Impact of Node Mobility on Link Duration in Multi-hop Mobile Networks

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Abstract-- In this paper, we study the impact of node mobility on link duration in multi-hop mobile networks. In multi-hop mobile networks, each node is free to move, and each link is established between two nodes. A link between two nodes is established when one node enters the transmission range of the other node, and the link is broken when either node leaves the transmission range of the other. The time interval during which the link remains active is referred to as the link duration. We develop an analytical framework for link duration in multi-hop mobile networks. We find that the link duration for two nodes is determined by the relative speed between the two nodes and the distance during which the link is connected, which are in turn determined by the angles between the two nodes' velocities and the angle of one node incident to the other node's transmission range, respectively. The analytical result is extended to model multipoint links which appear in the existing group mobility models. The accuracy of our framework is validated by simulations based on existing mobility models. The results show our model can describe accurately the link duration distribution for both types of links in multi-hop mobile networks, especially when the transmission range of each node is relatively smaller than the entire network coverage.

***Index Terms*—multi-hop mobile networks, link duration**

I. Introduction

Mobility management in wireless networks has been an active research topic for years [1-6]. Research efforts on single hop mobile networks include the results for cellular networks [1-4] or Mobile IP networks [5-6], i.e., only the last hop to each mobile node is wireless, and communications between nodes must go through the associated base stations (BSs). Each wireless link is established only between the BS and a node in a cell. When the node roams to another cell, the link is handed over to the respective BS so as to retain the on-going connection.

In a multi-hop mobile network, each node plays both roles of a router and an end-point, and no pre-deployed infrastructure such as BS is available for node communications. Data are relayed by intermediate nodes if the receiver node is beyond the transmission range of the sender node. As a result, a wireless link is established between two nodes. Each node is free to move arbitrarily. A link between two nodes is activated when one node enters the transmission range of the other, and the link is broken when either node leaves the transmission range of the other node. When a link on a routing path is broken, a rerouting process is initiated so as to reduce service disruption in the network.

In this paper, we study the impact of node mobility on link duration in multi-hop mobile networks. Specifically, we develop an analytical framework to evaluate link duration in such networks. In our framework, each node may move at a different speed, pause for a while, and then move again. The link duration here then corresponds to the time interval in which two nodes stay within transmission range of each other. The importance of link duration on system performance has been identified in the literature [7-8]. For example, the link duration may affect the lifetime of the routing path, which in turn determines the packet delivery ratio or per-connection throughput for an *S-D* pair. An analytical link duration model can also help determine the timer setting in ad hoc routing [7] or even on the design of better routing protocols to cope with link breakage caused by node mobility [9].

The framework we develop is based on the relative movement behavior of one node observed by the other node (i.e., from the perspective of the observer node). We find that the duration of a link between two nodes is determined by their relative speed and the distance traversed within the transmission range of the observer node during the link activation, which are in turn determined by the angle between the two nodes' velocities and the angle (formed by the relative velocity) incident into the observer node's transmission range, respectively. We also consider the link formed among a group of nodes, referred to as the multipoint link in this paper, in light of group mobility models (e.g., Reference Point Group Mobility Model [10] and Reference Velocity Group Mobility Model [11]) widely discussed in the literature for mobile ad hoc networks. The accuracy of our model is validated by simulations based on existing mobility models (e.g., random waypoint models [12], the random walk [13], and group mobility model [10-11]). We also demonstrate the usability of the derived model for different applications.

The rest of the paper is organized as follows. In Sec. II, the analytical framework for link duration of point-to-point links and multipoint links in multi-hop mobile network is developed. In Sec. III, the simulation results are provided to validate the analytical model. Finally, the paper is concluded in Sec. IV.

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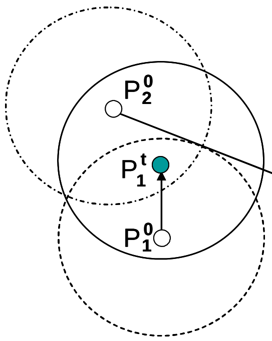
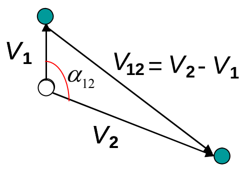
II. Analytical Model of Link Duration

In this section, we develop the probability distribution function of link duration for multi-hop mobile networks. Each node is assumed to use the same transmission power and move independently. We do not assume any specific mobility model for each node in this paper. Later in the simulation section, we will validate our model based on the existing mobility models such as the random waypoint model, random walk model, and group mobility models.

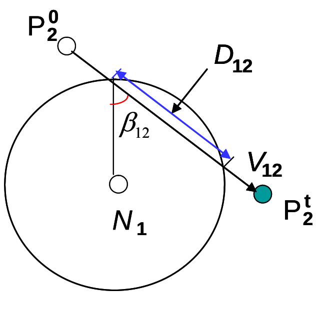
A. Link Duration between Two Nodes



(a) Absolute viewpoint of N_1 and N_2



(b) Relative velocity of N_1 and N_2



(c) Relative movement of N_2 observed by N_1, and D_{12} is the active distance between N_1 and N_2.

Figure 1. The relations between two nodes in terms of velocity and position in the network, where a circle represents the transmission range of the node centered at the circle.

Consider nodes N_i and N_j moving in the network. Let $\overrightarrow{V_i}$ and $\overrightarrow{V_j}$ denote the velocities of node N_i and node N_j, respectively; let P_i^0 denote the initial position of node N_i, and P_i^t, the position of N_i after time *t* with velocity $\overrightarrow{V_i}$. Fig. 1 shows the movements of N_1 and N_2 for time *t*. In Fig. 1 (a), the dotted circles represent the transmission ranges of the nodes centered at the circles; the solid circle represents the relationship between nodes N_1 and N_2. We are particularly interested in the relative movement behavior of N_2 observed by N_1, as shown in Fig. 1 (b), instead of the absolute viewpoints of both nodes as shown in Fig. 1 (a). From the perspective of N_1, N_2 is moving toward/away N_1 with relative velocity $\overrightarrow{V_{12}} = \overrightarrow{V_2} - \overrightarrow{V_1}$, and the relative speed is $V_{12} = |\overrightarrow{V_{12}}|$.

Let α_{ij} denote the angle between $\overrightarrow{V_i}$ and $\overrightarrow{V_j}$ given that the link between N_i and N_j can be established[1] (as shown in Fig. 1 (b)), and β_{ij} represent the incident angle, the relative velocity $\overrightarrow{V_{ij}}$, to N_i's transmission range (as shown in Fig. 1 (c)). Therefore, random variable α_{ij} ranges over [0, π], and β_{ij}, over [0, π/2]. Since N_i and N_j can only communicate part of the time, the distance traversed by N_j with relative velocity $\overrightarrow{V_{ij}}$ during which the link is activated is referred to as the *active distance* between N_i and N_j, denoted by D_{ij}. Fig. 1(c) illustrates D_{12}.

In Fig. 1(b), the value of angle α_{ij} determines the magnitude of V_{ij}, i.e.,

$$V_{ij} = \sqrt{V_i^2 + V_j^2 - 2V_iV_j \cos\alpha_{ij}} \,. \tag{1}$$

In Fig. 1(c), the value of angle β_{ij} determines the magnitude of D_{ij}, i.e.,

$$D_{ij} = 2r\cos(\beta_{ij}) \,. \tag{2}$$

Since the active distance D_{ij} and the relative speed V_{ij} are mutually independent (which will be proved shortly in the paper), the link duration T_{ij} between N_i and N_j can be expressed by

$$T_{ij} = \frac{D_{ij}}{V_{ij}} \,. \tag{3}$$

Since the probability distributions of V_{ij} and D_{ij} are determined by those of α_{ij} and β_{ij}, respectively, in what follows, we first develop the probability distributions of α_{ij} and β_{ij}, and then those of V_{ij} and D_{ij}, based on which the

[1] α_{ij} is a conditional random variable defined as the angle between velocities $\overrightarrow{V_i}$ and $\overrightarrow{V_j}$ given that the link between nodes N_i and N_j can be activated. In other words, α_{ij} is not the random variable representing the angle between any two nodes in the network, as a link may not be realized between any two arbitrary nodes.

distribution of T_{ij} can be derived. Note that since all nodes move independently and behave statistically identically, we will omit the argument *ij* of α_{ij}, β_{ij}, V_{ij}, D_{ij}, and T_{ij} in the subsequent analysis.

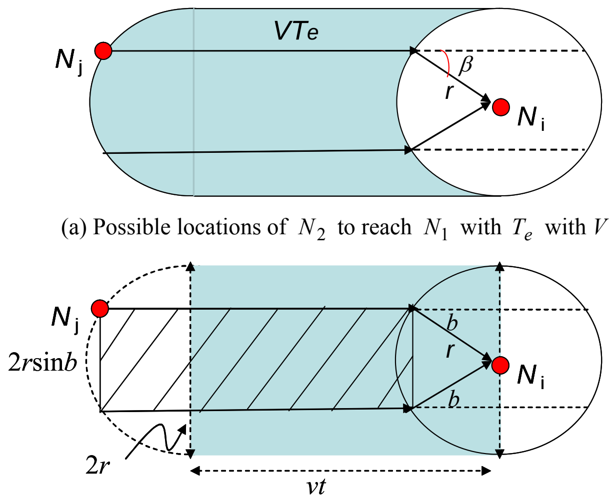
We start with the simplest case, i.e., each node is continuously roaming and moves at the same speed (i.e., $|\vec{V_i}| = |\vec{V_j}| = v_{fix}$). Later, we will extend it to the case with nodes moving at different speeds, and discuss the impact of node pause on link duration.

B. Probability Distribution of Link Duration for Point-to-Point Links without Pause

1) $|\vec{V_1}| = |\vec{V_2}| = v_{fix}$

Since each node moves at the same speed v_{fix}, the relative speed between two nodes given that the link in between can be established is given by

$$V = 2v_{fix}\sin(\alpha/2). \tag{4}$$



(a) Possible locations of N_2 to reach N_1 with T_e with V

(b) $\Pr\{\beta \le b \mid V = v, T_e = t\}$

Figure 2. An illustration of calculating $\Pr\{E_{LA} \mid V, T_e\}$

For ease of explanation, we assume that nodes outside the transmission range of node N_i are uniformly[2] distributed in the network. Let E_{LA} denote the event that the link between N_i and N_j is activated, and T_e, the random variable representing the elapsed time from when N_j starts to move with relative speed V to when the transmission range of N_i is reached. Given that N_j moves at relative speed $V = v$ toward N_i with elapsed time $T_e = t$, the probability that the link between N_i and N_j can be activated is equal to the probability that N_j starts moving from any point located in the shaded area as shown in Fig. 2 (a). The area of the shaded region can be obtained by $vt \cdot 2r$ as illustrated in Fig. 2 (b). Therefore, $\Pr\{E_{LA} \mid V = v, T_e = t\} = 2rvt$, where r is the transmission range of node N_i.

The probability $\Pr\{\beta \le b \mid V = v, T_e = t\}$ corresponds to the probability that node N_j falls inside the striped area as shown in Fig. 2 (b), i.e., the area from which N_j moving with $V = v$ and $T_e = t$ would reach N_i's transmission range with an angle no more than b, where $0 \le b \le \pi/2$. Therefore, the cumulative distribution function (*cdf*) of β can be expressed by

$$\Pr\{\beta \le b \mid V = v, T_e = t\} = 2rvt\sin b. \tag{5}$$

From (4), we have

$$\Pr\{\beta \le b \mid \alpha = a, T_e = t\} = 2v_{fix}t\sin\left(\frac{a}{2}\right)\sin b.$$

Thus,

$$\Pr\{\alpha \le a, \beta \le b \mid T_e = t\} = t\int_{x=0}^{a} f_{\alpha'}(x)\cdot\sin\left(\frac{x}{2}\right)\sin b\,dx$$
$$\propto \int_{x=0}^{a} f_{\alpha'}(x)\cdot\sin\left(\frac{x}{2}\right)\sin b\,dx. \tag{6}$$

where $f_{\alpha'}(.)$ is the *pdf* of the random variable α' denoting the angle between two arbitrary nodes.

Since, when the moving directions of N_i and N_j are assumed independent and nearly uniformly distributed from 0 to 2π, the distribution of angles between two arbitrary nodes can be approximated by a uniform distribution over [0,π], (6) can be re-expressed by

$$\int_{x=0}^{a} \frac{1}{\pi}\cdot\sin\left(\frac{x}{2}\right)\sin b\,dx = \frac{2}{\pi}\left[1-\cos\left(\frac{a}{2}\right)\right]\sin b. \tag{7}$$

The joint probability distribution of α and β in (7) is independent of the value taken by T_e, and can be expressed by $F_{\alpha,\beta}(a,b) = \Pr\{\alpha \le a, \beta \le b\} \propto \frac{2}{\pi}\left[1-\cos\left(\frac{a}{2}\right)\right]\sin b$, i.e.,

$$F_{\alpha,\beta}(a,b) = \frac{2}{\pi}\left[1-\cos\left(\frac{a}{2}\right)\right]\sin b\cdot K.$$

Since $F_{\alpha,\beta}(\pi, \frac{\pi}{2}) = 1$, this yields $K = \frac{\pi}{2}$. Thus,

$$F_{\alpha,\beta}(a,b) = \left[1-\cos\left(\frac{a}{2}\right)\right]\sin b. \tag{8}$$

From the joint *cdf* $F_{\alpha,\beta}(.)$, we can further obtain

$$F_{\alpha}(a) = 1-\cos\left(\frac{a}{2}\right), \tag{9}$$

[2] When this assumption does not hold, the derivation must be replaced with an integral of the grey area, instead of the area coverage itself as in the derivation.

$$F_\beta(b) = \sin b. \tag{10}$$

Thus, it follows that

$$f_\alpha(a) = \frac{dF_\alpha(a)}{da} = \frac{1}{2}\sin\left(\frac{a}{2}\right),$$

$$f_\beta(b) = \frac{dF_\beta(b)}{db} = \cos b.$$

From (8), (9) and (10), we have $F_{\alpha,\beta}(a,b) = F_\alpha(a)\cdot F_\beta(b)$. Thus, random variables α and β are independent.

Based on the distributions of α and β in (9) and (10), we can further derive the distributions of the active distance D and the relative speed V. From (4) and (9), we obtain

$$F_V(v) = \Pr\{V \le v\} = \Pr\left\{2v_{fix}\sin\left(\frac{\alpha}{2}\right) \le v\right\}$$
$$= F_\alpha\left(2\sin^{-1}\left(\frac{v}{2v_{fix}}\right)\right) = 1-\sqrt{1-\left(\frac{v}{2v_{fix}}\right)^2}. \tag{11}$$

$$f_V(v) = \frac{dF_V(v)}{dv} = \frac{v}{2v_{fix}\sqrt{4v_{fix}^2 - v^2}}. \tag{12}$$

From (3) and (10), we obtain

$$F_D(d) = \Pr\{D \le d\} = \Pr\{2r\cos\beta \le d\}$$
$$= \Pr\left\{\beta \ge \cos^{-1}\left(\frac{d}{2r}\right)\right\} = 1 - F_\beta\left(\cos^{-1}\left(\frac{d}{2r}\right)\right) \tag{13}$$
$$= 1-\sqrt{1-\left(\frac{d}{2r}\right)^2}.$$

$$f_D(d) = \frac{dF_D(d)}{dd} = \frac{d}{2r\sqrt{4r^2-d^2}}. \tag{14}$$

Substituting the derived distributions of D and V in (11) and (13) into (3), we obtain the distribution of T accordingly, i.e.,

$$F_T(t) = \Pr\{T \le t\} = \Pr\left\{\frac{D}{V} \le t\right\} = \Pr\left\{\frac{2r\cos\beta}{2v_{fix}\sin\left(\frac{\alpha}{2}\right)} \le t\right\}$$
$$= \int_{b=0}^{\frac{\pi}{2}} f_\beta(b)\cdot\Pr\left\{\frac{2r\cos b}{2v_{fix}\cdot\sin\left(\frac{\alpha}{2}\right)} \le t\right\}db$$
$$= \int_{b=0}^{\frac{\pi}{2}} f_\beta(b)\cdot\Pr\left\{\sin\left(\frac{\alpha}{2}\right) \ge \frac{r\cos b}{v_{fix}\cdot t}\right\}db.$$

Since $\Pr\left\{\sin\left(\frac{\alpha}{2}\right) \ge \frac{r\cos b}{v_{fix}\cdot t} \mid t \le \frac{r}{v_{fix}}, b \le \cos^{-1}\left(\frac{v_{fix}\cdot t}{r}\right)\right\} = \Pr\left\{\sin\left(\frac{\alpha}{2}\right) \ge \frac{r\cos b}{v_{fix}\cdot t} \ge 1\right\} = 0$, we can further decompose the distribution $F_T(t)$ into two cases.

Case 1: $t \le \frac{r}{v_{fix}}$,

$$F_T(t) = \int_{b=\cos^{-1}\left(\frac{v_{fix}\cdot t}{r}\right)}^{\frac{\pi}{2}} f_\beta(b)\cdot\Pr\left\{\sin\left(\frac{\alpha}{2}\right) \ge \frac{r\cos b}{v_{fix}\cdot t}\right\}db$$
$$= \int_{b=\cos^{-1}\left(\frac{v_{fix}\cdot t}{r}\right)}^{\frac{\pi}{2}} \cos b\cdot\sqrt{1-\left(\frac{r\cos b}{v_{fix}\cdot t}\right)^2}\,db \tag{15}$$
$$= \frac{1}{2} - \frac{r^2 - v_{fix}^2 t^2}{2rv_{fix}t}\ln\left(\frac{r+V_{fix}t}{\sqrt{r^2 - v_{fix}^2 t^2}}\right).$$

Case 2: $t > \frac{r}{v_{fix}}$,

$$F_T(t) = \int_{b=0}^{\frac{\pi}{2}} \cos b\cdot\sqrt{1-\frac{r^2\cos^2 b}{v_{fix}^2\cdot t^2}}\,db$$
$$= \frac{1}{2} - \frac{r^2 - v_{fix}^2 t^2}{2rv_{fix}t}\ln\left(\frac{r+v_{fix}t}{\sqrt{v_{fix}^2 t^2 - r^2}}\right). \tag{16}$$

From (15) and (16), we can obtain the probability distribution of link duration for point-to-point links as follows.

$$F_T(t) = \frac{1}{2} - \frac{r^2 - v_{fix}^2 t^2}{2rv_{fix}t}\ln\left(\frac{r+v_{fix}t}{\sqrt{\left|r^2 - v_{fix}^2 t^2\right|}}\right) \tag{17}$$

$$f_T(x) = \frac{dF_T(t)}{dt} = \frac{r^2 + v_{fix}^2 t^2}{2rv_{fix}t^2}\ln\left(\frac{r+v_{fix}t}{\sqrt{\left|r^2 - v_{fix}^2 t^2\right|}}\right) - \frac{1}{2t}$$

2) $\left|\vec{V_i}\right| = v_i$ and $\left|\vec{V_j}\right| = v_j$

Next, we consider the case that the velocities of N_i and N_j are no longer fixed. Let $|\vec{V_i}| = v_i$ and $|\vec{V_j}| = v_j$. The relative speed between N_i and N_j can be expressed by

$$V = \sqrt{v_i^2 + v_j^2 - 2v_i v_j \cos\alpha} = \sqrt{v_i^2 + v_j^2} \cdot \sqrt{1 - \frac{2v_i v_j}{v_i^2 + v_j^2} \cdot \cos\alpha},$$

which can be approximated by

$$\sqrt{v_i^2 + v_j^2} \cdot \left(1 - \frac{v_i v_j}{v_i^2 + v_j^2} \cdot \cos\alpha\right). \tag{18}$$

Accordingly, we have

$$\Pr\left\{ \alpha \le a, \beta \le b \mid T_e = t,\ \left|\vec{V_i}\right| = v_i,\ \left|\vec{V_j}\right| = v_j \right\}$$

$$\propto \int_{x=0}^{a} \frac{1}{\pi} \cdot \sqrt{v_i^2 + v_j^2} \cdot \left(1 - \frac{v_i v_j}{v_i^2 + v_j^2} \cdot \cos\alpha\right) \sin b dx$$

$$= \frac{1}{\pi} \sqrt{v_i^2 + v_j^2} \cdot \left(\alpha - \frac{v_i v_j}{v_i^2 + v_j^2} \cdot \sin\alpha\right) \sin b,$$

and

$$F_T(t) = \Pr\left\{ T \le t \mid \left|\vec{V_i}\right| = v_i,\ \left|\vec{V_j}\right| = v_j \right\}$$

$$= \int_0^{\frac{\pi}{2}} \cos b \cdot \Pr\left\{ V > \frac{2r\cos b}{t} \right\}$$

$$= \int_{\cos^{-1}\left(\frac{tk_i(k_i - k_j)}{2r}\right)}^{\cos^{-1}\left(\frac{tk_i(k_i + k_j)}{2r}\right)} \cos b \left[1 - \frac{1}{\pi}\left(-k_j x_i + \cos^{-1} x_j\right)\right] db,$$

where

$$k_i = \sqrt{v_i^2 + v_j^2},\ k_j = \frac{v_i v_j}{v_i^2 + v_j^2},\ x_i = \sqrt{1 - \left(\frac{k_i - \frac{2r\cos b}{t}}{k_1 k_j}\right)^2},\ \text{and}$$

$$x_j = \left(\frac{k_i - \frac{2r\cos b}{t}}{k_i k_j}\right).$$

Denote the speed of each node by V_I, which is a random variable distributed over [$V_{I_{\min}}, V_{I_{\max}}$]. Thus, we have

$$F_T(t) = \int_{V_{I_{\min}}}^{V_{I_{\max}}} \int_{V_{I_{\min}}}^{V_{I_{\max}}} f_{V_I}(v_i) \cdot f_{V_I}(v_j) \cdot \int_{\cos^{-1}\left(\frac{tk_i(k_i - k_j)}{2r}\right)}^{\cos^{-1}\left(\frac{tk_i(k_i + k_j)}{2r}\right)} \cos b \left[1 - \frac{1}{\pi}\left(-k_j x_i + \cos^{-1} x_j\right)\right] db dv_i dv_j. \tag{19}$$

C. Probability Distribution of Link Duration with Pause

We now study the impact of node pause on link duration. Suppose that each node may be either in the state of movement or in pause state. When either node enters the pause state, the link duration is determined by the moving node's velocity.

We model link duration with two components[3]: both nodes are in movement and either node is in pause state. The weights of the two components can be decided by the percentage of the time each node is in motion or in pause state. Let M denote the mean movement duration and P be the duration of pause time. Then, a node is in movement with a probability of $\frac{M}{M+P}$ and in pause state with a probability of $\frac{P}{M+P}$. Thus, the *pdf* of the link duration with the consideration of pause can be expressed by $f_{T_P}(t) = \frac{M}{M+P} \cdot f_T(t) + \frac{P}{M+P} \cdot f_{T_s}(t)$, where $f_T(t)$ is the derivative of (19) (i.e., the case in which both nodes are in movement) and $f_{T_s}(t)$ is the case that either node is in pause state during the link activation. Thus, we obtain

$$F_{T_S}(t \mid V_I = v_i) = \Pr\{\frac{2r\cos\beta}{v_i} \le t\} = \Pr\{\beta \le \cos^{-1}(\frac{v_i t}{2r})\}$$

$$= \sin(\cos^{-1}(\frac{v_i t}{2r})) = 1 - \sqrt{1 - \left(\frac{v_i t}{2r}\right)^2},$$

leading to

$$F_{T_S}(t) = \int_{V_{I_{\min}}}^{V_{I_{\max}}} f_{V_I}(v_i) \cdot (1 - \sqrt{1 - \left(\frac{v_i t}{2r}\right)^2}) dv_i. \tag{20}$$

D. Link Duration of Multi-point Link

Finally, we extend the derivation to capture the behavior of a multipoint link shared by multiple nodes. The derivation is similar to that of the Reference Point Group Model described in [10]. In this model, a multipoint link is formed among a group, led by a group leader. All member nodes move along with the leader according to a group mobility model, and each node moves independently by following a node mobility model. The link duration of a multipoint link is then referred to as the time interval in which all member nodes have left the transmission range of the leader node.

Suppose that there are m member nodes in a group (denoted by $N_1, N_2, \ldots, N_m$) led by a group leader N_0 and all member nodes in the group are initially inside the transmission range of N_0. We assume that all member nodes move independently

[3] We do not consider both nodes are in pause states in this paper due to space limitations. Later in the simulation, we will show that the probability of both nodes in pause states is small especially when the transmission range of each node is relatively smaller compared to the entire network coverage.

guided by their node mobility models and all nodes are continuously moving. Thus, all nodes will eventually move out of the transmission range of the leader nodes unless they move toward the same direction as the leader node. The link duration of a multipoint link corresponds to the longest link duration between each member node and the leader node, which can be expressed by

$$T_M = max\ \{T_1, T_2, \cdots, T_m\}, \tag{20}$$

where T_i is the link duration between nodes N_i and N_0.

Let $F_M(t)$ denote the *cdf* of the link duration for a multi-point link shared by *m* nodes, and $F_i(t)$ is the *cdf* of the link duration between the leader N_0 and the member node N_i. Since all nodes move mutually independently by their node mobility models and the moving direction of each node is uniformly distributed over [0, 2π]. We obtain

$$\begin{aligned} F_M(t) &= \Pr\{T_M < t\} = \Pr\{T_1 < t, T_2 < t, T_3 < t \ldots\ldots T_m < t\} \\ &= \Pr\{T_1 < t\} \cdot \Pr\{T_2 < t\} \cdot \Pr\{T_3 < t\} \ldots\ldots \cdot \Pr\{T_m < t\} \\ &= F_1(t) \cdot F_2(t) \cdot F_3(t) \ldots\ldots \cdot F_m(t) \\ &= (F_T(t))^m, \end{aligned} \tag{21}$$

where $F_i(t)$ is the conditional *cdf* of member node N_i, given $\overrightarrow{V_{0i}}$ and $\overrightarrow{R_{0i}}$.

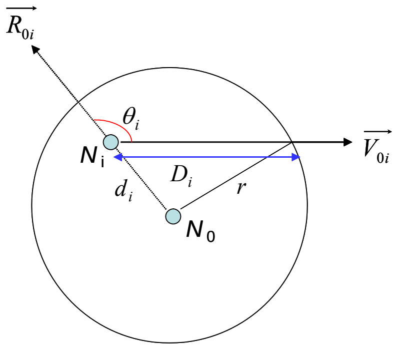


Figure 3. Relations between member node N_i and leader node N_0

The distribution of $F_i(t)$ is obtained as follows. In Fig. 3, θ_i is the angle formed by the relative velocity $\overrightarrow{V_{0i}}$ and the relative position vector $\overrightarrow{R_{0i}}$, and $d_i = \left|\overrightarrow{R_{0i}}\right|$. The active distance traversed by node N_i is

$$D_i = \sqrt{r^2 - d_i^{\ 2} \sin^2(\pi - \theta_i)} + d_i \cos(\pi - \theta_i).$$

From (3) we have

$$\begin{aligned} F_i(t) &= \Pr\left\{ \frac{D_i}{V_{0i}} \le t \,|\, \left|\overrightarrow{V_0}\right| = v_0, \left|\overrightarrow{V_i}\right| = v_i, \left|\overrightarrow{R_{0i}}\right| = d_i \right\} \\ &= \Pr\left\{ \frac{\sqrt{r^2 - d_i^{\ 2} \sin^2(\pi - \theta_i)} + d\cos(\pi - \theta_i)}{\sqrt{v_0^2 + v_i^2 - 2v_0 v_i \cos\alpha}} \le t \right\}, \end{aligned}$$

where V_{0i} denotes the relative speed between N_0 and N_i.

Let $F_{R_{0i}}(.)$ be the *cdf* of $\left|\overrightarrow{R_{0i}}\right|$. Thus,

$$F_i(t) = \begin{cases} \dfrac{1}{\pi^2} \cdot \displaystyle\int_0^{\pi}\!\!\int_0^{\pi} \left(1 - F_{R_{0i}}\left(\sqrt{\mathrm{r}^2 - v^2 t^2 \sin^2(\pi - \vartheta)} + vt\cos(\pi - \vartheta)\right)\right) da\, d\vartheta, \\ \text{if } t < \dfrac{r}{v}; \\ \dfrac{1}{\pi^2} \cdot \displaystyle\int_0^{\pi}\!\!\int_0^{\pi} \left(\begin{aligned} &1 - F_{R_{0i}}\left(\sqrt{\mathrm{r}^2 - v^2 t^2 \sin^2(\pi - \vartheta)} + vt\cos(\pi - \vartheta)\right) \\ &+ F_{R_{0i}}\left(-\sqrt{\mathrm{r}^2 - v^2 t^2 \sin^2(\pi - \vartheta)} + vt\cos(\pi - \vartheta)\right) \end{aligned} \right) da\, d\vartheta, \\ \text{if } t \ge \dfrac{r}{v}. \end{cases}$$

where $v = \sqrt{v_0^2 + v_i^2 - 2v_0 v_i \cos a}$.

III. Performance Evaluation

In this section, we validate our analytical results via an event-driven C++ simulator. In our simulations, 300 nodes are uniformly distributed in a unit-square area. Each node moves independently, including both speed and the moving direction, guided by a mobility model. The result in each figure is an average of 100 runs. We validate the link duration for both point-to-point links and multipoint links as follows.

A. Link Duration of Point-to-Point Links

We consider two models, described as follows.

i) The random waypoint model: each node selects a target location to move at a speed selected from a uniformly distributed interval [$V_{I_{\min}}, V_{I_{\max}}$]. Once the target is reached, the node pauses for a random time with probability *p* and then selects another target with another speed to move again.

ii) The random walk model: each node selects a direction and a speed pair [θ, *v*] from uniformly distributed intervals [0, 2π] and [$V_{I_{\min}}, V_{I_{\max}}$], respectively, and then starts to move for a time *t* uniformly selected from [0, $t_{\max}$]. Once the node has moved for *t* units of time, it pauses for a random time with probability *p* and then starts a new movement again.

1) Nodes with Fixed Speed

We first simulate the case with nodes moving at fixed speed $v_{fix} = 0.01$ unit per second under both mobility models (denoted by Sim-RWP and Sim-RW). The *pdf* of *T* is plotted in Fig. 4 with transmission range *r* =0.15, where RWP is for the random waypoint model, and RW, for the random walk model. The figure shows that link duration has the highest probability around the value r / v_{fix} for both models, explained as follows. A larger *T* results from a larger *D* and/or smaller *V*, which in turn results from a larger β and/or smaller α, and vice verse. Let us give a more intuitive explanation with the following two facts. First, two nodes moving in a similar direction have a smaller probability to meet and form a link. Second, with a given velocity, the point at which one node enters the other node's transmission range is uniformly distributed (denoted by x, 2x, and 3x in Fig. 5). However, when the incident point is near the tangential line, the active distance D is shortened rapidly, i.e., $D(x) > D(2x) > D(3x)$. Since T=D/V, the probability of large T is determined by the nodes with similar moving direction, which has a smaller chance for nodes to meet. On the other hand, the case of small T is determined by the low probability of a short active distance. Since the probability T being small or large is low, the peak is at the point where both α and β have the highest probabilities, i.e., $T = 2r\cos(0)/2v_{fix}\sin(\frac{\pi}{2}) = r/v_{fix}$.

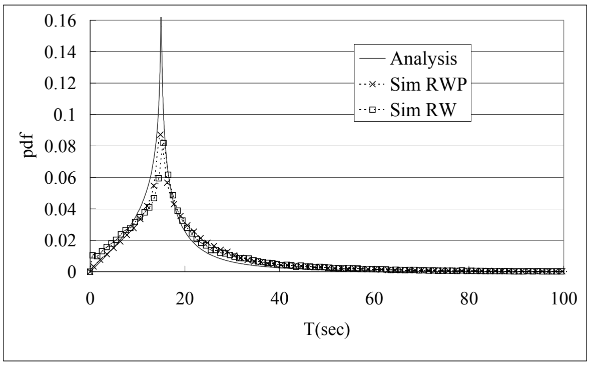


Figure 4. The *pdf* of T with fixed moving speed $v_{fix} = 0.01$, r=0.15

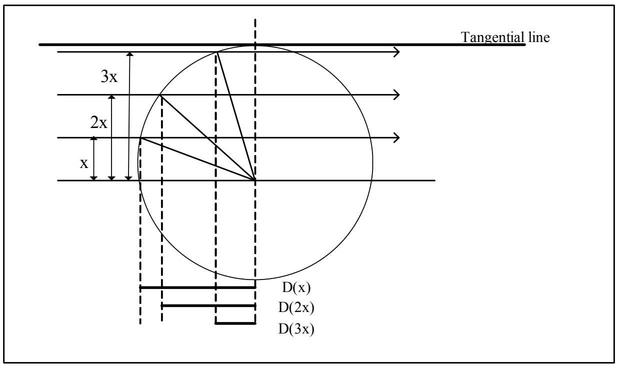


Figure 5. The active distances with different incident angles (i.e., at points x, 2x, and 3x)

2) Nodes with Different Speeds

Next, we show the simulation results when the speed of each node is uniformly selected over [0.005, 0.015] unit per second, again under the two models. The curve with "Ana-uniform" depicts the *pdf* of T with transmission range r=0.15 as in (19). The mismatch between the analytical curve and simulation curve is due to the fact that the speeds of both mobility models are not uniformly distributed [14]. To fix this problem, we let the *pdf* of nodal moving speeds to be inversely proportional to the speed itself, i.e., $f_{V_I}(v) = k \cdot \frac{1}{v}$. Since $\int_{V_{\min}}^{V_{\max}} f_{V_I}(v)dv = 1$, we obtain

$$f_{V_I}(v) = \frac{1}{v \cdot \ln \frac{V_{I_{\max}}}{V_{I_{\min}}}}. \tag{21}$$

With the revised density function (21), we integrate (19) over the range [$V_{I_{\min}}, V_{I_{\max}}$] to obtain the new probability density function of the link duration for point-to-point links. The curve in Fig. 6 with "Ana-non-uniform" shows the analytical result based on (21). The analytical result has an apparent accuracy improvement as compared to the uniformly distributed one.

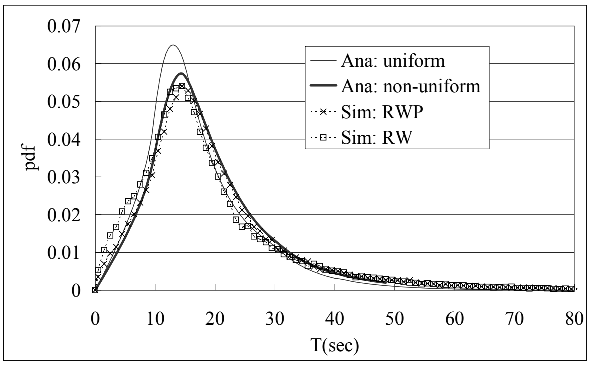


Figure 6. The *pdf* of T, where [$V_{I_{\min}}, V_{I_{\max}}$] =[0.005, 0.015], r=0.15

B. Link Duration with Pause

Then, we show the results when there are pauses between movements. Here we run simulations with the random waypoint model and the speed is fixed at 0.01 units per second with different pause times. Fig. 7 shows the comparison of the simulation and analysis results with a pause time of 50 seconds. We see that there are two peaks, i.e., at T=15 and 30 seconds, which are contributed by the two components (i.e., both nodes are in motion, and either is in pause), respectively. In this scenario, since the transmission range of each node is relatively smaller as compared to the entire network coverage, our model can accurately capture the behavior of link duration.

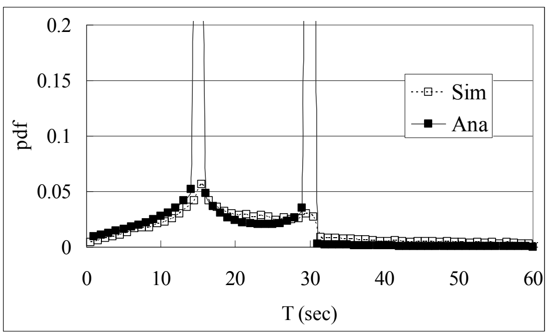


Figure 7. The pdf of *T* with pause time 50s, *r*=0.15

C. Boundary Effect on Link Duration

To observe the boundary effect, we simulate the random walk mobility model with a fixed movement interval of 100 seconds and a fixed speed of 0.01 units per sec. In this way, we can avoid having a movement ends during the link duration. From the simulation results in Fig. 8, we observe that when the ratio of transmission range to network size is below 0.2, the boundary effect is insignificant. However, when the ratio exceeds 0.25, the probability of short *T* for nodes near the boundary may have shorter link durations, so our analysis can better describe the link duration when the node transmission range is relatively smaller compared to the entire network size.

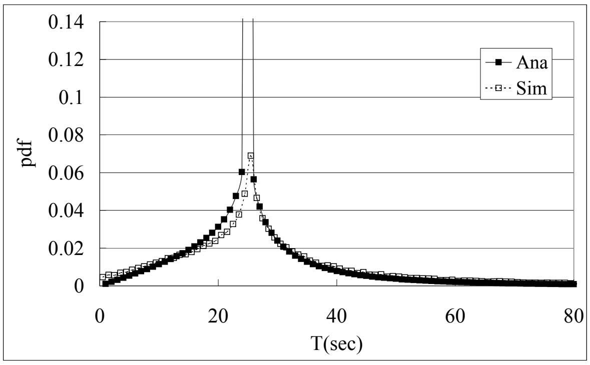


Figure 8. The *pdf* of *T* with fixed moving speed $v_{fix} = 0.01$, *r*=0.2

D. Link Duration of Multipoint Links

In this simulation, there are one leader node and 10 member nodes in a unit-square network. Each member node is initially located in the transmission range of the leader node, and its distance to the leader node is uniformly selected from the range [0, *r*] unit, where *r* is the transmission range of the leader node. All member nodes move with a fixed speed of 0.01 units per second, and the direction to move is uniformly distributed from [0, 2π]. Fig. 9 shows that compared to the distribution of point-to-point links, multi-point links have a higher probability for long link duration. This is due to the fact that as long as at least one of the member nodes chooses a similar direction as the leader node, the link will exist for a long time.

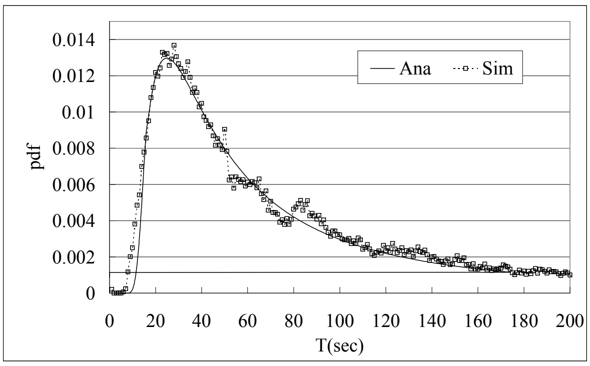


Figure 9. The *pdf* of multi-point link duration

IV. Conclusions and Future Work

In this paper, we propose an analytical framework to model the link duration for multi-hop mobile networks. We consider both point-to-point links between nodes and multipoint links shared by a group of nodes. Each node may be in movement or in pause state. The model starts with the derivation of the distributions of two parameters α and β, which in turn determine the distributions of the relative speed of the two considered nodes and the active distance between the two nodes for an activated link. Then, the *pdf* of link duration for point-to-point links can be obtained. The derived result is then extended to model multipoint links. We validate the analytical results via simulations with three different mobility models widely used in the literature, namely, the random waypoint, the random walk, and the group movement model.

In the future, we will further derive the path lifetime based on the analytical results developed in this paper. The probability distribution of the path lifetime in mobile ad hoc networks is even more difficult to derive than link duration since it is dependent on many system parameters such as spatial distribution, node density, and path connectivity.

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References

[1] Y. Fang, I. Chlamtac, and Y.-B. Lin, “Portable Movement Modeling for PCS Networks,” *IEEE Transactions on Vehicular Technology*, Vol. 49, Issue 4, 2000, pp.1356-1363.

[2] J. Li, H. Kameda, K. Li, “Optimal Dynamic Mobility Management for PCS Networks,” *IEEE Transactions on Networking*, Vol. 8, Issue 3, June 2000, pp. 319-327.

[3] Y. Fang, “Movement-based Mobility Management and Tradeoff Analysis for Wireless Mobile Networks,” *IEEE Transactions on Computers*, Vol. 52, Issue 6, June 2003, pp. 791-803.

[4] Y. -B. Lin and S. -R. Yang, “A Mobility Management Strategy for GPRS,” IEEE Transactions on Wireless Communications, Vol. 2, Issue 6, Nov. 2003, pp. 1178-1188.

[5] M. Wenchao and Y. Fang, “Dynamic Hierarchical Mobility Management Strategy for Mobile IP Networks,” *IEEE Journal on Selected Areas in Communications*, Vol. 22, Issue 4, May 2004, pp. 664-676.

[6] J. -T. Weng, W. Liao, and J. –R. Lai, “Modeling Node Mobility for Reliable Packet Delivery in Mobile IP Networks,” *IEEE Transactions on Wireless Communications*, Aug. 2006.

[7] J. Boleng, W. Navidi, and T. Camp, “Metrics to Enable Adaptive Protocols for Mobile Ad Hoc Networks,” *Proc. ICWN 2002*, Jun. 2002, pp.293-298.

[8] C. -L Chao, Y. -T. Wu, W. Liao, and J. -C. Kuo, “Link Duration of the Random Waypoint Model in Mobile Ad Hoc Networks,” *Proc. IEEE WCNC 2006*.

[9] J. Boleng and T. Camp, “Adaptive Location Aided Mobile Ad Hoc Network Routing,” Proc. IEEE IPCCC 2004, pp. 423-432, 2004.

[10] X. Hong, M. Gerla, G. Pei, and C.-C. Chiang, “A Group Mobility Model for Ad Hoc Wireless Networks,” *Proc. ACM international Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM)*, Aug. 1999.

[11] K.H. Wang and B. Li. "Group Mobility and Partition Prediction in Wireless Ad-Hoc Networks," *IEEE ICC 2002*, Apr. 2002, pp. 1017-1021.
[12] D. B. Johnson and D. A. Maltz, "Dynamic Source Routing in Ad Hoc Wireless Networks," *Mobile Computing*, Kluwer Academic Publishers, 1996, pp. 153-181.
[13] A. B. McDonald and T. Znati, "A Path Availability Model for Wireless Ad-Hoc Networks," *Proc. IEEE WCNC 1999*, Sep. 1999, pp.35-40.
[14] T. Camp, J. Boleng, and V. Davies, "A Survey of Mobility Models for Ad Hoc Network Research," *WCMC*, vol. 2, no. 5, 2002, pp. 483-502.

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