Estimation of Stationary Phase Noise by the Autocorrelation of the ICI Weighting Function in OFDM Systems

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***Abstract*— Phase noise resulting in Common Phase Error (CPE) and Inter-Carrier Interference (ICI) is a critical challenge to the implementation of OFDM systems. Modeling phase noise as a stationary Gaussian random process with the specified power spectrum density, different from conventional approaches which mostly relay on pilots to provide CPE estimation, we explore the statistical characteristics of the sufficient statistics then propose a pilot-aided decision-directed approach according to maximum-likelihood criterion. Numerical results demonstrate that the proposed algorithm enjoys 2dB gain at moderate SNR and is quite robust against possible model mismatch.**

***Index Terms*— Maximum-likelihood estimation, OFDM, phase noise.**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) is a spectrally efficient transmission technique which can combat dispersive channel by performing simple frequency domain equalization. However, OFDM is much more sensitive to synchronization errors than single carrier systems. Among them, phase noise which is mainly caused by the instability of the local oscillator is a crucial challenge. In [1], phase noise is classified into two categories. When the system is only frequency-locked, the resulting phase noise is slowly varying but not limited, and it is modeled as a zero-mean, nonstationary, infinite-power Wiener process. When the system is phase-locked, the resulting phase noise is low and modeled as a zero-mean, stationary, finite-power random process. Here, we concentrate on the suppression of the stationary phase noise process.

To eliminate the effect of phase noise, we propose a Pilot-Aided Decision-Directed (PADD) approach based on maximum likelihood criterion. Distinct from conventional phase noise suppression algorithms [2]–[4] which do not consider the statistical information of phase noise, we investigate the autocorrelation function of the ICI weighting function which is the kernel of the second order statistics of the ICI to enhance the performance. Under stationary phase noise model, the relation between the autocorrelation function of phase noise process and that of the ICI weighting function is analytically derived. Numerical results demonstrate the effectiveness of the proposed algorithm.

Manuscript received February 3, 2005; revised December 31, 2005; accepted March 1, 2006. The associate editor coordinating the review of this letter and approving it for publication was F. Daneshgaran. This research was supported by the National Science Council under the contract NSC 94-2219-E-002-022.

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Digital Object Identifier 10.1109/TWC.2006.05071

II. Signal Model

We start considering a general OFDM system using N-point inverse fast Fourier transform (IFFT) for modulation. Assume the frequency domain subcarrier index set is composed of three mutually exclusive subsets defined by

$$\begin{aligned} \mathcal{D} &\triangleq \{d_1, d_2, \cdots, d_{N_d}\} \\ \mathcal{P} &\triangleq \{p_1, p_2, \cdots, p_{N_p}\} \\ \mathcal{V} &\triangleq \{v_1, v_2, \cdots, v_{N_v}\}, \end{aligned} \quad (1)$$

where $\mathcal{D}$ denotes the set of indices for N_d data-conveying subcarriers, $\mathcal{P}$ is the set of indices for N_p pilot subcarriers and $\mathcal{V}$ stands for N_v virtual subcarriers. Then, the set of indices for N_u useful subcarriers $\mathcal{U}$ can be defined as

$$\mathcal{U} \triangleq \{u_1, u_2, \cdots, u_{N_u}\} = \mathcal{D} \cup \mathcal{P}, \quad (2)$$

where $N_u = N_d + N_p$. Let $X_m(k)$ be the modulated symbol on the k-th subcarrier of the m-th OFDM symbol. For $k \in \mathcal{U}$, $X_m(k)$ is taken from some constellation with zero mean and average power $\sigma_X^2 \equiv E\{|X_m(k)|^2\}$. The output of the IFFT has a duration of T seconds which is equivalent to N samples. A N_g-sample cyclic prefix longer than the channel impulse response is preceded to eliminate the inter-symbol interference (ISI).

At the receiver, timing and frequency offset is assumed to be recovered. Considering the multiplicative phase noise and the additive white noise, the received n-th sample of the m-th OFDM symbol can be written as

$$r_m(n) = [x_m(n) \otimes h_m(n)]\, e^{j\phi_m(n)} + \xi_m(n) \quad (3)$$

in which $\otimes$ is the circular convolution and $\{x_m(n)\}$, $\{h_m(n)\}$ and $\{\phi_m(n)\}$ with $n \in \{0, \cdots, N-1\}$ represent the transmitted signal, the channel impulse response and the phase noise respectively, while $\{\xi_m(n)\}$ denotes the AWGN. After removing the cyclic prefix and performing the FFT, the frequency domain symbol can be expressed by

$$R_m(k) = \Phi_m(0) H_m(k) X_m(k) + \underbrace{\sum_{\substack{l \in \mathcal{U} \\ l \neq k}} \Phi_m(k-l) H_m(l) X_m(l) + Z_m(k)}_{I_m(k)} \quad (4)$$

where $H_m(k)$ is the channel frequency response and $Z_m(k)$ denotes the frequency domain expression of $\xi_m(n)$. $\Phi_m(q)$ is the discrete Fourier transform of the phase noise process given by

$$\Phi_m(q) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi_m(n)} e^{-j2\pi \frac{qn}{N}}. \quad (5)$$

It can be viewed as a weighting function on the transmitted frequency domain symbols. In particular, when $q = 0$, $\Phi_m(0)$ is actually the time average of the phase noise process within one OFDM symbol duration. This term is usually known as the Common Phase Error (CPE) which causes the same phase rotation and amplitude distortion to each transmitted frequency domain symbol. On the other hand, when $q \neq 0$, the second term in (4) is the Inter-Carrier Interference (ICI) resulting from contributions of other subcarriers by the weighting of $\Phi_m(q)$ due to the loss of orthogonality. We call $\Phi_m(q)$ as the ICI weighting function.

Based on (4), the received frequency domain vector can be given by

$$\begin{aligned} \boldsymbol{r}_m &= \Phi_m(0)\boldsymbol{H}_m\boldsymbol{x}_m + \boldsymbol{\iota}_m + \boldsymbol{\zeta}_m \\ &= \Phi_m(0)\boldsymbol{H}_m\boldsymbol{x}_m + \boldsymbol{\varepsilon}_m, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{H}_m &\triangleq diag(H_m(0), H_m(1), \cdots, H_m(N-1)) \\ \boldsymbol{x}_m &\triangleq \begin{bmatrix} X_m(0) & X_m(1) & \cdots & X_m(N-1) \end{bmatrix}^T \\ \boldsymbol{\iota}_m &\triangleq \begin{bmatrix} I_m(0) & I_m(1) & \cdots & I_m(N-1) \end{bmatrix}^T \\ \boldsymbol{\zeta}_m &\triangleq \begin{bmatrix} Z_m(0) & Z_m(1) & \cdots & Z_m(N-1) \end{bmatrix}^T \end{aligned} \tag{7}$$

and $diag(\cdot)$ is a diagonal matrix. We add a second subscript to one of the vector or matrix variables defined in (6) and (7) to indicate its sub-vector or sub-matrix which is taken according to one of the subcarrier index sets in (1) and (2). The second subscript may be chosen from $\{p, d, v, u\}$ which relates to $\{\mathcal{P}, \mathcal{D}, \mathcal{V}, \mathcal{U}\}$ respectively. For example,

$$\boldsymbol{r}_{m,p} = \begin{bmatrix} R_m(p_1) & R_m(p_2) & \cdots & R_m(p_{N_p}) \end{bmatrix}^T \tag{8}$$

stands for the received pilot vector.

Conventionally, $\boldsymbol{r}_{m,p}$ is utilized to obtain the channel frequency response and the CPE to carry out equalization on $\boldsymbol{r}_{m,d}$ and finally send the equalized results to the detection block to get the decisions. Since accurate channel estimation in OFDM systems can be obtained by either preambles or pilot symbols [5], we assume that the channel frequency response is acquired by the receiver in subsequent sections.

III. Phase Noise Model

Accurate modeling of oscillator phase noise is a key factor to the analysis and simulation of the distortion and interference caused by phase noise. For a classical model of stationary phase noise, $\phi_m(n)$ can be modeled as a stationary Gaussian process with zero mean and the specified power spectrum density (PSD). Two types of phase noise PSD are given in the literature, they differ in the decay of the transition band. The well-known model proposed by [3] which has a linear decay is given by

$$S_\phi(f) = 10^{-c} + \begin{cases} 10^{-a} & |f| \leq f_l \\ 10^{-\frac{(f-f_l)b}{f_h - f_l} - a} & f_l < f \\ 10^{-\frac{(f+f_l)b}{f_h - f_l} - a} & f < -f_l \end{cases} \tag{9}$$

where c determines the white phase noise floor of the oscillator. The parameter a gives the phase noise level near the center frequency up to $\pm f_l$ and b is the steepness of noise reduction with increasing frequency distance up to $\pm f_h$ where the noise floor becomes dominant. Another important characterization of stationary phase noise is its Auto-Correlation Function (ACF). The ACF of a phase noise process can be obtained by performing the inverse Fourier transform of its PSD. In [6], the ACF of (9) is given by

$$\begin{aligned} R_\phi(\tau) =& 10^{-c}\delta(\tau) + 2 \cdot 10^{-a}\Bigg[f_l \cdot \mathrm{sinc}(2 f_l \tau) \\ &+ \frac{p\cos 2\pi\tau f_l - (2\pi\tau)\sin 2\pi\tau f_l}{p^2 + (2\pi\tau)^2} \Bigg] \end{aligned} \tag{10}$$

where

$$p = \frac{b \ln 10}{f_h - f_l}, \quad q = \frac{b \ln 10}{(f_h - f_l)T_s}.$$

An alternative PSD model [7] which has a logarithmic decay is given by

$$S_\psi(f) = 10^{-c} + \begin{cases} 10^{-a} & |f| \leq f_l \\ 10^{-a}(f/f_l)^{-b} & f_l < f \\ 10^{-a}(-f/f_l)^{-b} & f < -f_l \end{cases}, \tag{11}$$

and its corresponding ACF is

$$\begin{aligned} R_\psi(\tau) =& 10^{-c}\delta(\tau) + 2 \cdot 10^{-a} \cdot f_l \cdot \mathrm{sinc}(2 f_l \tau) \\ &+ 2 f_l^b \cdot 10^{-a} \int_{f_l}^{\infty} f^{-b} \cos 2\pi f \tau df. \end{aligned} \tag{12}$$

An example of the two phase noise PSD model where $a = 7$, $b = 4$, $c = 11$, $f_l = 10$ KHz and $f_h = 100$ KHz is illustrated in Fig. 1.

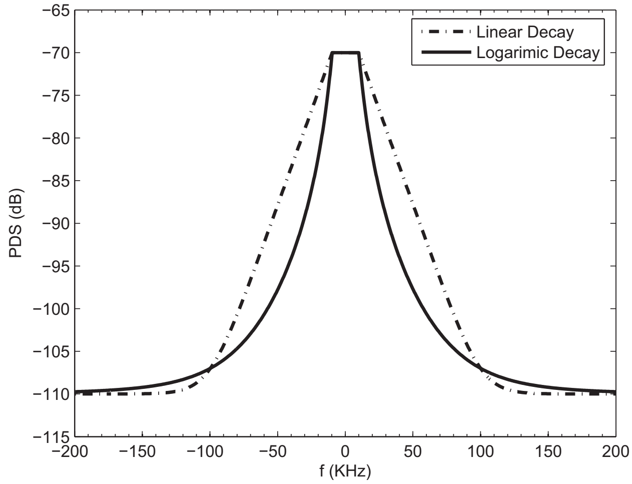


Fig. 1. PSD of phase noise processes.

IV. Data-Aided Decision-Directed CPE Estimation

Most OFDM systems employ pilots to facilitate receiver synchronization since data-aided estimation gives better and steadier estimate. However, since pilots cost system utilization, the number of pilots should be kept as small as possible which confines the performance of pilot-aided CPE correction algorithms. Comparatively, decision-directed approaches enjoy a larger observation space, however, to ensure the acceptable correctness of the decision, they can only be operated when phase noise is small. Therefore, one can use pilots to acquire the CPE and provide an initial compensation, then enlarge the observation space by including the tentative decisions as the sufficient statistics to perform the final estimate which can benefit from the advantages of both approaches. We refer this method as the Pilot-Aided-Decision-Directed (PADD) approach.

Considering the pilot-aided decision-directed approach, the sufficient statistics is $\boldsymbol{r}_{m,u}$ and can be modeled as a complex Gaussian random vector with mean vector $\Phi_m(0)\boldsymbol{H}_{m,u}\boldsymbol{x}_{m,u}$ and covariance matrix $\boldsymbol{C}_{\boldsymbol{r}_{m,u}}$. We will investigate of the statistical characteristics of the sufficient statistics $\boldsymbol{r}_{m,u}$ in next section. Assume that we have acquired the covariance matrix, the log-likelihood function of $\Phi_m(0)$ can be given by

$$\Lambda(\Phi_m(0)) = 2\Re\left\{\boldsymbol{x}_{m,u}^H\boldsymbol{H}_{m,u}^H\boldsymbol{C}_{\boldsymbol{r}_{m,u}}^{-1}\boldsymbol{r}_{m,u}\Phi_m^*(0)\right\} - \boldsymbol{x}_{m,u}^H\boldsymbol{H}_{m,u}^H\boldsymbol{C}_{\boldsymbol{r}_{m,u}}^{-1}\boldsymbol{H}_{m,u}\boldsymbol{x}_{m,u}|\Phi_m(0)|^2. \tag{13}$$

Differentiating (13) with respect to $\Phi_m(0)$, the ML estimation of the common phase error can be given by

$$\hat{\Phi}_m(0) = \frac{\boldsymbol{x}_{m,u}^H\boldsymbol{H}_{m,u}^H\boldsymbol{C}_{\boldsymbol{r}_{m,u}}^{-1}\boldsymbol{r}_{m,u}}{\boldsymbol{x}_{m,u}^H\boldsymbol{H}_{m,u}^H\boldsymbol{C}_{\boldsymbol{r}_{m,u}}^{-1}\boldsymbol{H}_{m,u}\boldsymbol{x}_{m,u}}. \tag{14}$$

Please note that we need the covariance matrix which requires the second order statistics of the ICI to perform this estimation.

V. Statistical Characteristics of the Sufficient Statistics

In the PADD approach, since the data and the channel frequency response are acquired, the statistical characteristics of the sufficient statistics mentioned above depend on that of the ICI and AWGN noise. The AWGN noise on each subcarrier can be modeled as a zero mean complex Gaussian random variable with variance σ_Z^2. As for the ICI, we also approximate its distribution as complex Gaussian. The argument of this reasonable approach is confirmed by our simulation which is not illustrated here due to the limit of space. In the following, we first show that the ICI has a zero mean, then, the second order statistics of the ICI will be investigated.

1) The Mean of the ICI: To investigate the statistical characteristic of $R_m(k)$, we need to analyze the ICI, namely, $I_m(k)$ first. The ICI can be modeled as a random variable which is independent of $Z_m(k)$, since $X_m(k)$ and $H_m(k)$ are assumed to be known in PADD circumstance, the mean of the ICI depends on the mean of the ICI weighting function $\Phi_m(q)$ which can be expressed as

$$E[\Phi_m(q)] = \frac{1}{N}\sum_{n=0}^{N-1} E\left[e^{j\phi_m(n)}\right]e^{j2\pi\frac{qn}{N}} \tag{15}$$

where $E[\cdot]$ is the expectation operation. Since $\phi_m(n)$ is stationary, $E[e^{j\phi_m(n)}]$ shall be a constant and can be taken out of the summation such that

$$\sum_{n=0}^{N-1} e^{j2\pi\frac{qn}{N}} = 0, \quad \forall q \neq 0. \tag{16}$$

Therefore, the mean of the ICI becomes zero.

2) The Second Order Statistics of the ICI: In a pilot-aided decision-directed scenario, the sufficient statistics are the received useful symbols given by

$$\boldsymbol{r}_{m,u} = \Phi_m(0)\boldsymbol{H}_{m,u}\boldsymbol{x}_{m,u} + \boldsymbol{\iota}_{m,u} + \boldsymbol{\zeta}_{m,u}. \tag{17}$$

Similarly, we may denote the covariance matrix of $\boldsymbol{\iota}_{m,u}$ by $\boldsymbol{C}_{\boldsymbol{\iota}_{m,u}}$, then the covariance matrix of $\boldsymbol{r}_{m,u}$ can be expressed as

$$\boldsymbol{C}_{\boldsymbol{r}_{m,u}} = \boldsymbol{C}_{\boldsymbol{\iota}_{m,u}} + \sigma_Z^2\boldsymbol{I}. \tag{18}$$

As for σ_Z^2, since it can be obtained by preamble signal and has been proposed in the literature[8], therefore, we may assume that σ_Z^2 is known by the receiver henceforth. Let the element of $\boldsymbol{C}_{\boldsymbol{\iota}_{m,u}}$ be denoted by $\sigma_i(k1,k2)$, it can be expressed as

$$\sigma_i(k_1,k_2) = \sum_{\substack{l_1\in\mathcal{U}\\ l_1\neq u_{k_1}}}\sum_{\substack{l_2\in\mathcal{U}\\ l_2\neq u_{k_2}}} H_m(l_1)H_m^*(l_2)X_m(l_1)X_m^*(l_2) E\left[\Phi_m(u_{k_1}-l_1)\Phi_m^*(u_{k_2}-l_2)\right]. \tag{19}$$

We can observe that $\sigma_i(k1,k2)$ depends on the autocorrelation function of the ICI weighting function $\Phi_m(q)$ which is defined as

$$R_\Phi(q_1,q_2) \equiv E[\Phi_m(q_1)\Phi_m^*(q_2)] = \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\frac{E[e^{j(\phi_m(n_1)-\phi_m(n_2))}]}{N^2}e^{-j2\pi\frac{n_1q_1-n_2q_2}{N}}. \tag{20}$$

Since $\phi_m(n)$ can be modeled as a stationary Gaussian random process, $\phi_m(n_1)-\phi_m(n_2)$ is a Gaussian random variable with zero mean and variance:

$$E[(\phi_m(n_1)-\phi_m(n_2))^2] = 2R_\phi(0) - 2R_\phi(n_1-n_2), \tag{21}$$

where $R_\phi(n)$ is the autocorrelation function of $\phi_m(n)$. Therefore,

$$E[e^{j(\phi_m(n_1)-\phi_m(n_2))}] = e^{R_\phi(n_1-n_2)-R_\phi(0)}. \tag{22}$$

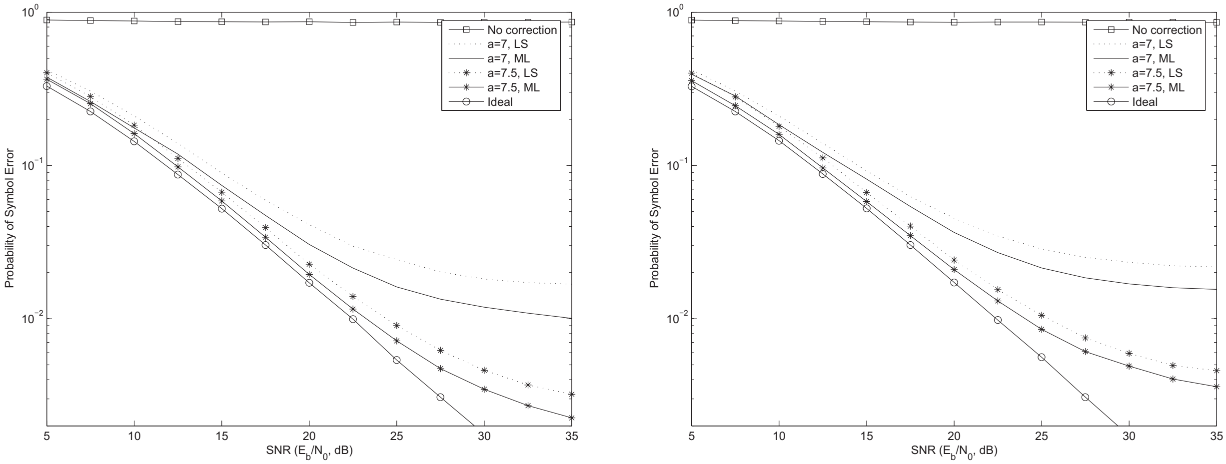
Substitute (22) into (20), we have

$$R_\Phi(q_1,q_2) = \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\frac{e^{R_\phi(n_1-n_2)-R_\phi(0)}}{N^2}e^{-j2\pi\frac{n_1q_1-n_2q_2}{N}}. \tag{23}$$

Because $R_\phi(n)$ is already available from (10) and (12), the ML CPE estimator (14) can be performed.

VI. Numerical Results

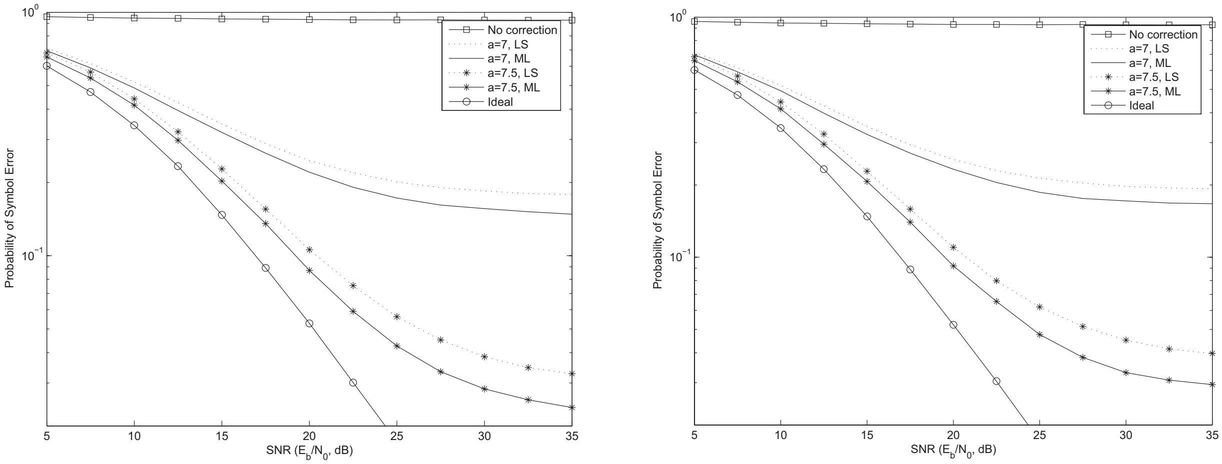
The proposed maximum-likelihood CPE estimator is evaluated in frequency selective slowly fading channels with 50 ns and 75 ns rms delay spread [9]. Channel impulse response remains static within a frame containing 16 symbols, but varies independently from frame to frame. Transmitted data is constructed according to IEEE 802.11a WLAN standard [10]. 16 QAM and 64 QAM which are more sensitive to phase noise than M-PSK, are used in the simulation to evaluate the performance of the proposed algorithm. Phase noise is generated using the linear-decay phase noise model with $b = 4$, $f_l = 100$ KHz and $f_h = 10f_l$ while a is chosen from



(a) r.m.s. delay spread = 50 ns

(b) r.m.s. delay spread = 75 ns

Fig. 2. Symbol error rate with 16 QAM.



(a) r.m.s. delay spread = 50 ns

(b) r.m.s. delay spread = 75 ns

Fig. 3. Symbol error rate with 64 QAM.

$\{7, 7.5\}$ and $c = a + 4$. Pilot-aided approach based on least-square criterion [4] is also simulated as a comparison. Each simulation point is generated using $3 \cdot 10^5$ OFDM symbols. The probability of symbol error (SER) with 16 QAM and 64 QAM in different channels are shown in Fig. 2 and Fig. 3 respectively.

About the effect of different channels, comparing Fig. 2(a) with Fig. 2(b), we can find that the shorter rms delay spread gives the better performance which is also evident in Fig 3. Observing Fig. 2 which corresponds to 16 QAM modulation, in general, the proposed ML estimator outperforms the LS estimator. The performance gaps between them at moderate SNR are 3 dB when $a = 7$ and 1 dB when $a = 7.5$ respectively. In Fig. 3 which corresponds to 64 QAM modulation, the performance gaps between the proposed ML estimator and the LS estimator at moderate SNR are both 2.5 dB when $a = 7$ and 7.5 respectively. In particular, to see the composite effect of different modulation schemes and phase noise severity, we first observe that under severer phase noise ($a = 7$), the performance gaps between the proposed ML estimator and the LS estimator in 64 QAM become smaller compared to 16 QAM. This phenomenon is mainly caused by the reduced correctness of the tentative decisions. Contrarily, under mild phase noise ($a = 7.5$), the performance gaps in 64 QAM become larger compared to 16 QAM. This phenomenon results from the closer Gaussian approximation of the distribution of the ICI when employing high order modulation which makes the estimation more accurate while the tantative decisons are still reliable under mild phase noise.

To examine the robustness of our proposed algorithm with respect to model mismatch, we discuss the situation when a parameter mismatch presents in the analytical phase noise model used at the receiver. The dominant parameter a is inspected and the results are illustrated in Fig. 4. The modulation used is 16 QAM and the channel r.m.s delay spread is 50 ns. The phase noise is generated by the aforementioned assumption except that a is fixed to 7. At the receiver, we vary the value of a which is used in the proposed CPE estimator to test the robustness. The operating SNR in the simulation are 15 dB, 20 dB and 25 dB. As we can see, under the parameter mismatch of a, our proposed algorithm is quite robust and can still outperform the conventional least-square based approach

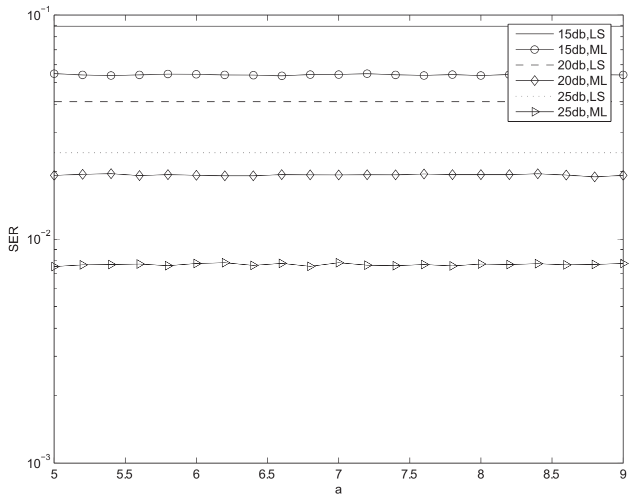


Fig. 4. The effect of model mismatch of a.

in the range of practical interest which justify the practicability of the proposed algorithm.

VII. CONCLUSION

In this letter, we propose a common phase error estimator based on maximum likelihood criterion to effectively remove the complex gain caused by stationary phase noise on the frequency domain transmitted symbols. We systematically derive the autocorrelation function of the ICI weighting function based on two stationary phase noise models. The second order statistics of the ICI can be calculated by this autocorrelation function. Then, different from conventional maximum likelihood approach that ideally assumes the inter-carrier interference observed on different subcarriers to be independent identically distributed, we combine pilot-aided and decision-directed approaches and investigate their covariances to yield the generalized maximum likelihood estimation scheme. The effectiveness of the proposed algorithm is manifested by computer simulations, moreover, the proposed algorithm is robust that in case of possible model mismatch, it still works and outperforms the conventional scheme.

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