Extended Perfect Difference Codes for SAC Optical CDMA PONs

Shin-Pin Tseng and Jingshown Wu

Abstract—**In this letter, we present a new code family named the extended perfect difference codes (EPDC) suitable for passive optical network (PON) using compact SAC OCDMA encoder and decoder. Applying a mapping technique, we construct the EPDC, which not only still retain the cyclic property but also can provide flexible code length. Because of the cyclic property, the optical line terminal (OLT) needs only one arrayed waveguide grating (AWG) router to generate each user's codeword effectively. In addition, the decoder in each optical network unit (ONU) using fiber Bragg gratings (FBGs) and the EPDC is also designed. We analyze the system performance with the consideration of phase-induced intensity noise (PIIN) and thermal noise. The numerical result shows that the proposed code has higher transmission capacity than other SAC OCDMA codes.**

Index Terms—**Arrayed waveguide grating (AWG) router, extended perfect difference code (EPDC), fiber Bragg grating (FBG), and spectral amplitude coding (SAC).**

I. Introduction

With rapid progress of multi-media communications, optical code division multiple access (OCDMA) techniques have attracted much attention because they can provide high capacity transmission and secure communication. Initially the non-coherent OCDMA systems were usually based on time spreading information signals by optical orthogonal codes. However, the lengths of the codes were long and the performance of these systems was limited by the influence of multiple-access interference (MAI). Several years ago, the researchers proposed the spectral amplitude coding (SAC) OCDMA systems using several code families [1]-[4] because the influence of MAI can be reduced.The systems using fiber Bragg gratings (FBGs) as codec devices can provide flexible configuration [1]-[2]. In addition, the SAC OCDMA system with shared codecs and array waveguide grating (AWG) routers was proposed to have compact configuration [4]. On the other hand, the perfect difference codes (PDC) were presented for the time spreading OCDMA system several years ago [5].

In this letter, we propose the extended PDC (EPDC) and combine the concept of sharing codec to construct the SAC OCDMA-based passive optical network (PON). The performance of the proposed SAC OCDMA-based PON is evaluated with the consideration of the effect of PIIN and thermal noise. The results show that the proposed PON has better performance than many other SAC OCDMA systems.

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II. Extended Perfect Difference Codes

As disclosed in [5], a family of the PDC with $(N, v, 1)$ is derived from the perfect difference set, which is defined as follows [6]. $Definition$: Let Q be the N-set of the integers $0, 1, \cdots, (N-1)$ modulo N. A set $D = \{d_1, d_2, \cdots, d_v\}$ is an v-subset of Q. For every $a \neq 0$ (mod N), there are exactly λ ordered pairs (d_i, d_j), $i \neq j$, such that

$$d_i - d_j \equiv a \pmod N. \quad (1)$$

A set D fulfilling these requirements is called a perfect difference set or (N, v, λ)-cyclic difference set modulo N. In accordance with this definition, the (N, v, λ)-cyclic difference sets $D_k = \{d_1 - k, d_2 - k, \cdots, d_v - k\}$ modulo N, in which $k = 0, 1, \cdots, (N-1)$, have the same property, where $D_0 = D$ and each $D_i \cap D_j$ for $i \neq j$ is a λ-subset of Q. In addition, since there are $v(v-1)$ pairs and the total number is $(N-1)$ for $a \neq 0$ (mod N), the relation among N,v, λ is given as

$$\lambda = v(v-1)/(N-1). \quad (2)$$

In [6], Singer proved and described the existence of the $(N, v = p+1, \lambda = 1)$-cyclic difference set, where N is the sequence length and equal to $v^2 - v + 1$ and p is a prime number. Because any two sequences of the set have exact one overlay, i.e., $\lambda = 1$, it can be used to construct the PDC with the fixed in-phase cross correlation equal to 1. Let $X_k = (x_{k,0}, x_{k,1}, \cdots, x_{k,N-1})$, $k \in Q$, be a unipolar PDC of length N, where k denotes the k-th cyclic-shift of X_0 and X_0 is the original code. The elements of the codeword X_k are constructed using the following rule:

$$x_{k,i} = \begin{cases} 1, & \text{for}\, i \in D_k \\ 0, & \text{otherwise} \end{cases}. \quad (3)$$

Each codeword can be generated by cyclically shifting the original codeword X_0 (e.g., $X_k = \mathrm{T}^k X_0$), where T^k is the operator that shifts codes cyclically to the right by k times (e.g., $X_1 = \mathrm{T}^1 X_0 = (x_{0,N-1}, x_{0,0}, \ldots, x_{0,N-2})$, $X_2 = \mathrm{T}^2 X_0 = (x_{0,N-2}, x_{0,N-1}, \ldots, x_{0,N-3})$, and so on) [5].

The k-th code of the m-th group in the EPDC's can be generated from the PDC by the following mapping:

$$\mathbf{H}_{W\times W} = \begin{bmatrix} x_{0,0} * E_M & \cdots & x_{0,N-1} * E_M \\ \vdots & \ddots & \vdots \\ x_{N-1,0} * E_M & \cdots & x_{N-1,N-1} * E_M \end{bmatrix}, \quad (4)$$
$$= [c_{m,k}(w)] = [X_k]_{N\times N} \otimes E_M$$

and

$$\bar{\mathbf{H}}_{W\times W} = [\bar{c}_{m,k}(w)] = [\bar{X}_k]_{N\times N} \otimes E_M, \quad (5)$$

where $c_{m,k}(w)$ is the w-th element of $C_{m,k}$ $(w = 0, 1, \ldots, MN-1)$, $C_{m,k} = (c_{m,k}(0), c_{m,k}(1), \ldots, c_{m,k}(W-$

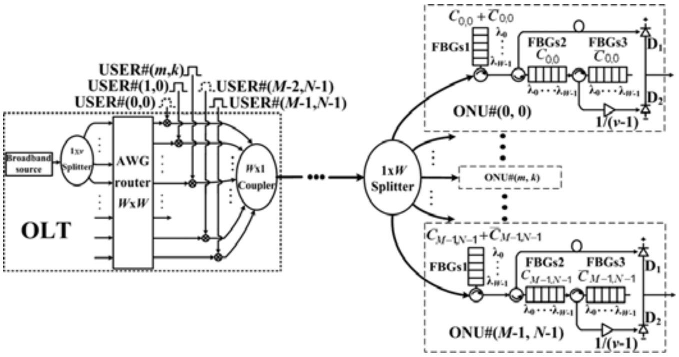

Fig. 1. The block diagram of the proposed system with EPDC's.

1)) ($m = 0, 1, \ldots, M-1$ and $k = 0, 1, \ldots, N-1$) is a unipolar EPDC of length W, $\bar{X}_k = (1-x_{k,0}, 1-x_{k,1}, \ldots, 1-x_{k,N-1})$ is the complementary of X_k, E_M is the M-by-M identity matrix, $\otimes$ is the Kronecker Tensor product, and $*$ is the multiplication. Note that $C_{m,k}$ is the $(kM+m+1)$-th row of $\mathbf{H}_{W\times W}$ and $\bar{C}_{m,k}$ is the $(kM+m+1)$-th row of $\bar{\mathbf{H}}_{W\times W}$. Thus, the code length and the number of maximum users (W) are equal to MN. Here, M represents the number of groups and is a positive integer. Due to the cyclic property of PDC's and the mapping formula of Eq. (4)-(5), the EPDC's also have the cyclic property. Table I lists the example of EPDC's with $M = 2$ and $N = 7$ from given $X_0 = 1101000$.

TABLE I
THE EXAMPLE OF EPDC FOR $M = 2$ AND $N = 7$.

m	k	codeword $C_{m,k}$	codeword $\bar{C}_{m,k}$
0	0	10100010000000	00001000101010
0	1	00101000100000	10000010001010
0	2	00001010001000	10100000100010
0	3	00000010100010	10101000001000
0	4	10000000101000	00101010000010
0	5	00100000001010	10001010100000
0	6	10001000000010	00100010101000
1	0	01010001000000	00000100010101
1	1	00010100010000	01000001000101
1	2	00000101000100	01010000010001
1	3	00000001010001	01010100000100
1	4	01000000010100	00010101000001
1	5	00010000000101	01000101010000
1	6	01000100000001	00010001010100

III. SYSTEM CONFIGURATION

Fig. 1 shows the proposed SAC OCDMA-based PON with EPDC's, which consists of one optical line terminal (OLT) in the central office, MN optical network units (ONUs) of end users, and one $1 \times W$ splitter with equal power ratio. Due to the cyclic property of AWG router, a compact AWG encoder with EPDC's is suitable for constructing the OLT. The OLT's structure consists of a broadband light source with flat power spectral density (PSD), a $1 \times v$ optical splitter, a $W \times W$ AWG router, MN modulators, and a $W \times 1$ coupler, where the v output ports of the splitter are connected to the input ports of the AWG router according to the cardinal of code weight of $C_{0,0}$. Then, these optical signals that are incident to different input ports of AWG router are demultiplexed into different output ports of AWG router and can be described by the following formula:

$$Input\ port \oplus Output\ port \equiv Wavelength, \qquad (6)$$

where $\oplus$ denotes modulo-W addition. Thus, it is found that all spectral components corresponding to the codeword $C_{m,k}$ appear at the $(kM+m)$-th output port of the AWG router. Then the on-off keying modulation is applied according to the data bit of each user to generate the SAC OCDM code. With the cyclical property, the spectrum of the broadband incoherent optical source must be limited within one free spectral range (FSR) of the AWG router. Obviously at the output port of the $W \times 1$ coupler, the signal r which is the sum of all users' coded spectral signals, can be expressed as

$$r = (r_0, r_1, \cdots, r_{W-1}) = \sum_{m=0}^{M-1}\sum_{k=0}^{N-1} b_{m,k} C_{m,k}, \qquad (7)$$

where $b_{m,k} \in \{0, 1\}$ denotes the binary data bit of the k-th user of the m-th group. Each of ONUs is composed of three sets of FBGs, three optical circulators, one attenuator, one balanced photodetector, and a delay line for compensating the delay caused by FBGs2 and FBGs3. The decoding process is similar to conventional SAC OCDMA systems and can be accomplished by respectively passing the receiving signal through FBGs2 and FBGs3; the balance detector is used to detect the signal. The spectral components of the received signals which match to "1s" of FBGs2 are reflected back and then pass to the upper photodiode via a delay line. Similarly, the remainder signals are input to FBGs3, the matched portion is reflected to the lower photodiode via one $1/(v-1)$ attenuator. In order to compensate the different round-trip delays of the matched spectral components, the ordinal arrangement of the gratings in FBGs1 is contrary to that in FBGs2 and FBGs3. Note that the bandwidth of each FBG is the same as the one of the wavelength channels of the AWG router. Based on the EPDC's property, the correlation of $C_{m,k}$ and $C_{j,s}$ is given by:

$$C_{m,k} \bullet C_{j,s} = \sum_{w=0}^{W-1} c_{m,k}(w) c_{j,s}(w) = \begin{cases} v, & \text{if } m = j,\ k = s \\ 1, & \text{if } m = j,\ k \neq s \\ 0, & \text{if } m \neq j \end{cases}, \qquad (8)$$

where $\bullet$ is the inner-product of two codes. The (j, s)-th ONU is implemented by using the balanced photodetector to eliminate the influence of MAI. We define the parameter Z, which is proportional to the input energy of the (j, s)-th ONU while we receive the codeword $C_{m,k}$, as

$$Z = (C_{m,k} \bullet C_{j,s}) - \left[\frac{(C_{m,k} \bullet \bar{C}_{j,s})}{(v-1)}\right] = \begin{cases} v, & \text{if } m = j,\ k = s \\ 0, & \text{otherwise} \end{cases}. \qquad (9)$$

The correlation values of and $C_{m,k} \bullet C_{j,s}$ and $(C_{m,k} \bullet \bar{C}_{j,s})/(v-1)$ are obtained in the upper and lower photodetectors, respectively. If the (m, k)-th user does not belong to the same group of the (j, s)-th ONU, i.e., the spectral code of the (m, k)-th user will pass through the grating in FBGs1 without contribution of PIIN in the photodetectors.

IV. Performance Analysis

For the SAC OCDMA system using broadband light source with flat PSD, the performance is mainly dominated by PIIN and thermal noise [4]. Assume that the light source is unpolarized and has flat PSD over a bandwidth Δf Hz with magnitude $P_s/\Delta f$. Note that P_{sr} represents the effective power of the light source at the input of each ONU and equals to P_s/W. P_{sr} is proportional to the value of r expressed in Eq. (7). Based on Eq. (9) the power of average photocurrent (P_{sig}) obtained from the balanced photodetector when the desired user transmits bit "1" can be expressed as [2]

$$P_{sig} = R^2 P_{sr}^2 v^2 / W^2, \tag{10}$$

where R is the responsivity of the photodiodes. The variance of total noise photocurrent, $\langle i^2 \rangle$, can be written as [4]

$$\langle i^2 \rangle = \sigma_{\text{PIIN}}^2/2 + 4\kappa_B T_n B/R_L, \tag{11}$$

and

$$\sigma_{\text{PIIN}}^2 = B(I_1^2 \tau_{c1} + I_2^2 \tau_{c2}), \tag{12}$$

where σ_{PIIN}^2 is the variance of PIIN, κ_B is the Boltzmann's constant, T_n is the absolute temperature in degrees Kelvin, B is the noise-equivalent electrical bandwidth, and R_L is the load resistance. Note that I_1 and I_2 are the average photocurrents, and τ_{c1} and τ_{c2} are the coherence times of the light incident on upper and lower photodiodes. Based on the analysis method in [2]-[4], we can deduce the average Signal-to-Noise Ratio (SNR) as

$$\text{SNR} = \frac{R^2 P_{sr}^2 v^2 / W^2}{\frac{BR^2 P_{sr}^2}{2\Delta f W}\left\{ v + \lfloor \frac{K-1}{M} \rfloor \left(\frac{3N-1}{N} + \frac{v^2 \lfloor \frac{K-1}{M} \rfloor}{N(v-1)} \right) \right\} + \frac{4\kappa_B T_n B}{R_L}}, \tag{13}$$

where K is the number of active users and $\lfloor\,\rfloor$ is the floor function. Table II shows these parameters used for our numerical result. The bit error rate (BER) can be expressed as [2]-[4]

$$\text{BER} = 1/2 \times \text{erfc}(\text{SNR}/8)^{1/2}, \tag{14}$$

where erfc($\cdot$) is the complementary error function. Fig. 2 depicts the BER versus the number of active users. At BER equal to 10^{-9} and having similar code length, the numbers of active users for the systems using EPDC's with $M = 9$ and $N = 21$, modified quadratic congruence (MQC) code, PDC's with $N = 183$, and complementary Walsh-Hadamard (CWH) code are 140, 90, 90, and 42 respectively. Moreover, it is observed that the performance of the PDC's is similar to that of the MQC's because the correlation values of the PDC's and MQC's are the same. However, the systems using EPDC's always obtain excellent performances because the correlation of codes in different groups is equal to zero. Hence, the average correlation is approximately equal to $1/M$. Furthermore, it can be found that the effect of PIIN can be further mitigated by increasing the number of groups. Note that the curve of PDC's ($N = 183$) almost coincides with that of MQC's ($N = 182$).

TABLE II
The Parameters Used For Performance Evaluation.

Symbol	Name	Value
P_{sr}	Power of the light source at each ONU	$100\mu W$
R	PD responsivity for InGaAs at 1550nm	$0.93A/W$
B	Noise-equivalent electrical bandwidth	$80MHz$
κ_B	Boltzmann's constant	$1.38 \times 10^{-23} J/^\circ K$
T_n	Absolute temperature	$300^\circ K$
R_L	Load resistor	1030Ω
Δf	Spectral widths of light source	$40nm$

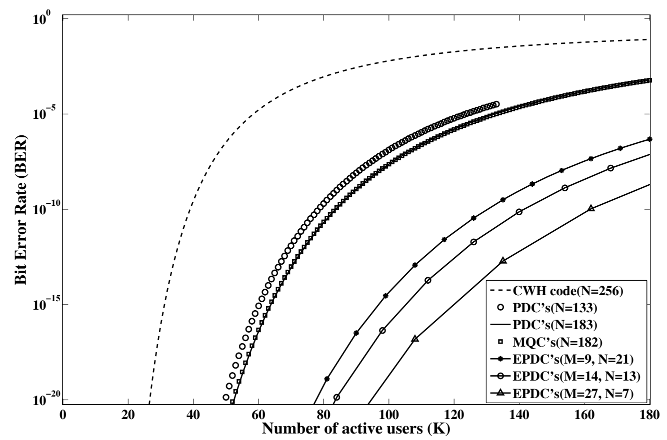

Fig. 2. The BER versus the number of active users.

V. Conclusion

In this letter, we present a family of new codes named EPDC for the SAC OCDMA-based PON applications. The EPDC's not only provide smaller in-phase cross correlation with more flexible code lengths but also still retain the cyclic property. Due to its excellent property, a compact OLT at the central office can be constructed by using an AWG router with even port numbers. The numerical results show that the proposed PON has more active users than the systems using other codes at BER $= 10^{-9}$.

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