Revisiting Topology Control for Multi-Hop Wireless Ad Hoc Networks

Kun-Da Wu, *Student Member, IEEE,* and Wanjiun Liao, *Senior Member, IEEE*

***Abstract*—This paper revisits the topology control problem for multi-hop wireless ad hoc networks. It accounts for the maximum number of links interfering with each node in the network, which is referred to as the interference load in the paper. Finding a topology with minimum interference load is an NP-complete problem. In this paper, we develop the essential theorems and lemmas for constructing low interference load topologies for wireless ad hoc networks. Based on the theoretical results, we provide the design guidelines for topology construction with low interference load, and propose a polynomial-time solution accordingly. The performance of the proposed mechanism is evaluated via simulations. The results show that the proposed mechanism outperforms existing work in terms of lower interference load even using the maximum transmission power in the generated topology.**

***Index Terms*—Interference, topology control, multi-hop wireless networks.**

I. Introduction

CONSERVING energy consumption is a key issue for wireless ad hoc network deployment. For each node, instead of using its maximum power to transmit, the actual transmission power can be determined collaboratively with other nodes of the network, and hence the topology is constructed. A topology may be flat [1]–[9] or cluster-based [10]–[12]. In either case, many factors (e.g., interference, throughput, delay, robustness) should be considered in constructing a network topology. Among them, interference is a fundamental issue to address for topology control. Interference may cause collisions and consequently data retransmissions at the medium access control layer. Accordingly, network throughput may degrade, channel access delay may increase, and energy may be wasted, due to packet retransmissions.

The interference problem is often implicitly claimed to be mitigated in most existing topology-control algorithms for wireless ad hoc networks. The typical approach is either 1) to lower the node degree of each node [1], [2] or 2) to establish connections to each node's nearest neighbors [3], [4]. These approaches, however, are disapproved in [5] for networks with bi-directional links. With bi-directional links, the networks can support data link level acknowledgements and channel acquisition mechanisms like RTS/CTS in IEEE 802.11. In this paper, we study the topology control problem for wireless ad hoc networks with bi-directional links. We aim to develop essential theoretical results and provide design guidelines for this problem. In what follows, we first overview the related work on topology control with interference considerations for wireless ad hoc networks, and then describe the motivation and contribution of this paper.

Manuscript received March 16, 2007; revised June 13, 2007; accepted August 30, 2007. The associate editor coordinating the review of this paper and approving it for publication was X. Zhang. This work was supported by National Science Council (NSC), Taiwan, under a Center Excellence Grant NSC95-2752-E-002-006-PAE, and under Grant Number NSC95-2221-E-002-066.

The authors are with the Department of Electrical Engineering and the Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan (e-mail: wjliao@ntu.edu.tw).

Digital Object Identifier 10.1109/TWC.2008.070292.

A. Related work on interference-based topology control in wireless ad hoc networks

In [6], the author proposes a heuristic algorithm based on Delauney triangulation. This algorithm first constructs planar triangulations based on the locations of all nodes in the network, and then reduces the number of links for each node to maximize the minimum angles of all triangulations. This algorithm, however, cannot guarantee the network connectivity. In [7], Ramanathan and Rosales-Hain forumlate the topology control problem as an optimization problem in which the maximum transmission power is minimized under two constraints: connectivity and bi-connectivity. They then propose a centralized algorithm based on the spanning tree property to construct bi-connected static networks. In addition, they design two distributed algorithms, namely, Local Information Node Topology (LINT) and Local-Information Link-state Topology (LILT). The LINT algorithm is governed by three parameters: the desired node degree for each node, and two thresholds for the node degree, d_h and d_l. If the degree exceeds d_h, the node reduces its transmission power; if the degree is lower than d_l, the node increases its power. LINT, however, cannot guarantee the network connectivity. LILT can guarantee the network connectivity, but requires global information stored in each node for topology computation. Thus, it is impractical for deployment. In [2], a cone-based topology control algorithm is proposed. This algorithm requires directional information (i.e., an angle α) to build the network topology. It claims that taking $\alpha = 5/6$ is necessary and sufficient to preserve the network connectivity. Some distributed algorithms are then proposed based on geometric construction properties such as Delaunay triangulation [8] or the minimum spanning tree [9].

The topology control algorithms described above all attempt to save energy, prolong the lifetime, or increase the throughput by either lowering the node degree or establishing connections to each node's nearest neighbors. The explicit notion of an interference model is first proposed in [13]. In this model, a packet can be transmitted over a link successfully only when there are no interferring links in the vicinity being activated simultaneously. One recent study [5] further disproves the assumption that low interference can be achieved by constructing

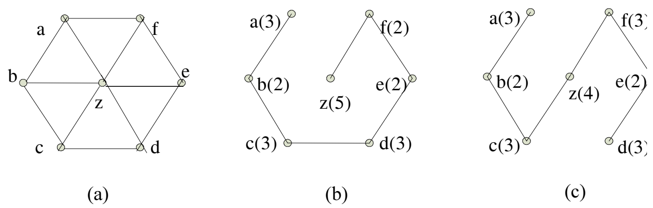

Fig. 1. Example topologies for a network of seven nodes. (a) A network with seven nodes. (b) A topology in which there are five links interfered with node z. (c) A topology in which there are four links interfered with node z.

sparse topologies. The authors argue that 1) existing topology control algorithms cannot yield nontrivial interference approximation to the optimal solution for networks with only bi-directional links, and 2) there exists a class of graphs in which no locally optimal algorithm can approximate the optimal interference while maintaining the network connectivity. They then propose an interference-minimal connectivity-preserving algorithm (called LIFE) and a spanner construction algorithm (called LISE). These two algorithms outperform existing work which is based on Gabriel Graph [14] or Relative Neighborhood Graph [3]. The algorithms are also proved to be optimal if the interference of a network is defined as the maximum number of nodes interfered by a particular link.

B. Motivation of this work

The work in [5] aims to minimize the maximum number of nodes interfered by a particular link during topology construction. The key idea is to avoid including a link which interferes with the most number of the nodes in the network. These mechanisms, however, may not work efficiently when a particular node is interfered by multiple links with smaller costs. The cost here is referred to as the number of nodes affected by a transmission over a given link. For example, consider a network of seven nodes, each of which has the same maximum transmission power, as shown in Fig. 1 (a). Fig 1 (b) shows a topology which is constructed according to [5], i.e., the links that interfere with the least number of nodes are selected. In this topology, node z is interfered by five links, i.e., $\overline{ab}$, $\overline{bc}$, $\overline{cd}$, $\overline{de}$, and $\overline{ef}$; node a and c are interfered by three links, and nodes b, e and f are all all interfered by two links. Therefore, node z is the node which is interfered by the most number of links in this topology. Now consider another topology shown in Fig. 1 (c), where link $\overline{cz}$ is selected instead of $\overline{cd}$. In this topology, node z is still the node interfered by the maximum number of links in the network, but this time it is interfered by four links, instead of five links as in Fig. 1 (b).

To address this problem, we define the *interference load* for a node as the number of links which cause interference to the node. The objective of this paper is to minimize the maximum per-node interference load and to balance the interference loads among all nodes in the network. The maximum per-node interference load in the generated topology is referred to as the interference load of this topology. The interference load of a topology has significant impact on the network performance, explained as follows. A node with higher interference load may suffer more irrelevant communications contending for bandwidth with the node. The more the contending communications, the less the throughput (or the more forwarding delay) this node may have, and therefore the lower the availability of this node. The interference load experienced by each node may vary greatly, depending on its location and the link selection process during topology formulation. Such unbalanced interference load among nodes may result in virtual network partitioning in the constructed topology once the availabilities of some nodes are reduced dramatically even though they are connected from the point of view of physical topology. This calls for a solution that reduces the maximum per-node interference load in topology formulation. With such a solution, the network interference load can be reduced and the interference loads among all nodes can also be balanced.

C. Contributions of this work

In this paper, we study the topology control problem with *interference load* considerations. The objective is to find a network topology whose interference load is minimized under the constraint that the network is connected (referred to as the *connectivity* constraint). With the connectivity constraint, there exists at least one routing path for any pair of nodes in the network. We refer to this problem as the *minimum interference load* problem, denoted as **MIN_IFLOAD**, in this paper.

The problem **MIN_IFLOAD** is shown to be NP-complete in our previous work [15]. In this paper, we further develop essential theorems and lemmas to construct a network topology with low interference load. Specifically, we show that the interference load of a network can be minimized if the generated topology satisfies the defined *isolated* property. Based on the theoretical results, we provide the design guidelines for constructing low interference load topologies, and propose a polynomial-time solution accordingly. The performance of the proposed algorithm is evaluated via simulations. The results show that the proposed algorithm is superior to existing solutions.

The rest of the paper is organized as follows. In Sec. II, the key properties for low interference load topology generation are defined and the essential theorems for the problem are developed. In Sec. III, a topology control algorithm with low interference load is designed. In Sec. IV, the performance of the proposed algorithm is evaluated via simulations. In Sec. V, the discussions of communication overhead, timing restriction and the effect of interference load are provided. Finally, the paper is concluded in Sec. VI.

II. Low Interference Load Topology Control

Since the **MIN_IFLOAD** problem is NP-complete, we attempt to develop a heuristic algorithm to provide a feasible solution to this problem. In this section, we develope the necessary conditions for low interference load topology construction.

A. System Model and Definitions

The network is modeled as a connected graph $G_{max} = (V, E_{max})$, where $V = \{v_1, v_2, ..., v_n\}$ is the set of nodes

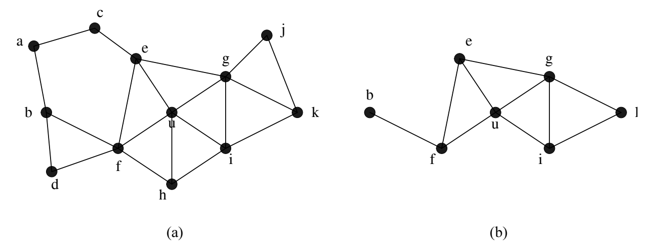

Fig. 2. An example of a dense cluster and its core. (a) A given topology G_{max}. (b) The projection of V_D on G_{max}.

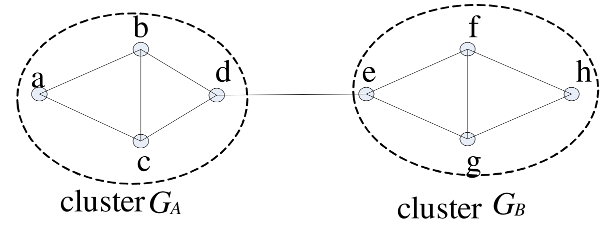

Fig. 3. The topology G_{max} with two isolated clusters G_A and G_B.

and E_{max} is the set of edges in the network. Each node v_i is located at a coordinate (x_i, y_i) inside the network, with the maximum transmission range r_i. For any two nodes v_i and v_j in V, $i \neq j$, they are connected by an edge (v_i, v_j) if $d(v_i, v_j) \leq r_i$ and $d(v_i, v_j) \leq r_j$, where $d(v_i, v_j)$ is the distance between nodes v_i and v_j. Then, the edge (v_i, v_j) is included in E_{max}. We consider static nodes only, and assume that at most one node is located at a coordinate. The topology formulation results from the process of finding a sub-graph G of G_{max}, where G consists of all nodes in G_{max} and satisfies certain constraints. Let $D(v_k, r)$ denote the circle centered at node v_k with radius r. The coverage of an edge $e = (v_i, v_j)$, denoted by $Cov(e)$, refers to the region $D(v_i, d(v_i, v_j)) \cup D(v_j, d(v_j, d_i))$. $|Cov(e)|$ represents the number of nodes in $Cov(e)$.

A node v_k in G_{max} is interfered by edge (v_i, v_j) if node v_k can receive messages transmitted by node v_i or v_j, although these messages are not destined to it. The *interference load* of a node is defined as the number of links that cause interference to the node when activated. The interference load of a node is formally defined as follows.

Definition 1. Given a topology $G = (V, E)$, the interference load of node w in G, denoted as $IFload(w)$, is defined as the maximum number of links interfering with node w when activated, i.e.,

$$IFload(w) := |\{(u, v) \in E | w \in D(u, d(u, v)) \cup D(v, d(v, u))\}| \quad (1)$$

Definition 2. Given a topology $G = (V, E)$, node v in G is said to have an interference load of k if node v is interfered by k edges in topology G.

Definition 3. Given a topology $G = (V, E)$, a node $v \in V$ is *k-hop interfered* by an edge $e = (w, u) \in E$ if node v is located inside the area $Cov(e)$ and with path length $k = min\{|p_1|, |p_2|\}$, where p_1 is the shortest path from w to v and p_2 is the shortest path from u to v in G. Node v is *multi-hop interfered* by edge $(w, u) \in E$ if $k > 1$.

Definition 4. Let V_p be a subset of V in $G_{max} = (V, E_{max})$. A sub-graph of G_{max} is a projection of V_p on G_{max} if $E_p = \{(u, v) | (u, v) \in E_{max}, u \in V_p, v \in V_p\}$.

Given a network $G_{max} = (V, E_{max})$, $G_w = (W, E_w)$ is a cluster of G_{max} if G_w is a projection of W on G_{max}. A cluster $G_w = (W, E_w)$ is connected if there exists at least one path for each pair of nodes in G_w. Two particular types of connected clusters are defined as follows.

Definition 5. (**Dense Property**) Consider a set of nodes $V_D \subseteq V$ in $G_{max} = (V, E_{max})$. The cluster $G_D = (V_D, E_D)$ of G_{max} is referred to as a dense cluster if 1) G_D is connected, and 2) there exists one and only one node $u \in V_D$ which is interfered by all edges E_D except those edges connecting node u directly. Node u is referred to as the core of the dense cluster G_D.

Fig. 2 (a) gives an example topology $G_{max} = (V, E_{max})$, which is constructed with the maximum transmission power of a set of nodes $V = \{a, b, c, d, e, f, g, h, i, j, k, u\}$. Fig. 2 (b) shows G_D, the projection of V_D on G_{max}, where $V_D = \{b, f, e, g, i, k, u\}$ is a subset of V. Since node u is interfered by all edges in G_D except those edges connecting it and it is the only node that satisfies this constraint, G_D is a dense cluster and node u is the core of G_D.

Definition 6. (**Isolated Property**) Consider a set of nodes $V_I \subseteq V$ in $G_{max} = (V, E_{max})$. A cluster $G_I = (V_I, E_I)$ of G_{max} is referred to as an isolated cluster if 1) G_I is connected, 2) each node u in G_I is interfered only by the edges $e \in E_I$ or the edges $e' \in E_{max} - E_I$ connecting to other clusters. The node w with the largest interference load caused by the edges in E_I is referred to as the *core* of this *isolated cluster* G_I.

Note that by definition, a dense cluster is a special case of an isolated cluster. Fig. 3 shows two isolated clusters in $G_{max} = (V, E_{max})$, where $V = \{a, b, c, d, e, f, g, h\}$. The set of nodes V can be partitioned into two disjoint subsets C_1 and C_2, where $C_1 = \{a, b, c, d\}$ and $C_2 = \{e, f, g, h\}$, and the projections of C_1 and C_2 on G_{max}, say G_A and G_B, respectively, are two isolated clusters of G_{max}. Nodes d and e are the cores of G_A and G_B, respectively.

Definition 7. The interference load of a cluster $G_w = (W, E_w)$ is the largest per-node interference load among all nodes in this *cluster*, i.e., $IFload(G_w) := max_{\forall w \in W}\{IFload(w) | w \in W\}$.

Definition 8. The coverage of a cluster $G_w = (W, E_w)$, denoted by $Cov(G_w)$, is define as $\bigcup_{e \in E_w} Cov(e)$. $|Cov(G_w)|$ represents the number of nodes in $Cov(G_w)$.

Let $G_1 = (W_1, E_1)$, $G_2 = (W_2, E_2)$, $\ldots$, and $G_k = (W_k, E_k)$ denote k clusters constructed by k disjoint sets of nodes $W_1, W_2, ..., W_k$ in G_{max}. For all G_i, $i = 1, 2, ..., k$, if they all satisfy the *isolated property*, then we have 1) $|Cov(G_i)| = |W_i|$, and 2) $Cov(G_i) \cap Cov(G_j) = \phi, \forall i, j \in \{1, 2, ..., k\}$, and $i \neq j$.

Definition 9. Let $e(G_i, G_j)$ be an edge connecting two nodes located in two isolated clusters G_i and G_j composed of two disjoint sets of nodes W_i and W_j, respectively. The cluster G_i (and G_j) is *1-hop interfered* by $e(G_i, G_j)$ if and only if all nodes $w \in W_i$ (and $u \in W_j$) are located in the area $Cov(e(G_i, G_j))$.

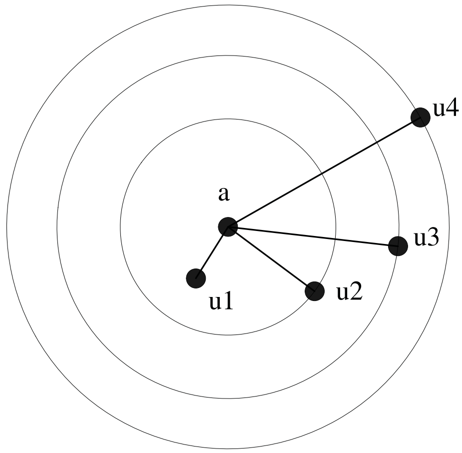

Fig. 4. Node u_1 is interfered by three edges.

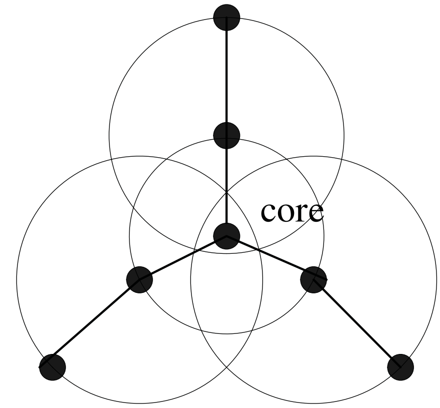

Fig. 5. Minimum interference load of G is 3.

B. Low Interference Load Topology Control

Lemma 1. Let k be the degree of node v (i.e., there are k edges incident on node v). There exists at least one of node v's neighbors whose interference load is at least $k-1$.

Proof. Let $u_1, u_2, ..., u_k$ be the set of nodes each connected to node v by an edge. Without loss of generality, we assume that the lengths of these edges are ordered non-decreasingly, i.e., $d(v, u_1) \leq d(v, u_2) \leq \cdots \leq d(v, u_k)$. Then, node u_1 is interfered by edges $(v, u_2), (v, u_3), ..., (v, u_k)$. Hence, the interference load of node u_1 is at least $k-1$.

Fig. 4 gives an example that node u_1 is interfered by the three edges $(a, u_2), (a, u_3)$ and (a, u_4). □

Lemma 2. If a cluster $G_w = (V_w, E_w)$ satisfies the *dense* property, then the minimum interference load of cluster G_w is given by $\lceil (|V_w| - 1)/2 \rceil$.

Proof. Since there are $|V_w|$ nodes in the dense cluster G_w, it is obvious that at least $|V_w|-1$ edges must be selected from E_w to ensure the connectivity of the resulting topology. By Definition 5, there exists a node u that is always interfered no matter which edge is selected. If the degree of node u is b, where $1 \leq b \leq |V_w| - 1$, then the interference load of u is $|V_w| - b - 1$. Based on Lemma 1, there exists a node y in G_w which is connected to node u by an edge and has an interference load of $b - 1$. The maximum interference load of the dense cluster G_w is determined either by node u or node y, depending on how the $|V_w| - 1$ interference loads is distributed to nodes u and y. The best distribution is an equal split of $|V_w| - 1$ between u and y, and hence the optimal interference load of the dense cluster G_w is $\lceil (|V_w| - 1)/2 \rceil$. □

Fig. 5 shows a dense cluster G of seven nodes, and the minimum interference load of G is 3 no matter how the topology is constructed. Note that in a k-node cluster without the dense property, edges not interfering with the core may be included in the topology, and the interference load of the cluster is smaller than $\lceil (k-1)/2 \rceil$. Hence, Lemma 2 provides an upper bound for the minimum interference load of a cluster in the network. Irrespective of the placement of nodes, for each k-node cluster of the network G_{max}, we can find a connected topology with interference load equal to or smaller than $\lceil (k-1)/2 \rceil$.

Corollary 3. If a cluster $G_w = (V_w, E_w)$ satisfies the isolated property, then the minimum interference load of G_w is at most $\lceil (|V_w| - 1)/2 \rceil$.

Lemma 4. Consider two sets of nodes W_1 and W_2. Let $G_1 = (W_1, E_1)$ and $G_2 = (W_2, E_2)$ denote two clusters that satisfy the dense property. If the interference loads of G_1 and G_2 are k_1 and k_2, respectively, then the minimum interference load by connecting G_1 to G_2 with an edge e is given by $max(k_1, k_2)$.

Proof. Let $W_1 = \{n_1, n_2, n_3, .., n_k\}$ and $W_2 = \{m_1, m_2, m_3, ..., m_l\}$. Suppose that nodes n_i and m_j are the cores of G_1 and G_2, respectively. By Definitions 5 and 7, the interference loads of the cores of G_1 and G_2, namely, n_i and m_j, are k_1 and k_2, respectively. If there exists an edge, say (n_p, m_q), with which n_i and m_j are not interfered, then connecting G_1 to G_2 by (n_p, m_q) will not increase interference load to either cluster. However, if no such edge exists, one must consider all possibilities of edge selections to connect G_1 to G_2. The number of choices for connecting G_1 to G_2 is $k \times l$. Of these choices, connecting node n_i to node m_j will not increase the interference load to either node n_i or m_j. Since there is only one node suffering the largest interference load in a dense cluster by definition and the following statements are both true,

$$IFload(n_b) < IFload(n_i), \forall b \in \{1, 2, ..., k\} \backslash \{i\},$$

$$IFload(m_h) < IFload(m_j), \forall h \in \{1, 2, ..., l\} \backslash \{j\},$$

it follows that adding an edge (n_i, m_j) to connect G_1 to G_2 will not increase the interference load of either G_1 or G_2. Hence, the interference load of $W_1 \bigcup W_2$ is given by $max(k_1, k_2)$. □

Lemma 5. Let $G_0, G_1, ..., G_k$ denote $k+1$ clusters that satisfy the dense property. If cluster G_0 is connected to $G_1, G_2, ..., G_k$ by k edges, then there exists one node in G_0 which experiences k additional interference loads caused by these edges.

Proof. Let $n_i(G_a)$ denote node i in the connected cluster G_a, and $R_i(G_a)$ denote the longest edge connecting node i to some other node in G_a. Since $Cov(G_i) \cap Cov(G_j) = \phi, \forall i, j \in \{0, 1, 2, ..., k\}$, for any two connected clusters G_a and G_b, $d(n_i(G_a), n_j(G_b)) \geq max\{R_i(G_a), R_j(G_b)\}, \forall i \in$

$G_a, j \in G_b$. There are three cases in selecting edges to connect G_0 to the other k clusters. The first case is that none of the selected k edges connects the core of G_0, then these k edges will cause interference to the core of G_0 and therefore the core of G_0 will gain k additional interference loads. The second case is that the selected k edges all connect the core of G_0. Since $R_i(G_0) \leq d(n_i(G_0), n_j(G_b)), \forall b \in \{1, 2, ..., k\}$, there exists one node j in G_0 which experiences k additional interference loads due to these k edges according to Lemma 1. The third case is that there are only x out of k edges connecting the core of G_0, then there exists one node g which experiences x additional interference loads according to Lemma 1. Since $R_g(G_0) \leq d(n_g(G_0), n_j(G_b)), \forall b \in \{1, 2, ..., k\}$, node g is also interfered by those $k-x$ edges. Hence, node g experiences k interference loads in total. □

Let C denote a set of connected clusters $G_1, G_2, ..., G_k$ with disjoint sets of nodes $W_1, W_2, ..., W_k$, respectively. Let E^C_{max} be the set of edges projected from E_{max}. The edge connecting node u in cluster G_i to node v in G_j, denoted by $e(G^u_i, G^v_j)$, is included in set E^C_{max} if $(u, v) \in E_{max}$, where $u \in W_i$ and $v \in W_j$, $i \neq j$. If each cluster G_i is regarded as a node, then the graph $G^C_{max} = (C, E^C_{max})$ can be used as an input graph to the cluster-based topology generation. Our objective is to find a sub-graph $G_C = (C, E_C)$ of G^C_{max} such that any pair of clusters in C is connected and the resulting interference load of C is as small as possible.

Lemma 6. Let $G^C = (C, E^C)$, where $C = \{G_1, G_2, ..., G_k\}$ is a set of isolated clusters with disjoint sets of nodes $W_1, W_2, ..., W_k, W^C = \bigcup_{i=1,..,k} W_i$, and E^C is a set of edges connecting these clusters. If each cluster G_i is *1-hop interfered* by the edges in $E^C, \forall i \in \{1, 2, ..., k\}$, then the *isolated* property still holds for G^C.

Proof. By definition, if a cluster $G_i = (W_i, E_i)$ is 1-hop interfered by an edge e in E^C, then for any node u interfered by edge e, u must be in W_i, and hence u is in W^C. Therefore, the *isolated* property still holds for G^C. □

Theorem 7. Let $G_1 = (W_1, E_1)$, $G_2 = (W_2, E_2)$,..., $G_k = (W_k, E_k)$ be k clusters, and $W_1 \cup W_2 \cup \cdots \cup W_k = V$, and $W_i \cap W_j = \phi, \forall i, j \in \{1, 2, ..., k\}$. If there exists an edge $e(G_i, G_j)$ which is 1-hop interfering to G_i and G_j, $\forall i, j \in \{0, 1, 2, ..., k\}$, and all these k clusters satisfy the isolated property, then the expected minimum interference load of connecting these k clusters together is at most $max\{I_i | I_i = \lceil(|W_i| - 1)/2\rceil\} + k/2, \forall i \in \{1, 2, ..., k\}$.

Proof. Without loss of generality, assume G_i is the isolated cluster with the most number of nodes, i.e., $|W_i| \geq |W_j|, \forall j \in \{1, 2, ..., k\}$. By Corollary 3, the minimum interference load of G_i is at most $\lceil(|W_i| - 1)/2\rceil$. Obviously, at least $k - 1$ additional edges are needed to connect k clusters G_1, G_2,...,G_k together. Since an edge connecting two clusters will not cause interference to the other clusters, the interference load of the resulting topology G is determined by the number of edges connecting G_i to other clusters, which is in turn determined by the location of each cluster. By Lemma 5, if there are t edges connecting G_i to other clusters, $t < k$, then t additional interference loads will be contributed to G_i in the worst case. Each cluster G_j, $\forall j \in \{1, 2, ..., k\}$, may be connected to a subset of the k clusters, denoted by $U(G_i)$, depending on its location. Let m denote the number of clusters in $U(G_i)$. The probability $p(m)$ is given by

$$p(m) = \frac{C_m}{\sum_{i=1}^{k-1} C_i^{k-1}}. \quad (2)$$

Then, L, the expected additional interference load after these k clusters are connected, is given by

$$E(L) = \sum_{m=1}^{k-1} m \times p(m) = \sum_{m=1}^{k-1} \frac{m \times C_m^{k-1}}{\sum_{i=1}^{k-1} C_i^{k-1}} = \frac{(k-1) \times 2^{k-1}}{2^{k-1} - 1} < \frac{k}{2}. \quad (3)$$

It follows that the expected minimum interference load of the resulting topology G is at most $max\{I_i | I_i = \lceil(|W_i| - 1)/2\rceil\} + k/2, i = 1, 2, ..., k$. □

C. Design Guidelines for Low Interference-Load Topology

Based on the theoretical results in Sec. II-B, we suggest the following guidelines for constructing a low interference load topology.

Guideline 1: According to Theorem 7, the interference load of a network can be reduced if its topology is constructed hierarchically by connecting more clusters that satisfy the *isolated* property.

Guideline 2: To connect two isolated clusters in G, it is preferred to select the node with the maximum interference load in each cluster as this can avoid an increase in the interference load of the resulting cluster. For example, assume that u and v are the nodes with the highest interference load in isolated clusters A and B, respectively. Suppose that $e = (u', v')$ is the edge chosen to connect cluster A to B. If $u' \in \{u, v\}$ and $v' \in \{u, v\}$, then selecting edge (u, v) is guaranteed not to increase the interference load of the resulting network. Hence, the interference load of the resulting connected cluster remains the same as in the original.

Guideline 3: Selecting an edge e which interferes with more number of nodes causes more incremental changes in interference loads among nodes. It is preferred to select the edge that interferes with less number of nodes among all possible edges.

III. Two Topology Control Algorithms with Interference Load Consideration

Since finding a solution to the **MIN_IFLOAD** problem is NP-complete [15], we propose a heuristic topology control algorithm to find a feasible solution based on the design guidelines described in the previous section.

A. Low Interference Load Topology Control Algorithm

We propose a new topology control algorithm called Low Interference-Load Neighborhood Forest (LILNF) for **MIN_IFLOAD**. We define a selection metric in the algorithm, referred to as the *interference cost*. Given a topology $G = (V, E)$, the interference cost of an edge e in E is defined as follows.

$$Cost(e) := \sum_{w \in Cov(e)} IFload(w) + |Cov(e)|. \quad (4)$$

TABLE I
LOW INTERFERENCE-LOAD NEIGHBORHOOD FOREST

Input: A set of nodes V, the maximum transmission range r_v^{max} of each node $v \in V$.
Output: The graph G_{LILNF}.
1: For each edge $e = (u, v), E_{max} = E_{max} \cup \{e\}$ if $r_u^{max} > d(u, v)$ and $r_v^{max} > d(u, v), u, v \in V$;
2: For each node $v \in V, v.IFload = 0$;
3: For each edge $e \in E_{max}, e.mark = FALSE$;
4: $E = \phi$;
5: Sort edge $e \in E_{max}$ and $e.mark = FALSE$ by $Cost(e)$ in ascending order;
6: select $e = (u, v)$ with minimum $Cost(e)$;
7: **if** u and v are note connected in G **then**
8: $E = E \cup \{e\}$;
9: $w.IFload = w.IFload + 1, \forall w \in Cov(e)$;
10: **endif**;
11: $e.mark = TRUE$;
12: **if** $e.mark = TRUE, \forall e \in E_{max}$, **then** stop;
13: Otherwise, go to 4.

In LILNF, for each pair of nodes that are not connected, an edge with the least interference cost is added to the topology. As such, the connectivity of the given network can be maintained and the least possible interference load of the topology can be ensured. The LILNF algorithm is summarized in Table I. Note that the proposed algorithm can be extended to be a distributed version. The detailed description can be found in [16].

B. The Complexity of the Algorithm

The time complexity of LILNF can be expressed as a function of the number of edges ($|E_{max}|$) and the number of nodes ($|V|$) in $G_{max} = (V, E_{max})$. Since the iterations at Steps 5 and 6 each requires calculating the minimum interference costs of the remaining edges in E, we get a complexity of $O((|E|^2 + |E|)/2)$. At Step 7, the operation of verifying whether nodes u and v are connected in G is equivalent to finding a path from node u to v in G, which results in a complexity $O(|E|) \times O(|V|^2)$. Obviously, the operations at Steps 8, 9, and 11 take complexity $O(1)$. Finding a path from node u to v is the dominant operation when each edge is examined to be included in E. Therefore, the overall complexity of LILNF is $O(|E| \times |V|^2)$.

IV. PERFORMANCE EVALUATION

In this section, we compare the performance of the proposed topology control algorithm LILNF with the minimum interference-based algorithm LIFE [5] in wireless ad hoc networks with bi-directional links. In the simulations, nodes are randomly and uniformly distributed in a 20×20 square area. Each node uses the same maximum transmission power. We simulate several randomly generated networks with different number of nodes, and calculate their average case performance. Our objective is to find a connected topology whose maximum per-node interference load is minimized. Since a reduction in the interference load at a node may lead to an increase in interference load to some other nodes, our objective is to generate a topology which tends to balance the interference loads among nodes. However, there is a trade-off between balancing the interference loads among nodes and reducing the total network interference load. This is because attempting to balance interference loads among nodes may lead to an increase in each node's interference load. Consequently, the total interference load of all nodes may be increased dramatically. To measure how balanced the interference loads among nodes are in a constructed topology G, we define the Interference load variance $I(G)$ as follows.

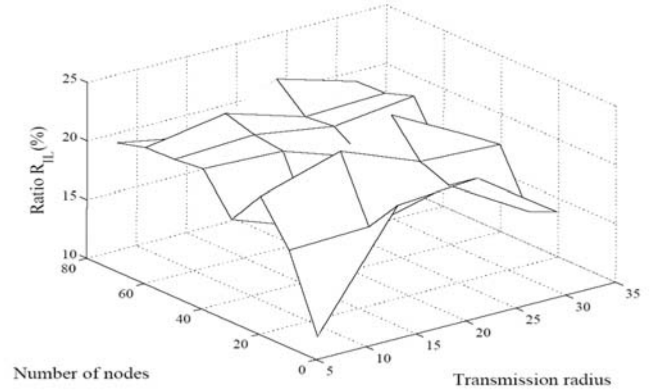

Fig. 6. Interference variance improvement for LILNF.

$$I(G) = \frac{\sum_{v \in V}(IFload(v) - \frac{1}{|V|}\sum_{v \in V} IFload(v))^2}{|V|}. \quad (5)$$

We further define a performance metric called the *Interference Load Variance Improvement Ratio* R_{IL} to compare the performance of interference load balancing for the algorithms as follows.

$$R_{IL} = \frac{I(G_1) - I(G_2)}{I(G_1)} \times 100\%, \quad (6)$$

where $I(G_1)$ is the interference load variance of the topology generated by LIFE, and $I(G_2)$, that obtained by LILNF. In addition, the maximum transmission range achieved by each node is an important metric to measure as it is related to the power consumption and the lifetime of the network operation. In our simulations, we compare the performance between algorithms in terms of 1) R_{IL}, 2) the total interference load, and 3) the maximum transmission range of a node.

We evaluate the average case performances of LILNF and LIFE. The number of nodes is varied from 20 to 80 and the maximum transmission range of each node is varied from 8 to 32 units. Fig. 6 shows the ratio R_{IL} of LILNF to LIFE. The results show that 1) the ratio R_{IL} increases with the number of nodes or the transmission range of each node; 2) the maximum ratio is 22.37%, which occurs with 40 nodes and a transmission range of 24.

Figs. 7 and 8 plot the total interference load of all nodes and the maximum transmission range used in the generated topology, respectively. The results show that compared with LIFE, LILNF can significantly reduce the interference load of a given network (Fig. 8) while having 1) the same total

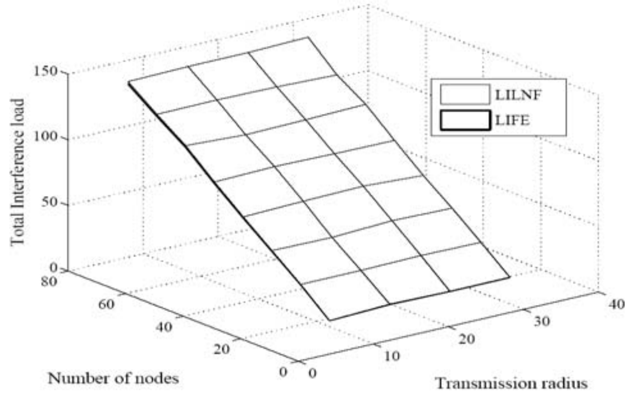

Fig. 7. Total interference loads of the LILNF and LIFE topologies.

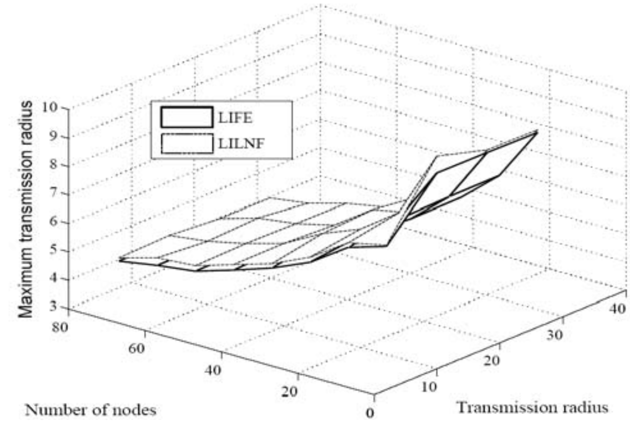

Fig. 8. Maximum transmission ranges for the LILNF and LIFE algorithms.

interference load of all nodes (Fig. 7), and 2) the same maximum transmission range of each node (Fig. 8) in the generated topology. Since the variance of the interference load of each node is improved significantly while the total interference load of all nodes in the network remains close to that in LIFE, it follows that LILNF generates a topology with lower interference load and more balanced interference loads among nodes. In addition, the maximum transmission range of each node in the topology generated by LILNF is larger that by LIFE. Thus, energy conservation can be achieved.

V. Discussions

A. Communication Overhead and Restrictions

The topology constructed by the LILNF algorithm is based on two types of information exchanges among neighboring nodes: the interference load of a node and the cost of an edge. The up-to-date interference load of each node w is recorded in $w.IFload$, and the up-to-date cost $Cost(e)$ for each edge e is calculated according to (4). The LILNF algorithm adds the edge whose cost is the minimum to the network at each iteration if the incident nodes of this edge have not been connected yet to the topology. Upon receiving the up-to-date interference load from all of the neighboring nodes, each node may update the cost of candidate edges for edge selection. With the connectivity information from its neighboring nodes, the node can enumerate a candidate set of un-connected edges in selection.

Exchanging interference load and connectivity information among neighboring nodes at each iteration incurs communication overheads. For a node to collect complete information from neighboring nodes, each of its neighboring nodes must announce its local information at least once during this period. A collision occurs when any two neighboring nodes send information at the same time. Hence, the period of information exchange needs to be sufficiently long so that all transmissions from neighboring nodes can be accommodated before a new edge is determined during topology construction.

B. Effect of Interference Load

Since the interference load of a node is related to the number of possible contentions for the wireless medium, a higher interference load of a node indicates that 1) the aggregate throughput of this node is lower; 2) more energy consumption for data re-transmissions is required, and hence, the lifetime of this node is reduced. Assume that the amounts of incoming and outgoing traffic on each edge are equal. Let C (Mbps) denote the capacity of the wireless channel. If the interference load of a node with degree g is k, then the maximum achievable throughput of this node is $C \times g/2(g+k)$. The effective portion of time for transmitting or receiving data is $g/(g+k)$ if the edges, interfering with each other, equally share the wireless medium. Therefore, for each transmission attempt, the probability of a successful transmission is $g/2(g+k)$. Let N be the number of retries for a successful data transmission. The probability mass function of random variable N and its expected value are given, respectively, by

$$P_N(n) = \left(\frac{g+2k}{2(g+k)}\right)^{n-1} \frac{g}{2(g+k)} = \frac{(g+2k)^{n-1}g}{2^n(g+k)^n},$$

$$E[N] = \sum_{n=1}^{\inf} nP_N(n) = \frac{2(g+k)}{g}.$$

For an edge to be included in a topology, a sufficiently large transmission range (i.e., transmission power) for a node is required to sustain the transmission over this edge. Hence, the transmission power $P(|e|)$ of the incident nodes of edge e is a function of the distance of edge e. The expected lifetime T of a node is $U_i/(E[N] \times t \times P(|e|))$, where U_i is the total energy of a node, and t is the time period of one transmission. Let $p(e) = P(|e|)/U_i$ be the ratio of power consumption for each data transmission. Thus, the normalized lifetime of a node is given by

$$E[T] = \frac{1}{E[N] \times t \times p(e)} = \frac{g}{2(g+k)p(e)t}.$$

For example, suppose that the wireless channel data rate is 11 Mbps, and the packet size for each transmission is 1500 bytes. Hence, the time period of each data transmission is 0.001091 seconds. Fig. 9 shows the effect of interference load on per node throughput with different node degrees. We observe that the per node throughput decreases as the interference load of a node increases. Fig. 10 plots the curves

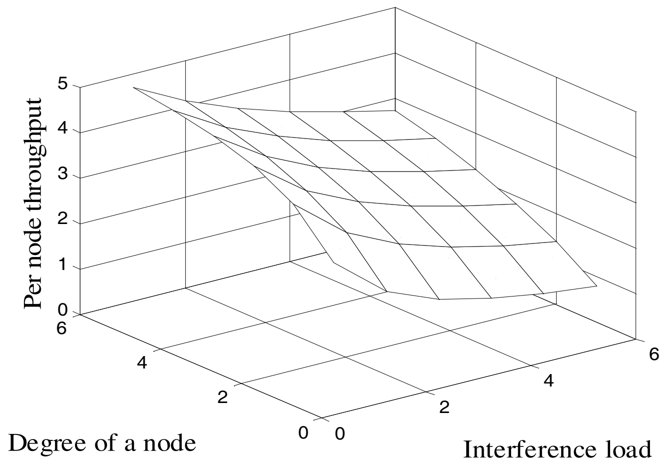

Fig. 9. Per node throughput versus interference load of a node with different degree.

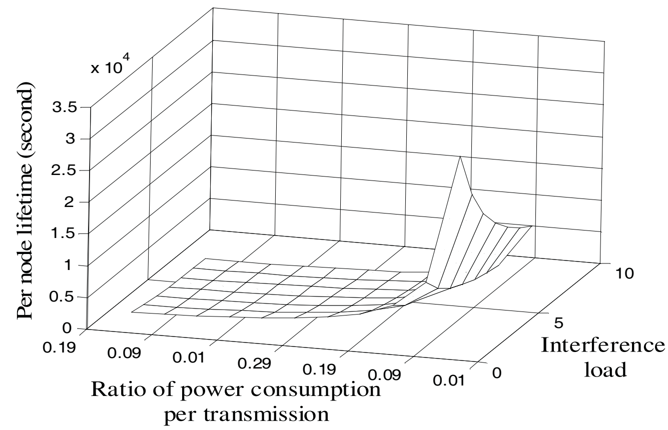

Fig. 10. Per node lifetime versus the interference load of a node and the ratio of power consumption for each transmission.

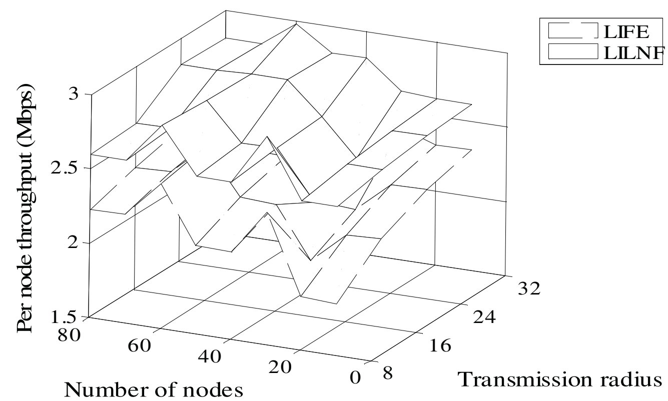

Fig. 11. The comparison of algorithms in terms of per node throughput versus the number of nodes and the transmission range.

of per node lifetime versus the per node interference load and the ratio of power consumption for a transmission $p(e)$ given that the degree of a node is 2. It shows that the per node lifetime decreases when either per node interference load or per transmission power consumption increases. The curves show that per transmission power consumption has larger impact than per node interference load on the lifetime of a node. The whole network may fail when one of them runs out of battery. As the transmission power of a node increases, the resulting transmission range increases and the lifetime of this node decreases. Hence, our measurement on the maximum transmission range in Fig. 8 implies that the lifetime of the network topology generated by LILNF is very close to that generated by LIFE. Fig. 11 shows the comparison of our proposed algorithm with LIFE in terms of per node throughput under different numbers of nodes and transmission ranges. We see that our algorithm can effectively improve per node throughput when the interference load of the network is reduced.

VI. Conclusion

In this paper, we study the topology control problem with interference load considerations in wireless ad hoc networks. Since finding a minimum interference load topology is NP-complete, we derive some essential theorems to serve as guidelines for constructing low interference load topologies in wireless ad hoc networks with bi-directional links. Based on the theoretical results, we propose a heuristic algorithm LILNF to determine network topologies with low interference loads. The time complexity of the algorithm is analyzed and the performance is evaluated via simulations. Compared with LIFE, the proposed LILNF algorithm has much improved performance. In particular, the interference load of our mechanism is reduced by up to 22.37% on average compared with existing work, while the maximum transmission power used in the topology is the same as that of existing work.

References

[1] Y. Wang and X. Y. Li, "Localized construction of bounded degree planar spanner," in *Proc. DIALM-POMC Joint Workshop on Foundations of Mobile Computing*, pp. 1035–1047, 2003.

[2] R. Wattenhofer, L. Li, P. Bahl, and Y. M. Want, "Distributed topology control for power efficient operation in multihop wireless ad hoc networks," in *Proc. IEEE INFOCOM 2001*, pp. 1388–1397, Apr. 2001.

[3] G. Toussaint, "The relative neighborhood graph of a finite planar set," *Pattern Recognition*, vol. 12, pp. 261–268, 1980.

[4] K. J. Supowit, "The relative neighborhood graph, with an application to minimum spanning trees," *J. ACM*, vol. 30, no. 3, pp. 428–448, July 1983.

[5] M. Burkhart, P. von Rickenbach, R. Wattenhofer, and A. Zollinger, "Does topology control reduce interference?" in *Proc. ACM MobiHOC*, pp. 9–19, 2004.

[6] L. Hu, "Topology control for multihop packet radio networks," *IEEE Trans. Commun.*, vol. 41, pp. 1482–1493, Oct. 1993.

[7] R. Ramanathan and R. Rosales-Hain, "Topology control of multihop wireless networks using transmit power adjustment," in *Proc. IEEE INFOCOM 2000*, pp. 90–100, 2000.

[8] X. Y. Li, G. Calinescu, and P. J. Wan, "Distributed construction of planar spanner and routing for ad hoc networks," in *Proc. IEEE INFOCOM 2002*, pp. 1268–1277, New York, June 2002.

[9] N. Li, J. C. Hou, and L. Sha, "Design and analysis of an MST-based topology control algorithm," in *Proc. IEEE INFOCOM 2003*, pp. 1702–1712, San Francisco, CA, USA, Apr. 2003.

[10] V. Kawadia and P. Kumar, "Power control and clustering in ad hoc networks," in *Proc. IEEE INFOCOM 2003*, pp. 459–469, San Francisco, CA, USA, Apr. 2003.

[11] L. Bao and J. Garcia-Luna-Aceves, "Topology management in ad hoc networks," in *Proc. ACM MobiHOC*, pp. 129–140, 2003.

[12] L. Jia, R. Rajaraman, and R. Suel, "An efficient distributed algorithm for constructing small dominating sets," in *Proc. ACM Symposium on Principles of Distributed Computing*, pp. 33–42, 2001.

[13] F. Meyer auf der Heide, C. Schindelhauer, K. Volbert, and M. Grunewald, "Energy, congestion and dilation in radio networks," in *Proc. ACM Symposium on Parallel Algorithms and Architectures*, pp. 230–237, 2002.

[14] K. Gabriel and R. Sokal, "A new statistical approach to geographic variation analysis," *System Zoology*, vol 18, pp. 259–278, 1969.

[15] K. D. Wu and W. J. Liao, "On constructing low interference topology in multihop wireless networks," in *Proc. IEEE ICC 2006*, pp. 3759–3764, Istanbul, Turkey, June 2006.

[16] K. D. Wu and W. J. Liao, "On constructing low interference topology in wireless sensor networks," *International J. Sensor Networks*, special issue on wireless ad hoc and sensor networks, vol. 2, no. 5/6, pp. 321–330, 2007.

Kun-Da Wu received his B.S. degree from the department of Computer Science and Information Engineering in National Chiao-Tung University, Taiwan, ROC in 1996, and M.S. degree from the department of Computer Science and Engineering in National Sun Yat-Sen University in 1998. He is currently a Ph.D. candidate in the department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, ROC. His research interests include wireless ad hoc networks, resource allocation, and network optimization.

Wanjiun Liao received the BS and MS degrees from National Chiao Tung University, Taiwan, in 1990 and 1992, respectively, and the Ph.D. degree in Electrical Engineering from the University of Southern California, Los Angeles, California, USA, in 1997. She joined the Department of Electrical Engineering, National Taiwan University (NTU), Taipei, Taiwan, as an Assistant Professor in 1997. Since August 2005, she has been a full professor in the EE department and the Graduate Institute of Communication Engineering at NTU. Her research interests include wireless networks, multimedia networks, and broadband access networks. Dr. Liao is currently an Associate Editor of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and IEEE TRANSACTIONS ON MULTIMEDIA. She served as the Technical Program Committee (TPC) chairs/co-chairs of many international conferences, including the Tutorial Co-Chair of IEEE INFOCOM 2004, the Technical Program Vice Chair of IEEE Globecom 2005 Symposium on Autonomous Networks, and the Technical Program Co-Chair of IEEE Globecom 2007 General Symposium. Dr. Liao has received many research awards. Papers she co-authored with her students received the Best Student Paper Award at the First IEEE International Conferences on Multimedia and Expo (ICME) in 2000, and the Best Paper Award at the First IEEE International Conferences on Communications, Circuits and Systems (ICCCAS) in 2002. Dr. Liao was the recipient of K. T. Li Young Researcher Award honored by ACM in 2003, and the recipient of Distinguished Research Award from National Science Council in Taiwan in 2006. She is a Senior member of IEEE.