b_i^T P_i e_i - \sigma_2 (\vartheta_i - \vartheta_{i0})] \\ =& \tfrac{1}{2} e_i^T (A_i^T P_i + P_i A_i) e_i - e_i^T P_i b_i \tilde{\theta}_i^T w_i \\ &+ e_i^T P_i b_i (\zeta_i - \vartheta_i w_i') \\ &+ \tilde{\theta}_i^T w_i b_i^T P_i e_i - \sigma_1 \tilde{\theta}_i^T (\theta_i - \theta_{i0}) \\ &+ \tilde{\vartheta}_i w_i' P_i e_i - \sigma_2 \tilde{\vartheta}_i (\vartheta_i - \vartheta_{i0}) \\ &- d_\vartheta \tilde{\vartheta}_i [w_i' b_i^T P_i e_i - \sigma_2 (\vartheta_i - \vartheta_{i0})] \\ &- d_\theta \tilde{\theta}_i^T \theta_{i\perp} \theta_{i\perp}^T [w_i b_i^T P_i e_i - \sigma_1 (\theta_i - \theta_{i0})] \end{aligned} \tag{A.2}$$

for $e^T P b b^T P e > d_0^2$. By (25), if $\theta_{i\perp}^T [w_i b_i^T P_i e_i - \sigma_1 (\theta_i - \theta_{i0})] \le 0$, we have $d_{\theta_i} = 0$ and the last term of the above equation is equal to zero. When $\theta_{i\perp}^T [w_i b_i^T P_i e_i - \sigma_1 (\theta_i - \theta_{i0})] > 0$ if $\theta_i \in M_{\theta_i}$ we also have $d_{\theta_i} = 0$ and the above conclusion holds. If $\theta_i \notin M_{\theta_i}$ and suppose that M_{θ_i} and M_{ϑ_i} are appropriately selected such that θ_i^* and ϑ_i^* are in the interior of M_{θ_i} and M_{ϑ_i} respectively, we obtain

$$\begin{aligned} \tilde{\theta}_i^T \theta_{i\perp} &= (\theta_i - \theta_i^*)^T \theta_i / |\theta_i| \\ &= \tfrac{1}{2} [(\theta_i - \theta_i^*)^T (\theta_i - \theta_i^*) + \theta_i^T \theta_i - \theta_i^{*T} \theta_i^* / |\theta_i| \\ &\ge 0 \end{aligned} \tag{A.3}$$

or

$$\tilde{\theta}_i^T \theta_{i\perp}^T [w_i b_i^T P_i e_i - \sigma_1 (\theta_i - \theta_{i0})] \ge 0. \tag{A.4}$$

In a similar way it can be shown that

$$\tilde{\vartheta}_i [w_i' b_i^T P_i e_i - \sigma_2 (\vartheta_i - \vartheta_{i0})] \ge 0. \tag{A.5}$$

Therefore

$$\begin{aligned} \dot{V}_i \le& \tfrac{1}{2} e_i^T (A_i^T P_i + P_i A_i) e_i + e_i^T P_i b_i (\zeta_i - \vartheta^M w_i') \\ &- \sigma_1 \tilde{\theta}_i^T (\theta_i - \theta_{i0}) - \sigma_2 \tilde{\vartheta}_i (\vartheta_i - \vartheta_{i0}). \end{aligned} \tag{A.6}$$

Using Assumption 1, the second term on the right-hand side satisfies the inequality

$$\begin{aligned} e_i^T P_i b_i(\zeta_i - \vartheta^M w_i') &\le |e_i^T P_i b_i| \vartheta_i^* \phi - e_i^T P_i b_i \vartheta^M w_i' \\ &\le \vartheta_i^M (|e_i^T P_i b_i| \phi - e_i^T P_i b_i w_i') \\ &= \vartheta_i^M \left(|e_i^T P_i b_i \phi| - e_i^T P_i b_i \phi \tanh\left(\frac{e_i^T P_i b_i \phi}{\varepsilon} \right) \right) \\ &\le \vartheta_i^M \kappa \varepsilon. \end{aligned} \tag{A.7}$$

Since the following fact can be shown easily by straightforward algebraic manipulation

$$0 \le |\gamma| - \gamma \tanh\left(\frac{\gamma}{\varepsilon}\right) \le \kappa\varepsilon \tag{A.8}$$

for any $\gamma \in R$. Furthermore, it can be readily shown that

$$\begin{aligned} \tilde{\theta}_i^T(\theta_i - \theta_{i0}) = \tfrac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i + \tfrac{1}{2}(\theta_i - \theta_{i0})^T(\theta_i - \theta_{i0}) \\ - \tfrac{1}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) \end{aligned} \tag{A.9}$$

$$\tilde{\vartheta}_i(\vartheta_i - \vartheta_{i0}) = \tfrac{1}{2}\tilde{\vartheta}_i^2 + \tfrac{1}{2}(\vartheta_i - \vartheta_{i0}) + \tfrac{1}{2}(\vartheta_i^* - \vartheta_{i0})^2. \tag{A.10}$$

Therefore

$$\begin{aligned} \dot{V}_i \le & -\frac{1}{2}e_i^T(Q_i')e_i - \frac{\sigma_1}{2}\tilde{\theta}_i^T\tilde{\theta}_i - \frac{\sigma_1}{2}\tilde{\vartheta}_i^2 \\ & + \frac{\sigma_1}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) + \frac{\sigma_2}{2}(\vartheta_i^* - \vartheta_{i0})^2 + \vartheta_i^M\kappa\varepsilon \\ \le & -a_iV_i + \lambda_i \end{aligned} \tag{A.11}$$

where

$$a_i \equiv \min\left\{ \frac{\lambda_{min}(Q_i')}{\lambda_{\max}(P_i)}, \frac{\sigma_1}{\lambda_{max}(R_i)}, \frac{\sigma_2}{\gamma_{\vartheta_i}} \right\} \tag{A.12}$$

and

$$\begin{aligned} \lambda_i = & \frac{\sigma_1}{2}(\theta_i^* - \theta_{i0})^T(\theta_i^* - \theta_{i0}) \\ & + \frac{\sigma_2}{2}(\vartheta_i^* - \vartheta_{i0})^2 + \vartheta_i^M\kappa\varepsilon \end{aligned} \tag{A.13}$$

or

$$\dot{V} \le -aV + \lambda \tag{A.14}$$

where $a = \min_i a_i$ and $\lambda = \Sigma_{i=1}^n \lambda_i$. The differential inequality (A.14) satisfies

$$0 \le V(t) \le \frac{\lambda}{\alpha} + \left(V(0) - \frac{\lambda}{\alpha} \right) e^{-at}. \tag{A.15}$$

Therefore, e_i, θ_i, and ϑ_i are uniformly ultimately bounded. Let $\rho^* = (2\lambda/a)$ then from (A.15) we readily obtain (30).

ACKNOWLEDGMENT

The authors would like to thank the reviewers for their valuable suggestions.

REFERENCES

[1] L. Guzzella and H. P. Geering, "Model following variable structure control for a class of uncertain mechanical systems," in *Proc. 25th IEEE Conf. Decision Contr.*, vol. 3, Dec. 1986, pp. 312–316.

[2] K.-K. D. Young, "A variable structure model following control design for robotic application," *IEEE Trans. Robot. Automat.*, vol. 4, pp. 556–561, Oct. 1988.

[3] T. P. Leung, Q. J. Zhou, and C. Y. Su, "An adaptive variable structure model following control design for robot manipulators," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 347–353, Mar. 1991.

[4] G. Ambrosino, G. Celentano, and F. Garofalo, "Robust model tracking control for a class of nonlinear plants," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 275–279, Mar. 1985.

[5] S. N. Singh, "Adaptive model following control of nonlinear robotic systems," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 1099–1100, Nov. 1985.

[6] T. C. Hsia, "Adaptive control of robot manipulators—a review," in *Proc. IEEE Conf. Robot. Automat.*, vol. 3, May 1986, pp. 2100–2105.

[7] H. Durrant-Shyte, "Practical adaptive control of actuated spatial mechanisms," in *Proc. IEEE Conf. Robot. Automat.*, May 1985, p. 650.

[8] Y. H. Chen, "Adaptive robust model-following controlof nonlinear robotic systems," *Trans. ASME J. Dynam. Syst., Measur., Contr.*, vol. 109, pp. 209–215, 1987.

[9] A. H. Lewis and S. I. Marcus, *et al.*, "Challenges to control: a collective view," *IEEE Automat. Contr.*, vol. AC-32, pp. 275–285, Apr. 1987.

[10] M. M. Polycarpou and P. A. Ioannou, "Robust adaptive nonlinear control design," *Automatica*, vol. 32, no. 1, pp. 423–427, 1996.

[11] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 1, pp. 146–149, May 1993.

[12] J. Nie and D. A. Linkens, "Learning control using fuzzifield self-organizing radial basis function network," *IEEE Trans. Fuzzy Syst.*, vol. 1, pp. 280–287, Nov. 1993.

[13] T.-K. John Koo, "Model reference adaptive fuzzy control of robot manipulator," in *Proc. IEEE Syst., Man, Cybern.*, Oct. 1995, pp. 424–429.

[14] S. S. Neo and M. J. Er, "Adaptive fuzzy controllers of a robot manipulator," *Int. J. Syst. Sci.*, vol. 27, no. 6, pp. 519–532, 1996.

[15] S. Commuri and F. L. Lewis, "Adaptive-fuzzy logic control of robot manipulators," in *Proc. IEEE Robot. Automat.*, vol. 3, Apr. 1996, pp. 2604–2609.

[16] S. Jagannathan, "Discrete-time fuzzy logic control of a mobile robot with an onboard manipulator," *Int. J. Syst. Sci.*, vol. 28, no. 12, pp. 1195–1209, 1997.

[17] R. P. Paul, *Robot Manipulators: Mathematics, Programming, and Control: The Computer Control of Robot Manipulators*. Cambridge, MA: MIT Press, 1992.

[18] H. R. Berenji and P. S. Khefkar, "Learning and tuning fuzzy logic controllers through reinforcements," *IEEE Trans. Neural Networks*, vol. 28, pp. 1195–1209, Dec. 1997.