WATER-RESOURCES ENGINEERING

Chapter 11 : Pressure Conduits

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Introduction

- 壓力管路是指滿流的管路。
- 壓力管路的費用常比渠道或渡漕高。
- 若水量很少，壓力管路可用來避免發生明渠中滲漏與蒸發的水量損失。
- 公共給水寧願採用壓力管路，因為減少了被污染的機會。
- 壓力管中的水流幾乎是亂流，故本章的討論將限於這種水流。

Hydraulics of Pressure Conduits

Definition sketch for pipe flow

- 下圖中斷面A與斷面B之間的能量方程式如下：

$$Z_A+\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+h_P=Z_B+\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+h_L$$

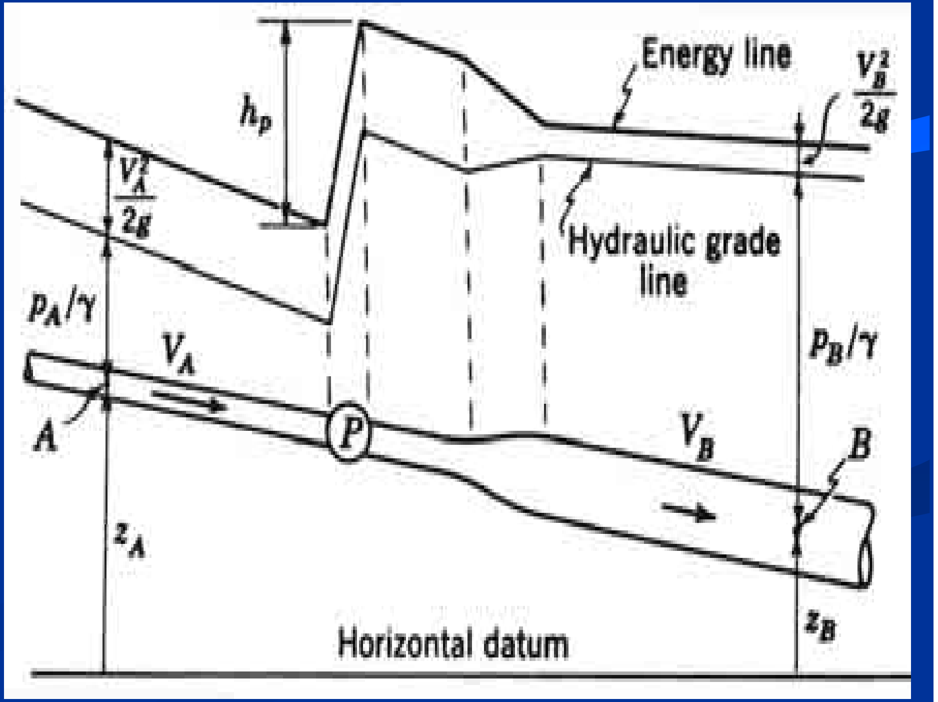
Z：高出任意水平基準的垂直距離

$\frac{P}{\gamma}$：壓力水頭

V：水流平均流速

h_P：抽水機(pump)加給水的能量水頭(若管線中為水輪機(turbine)，而非抽水機，則符號$+h_P$將被$-h_t$取代)

h_L：斷面A與斷面B之間的總水頭損失



Outline—Hydraulics of Pressure Conduits

- 11.1 Head Loss by Pipe Friction
- 11.2 Minor Losses in Pipelines
- 11.3 Flow with Negative Pressure
- 11.4 Flow in Branching Pipes
- 11.5 Flow in Pipe Systems
- 11.6 Pipe Networks
- 11.7 Contaminant Propagation in Distribution Systems
- 11.8 Power in Fluid Flow

11.1 Head Loss by Pipe Friction

- 管摩擦所損失水頭可從達西-威士巴(Darcy-Weisbach)方程式求出

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

f ：摩擦係數

L ：管的長度

D ：管的直徑

V ：水流平均流速

- 摩擦係數是管的相對糙度及管流雷諾數 $(N_R = DV/\nu)$ 的函數。

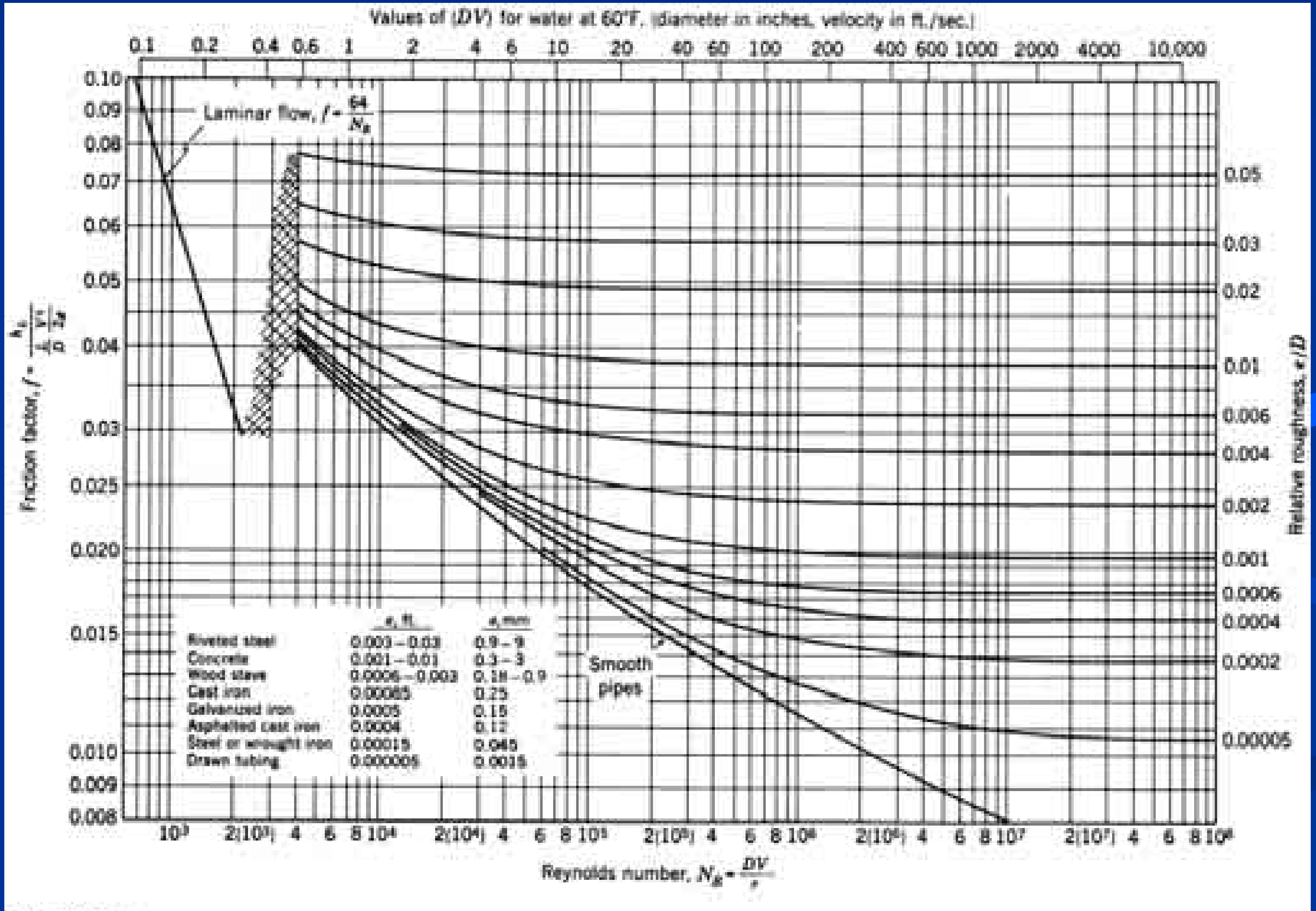
$N_R < 2100$ →層流(laminar flow)

$2100 < N_R < 4000$ →過渡型流態

$N_R > 4000$ →亂流(turbulent flow)

11.1 Head Loss by Pipe Friction

- 圖11.2所示為根據商業用管之廣泛試驗結果，所得到用來決定摩擦係數 f 的曲線。



	ϵ, ft	ϵ, mm
Riveted steel	0.003 – 0.03	0.9 – 9
Concrete	0.001 – 0.01	0.3 – 3
Wood stave	0.0006 – 0.003	0.18 – 0.9
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015

FIGURE 11.2
Moody diagram for estimating f for pipes.

11.1 Head Loss by Pipe Friction

- 曼寧(Manning)公式也可用於壓力管路中的水流計算。

英制單位 $V = \frac{1.49}{n} R^{2/3} S^{1/2}$

公制單位 $V = \frac{1}{n} R^{2/3} S^{1/2}$

- 另一廣泛使用的管流經驗公式為海森-威廉(Hazen-William)方程式。

英制單位 $V = 1.318\, C_H R^{0.63} S^{0.54}$

公制單位 $V = 0.85\, C_H R^{0.63} S^{0.54}$

式中
C_H ：係數(表11.1)
S ：能量線的坡度
R ：水力半徑

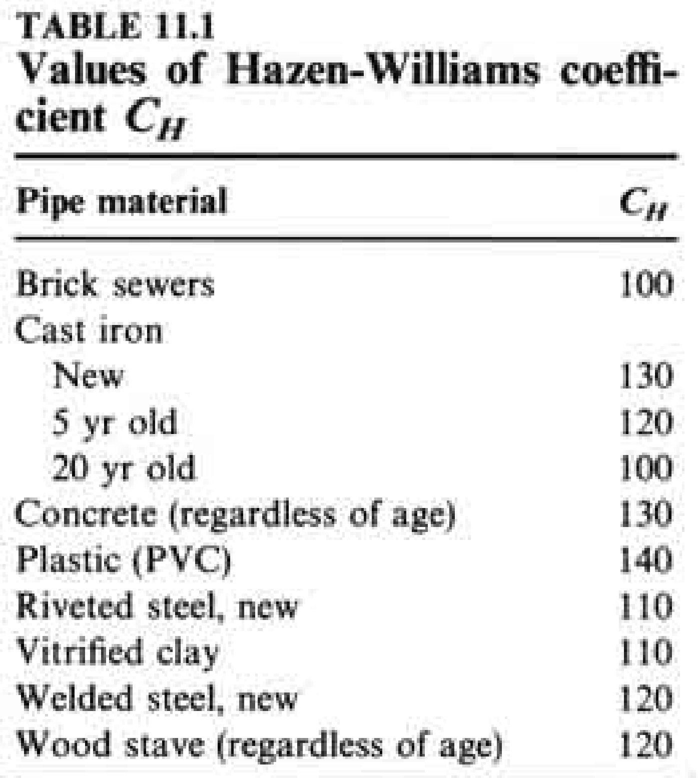
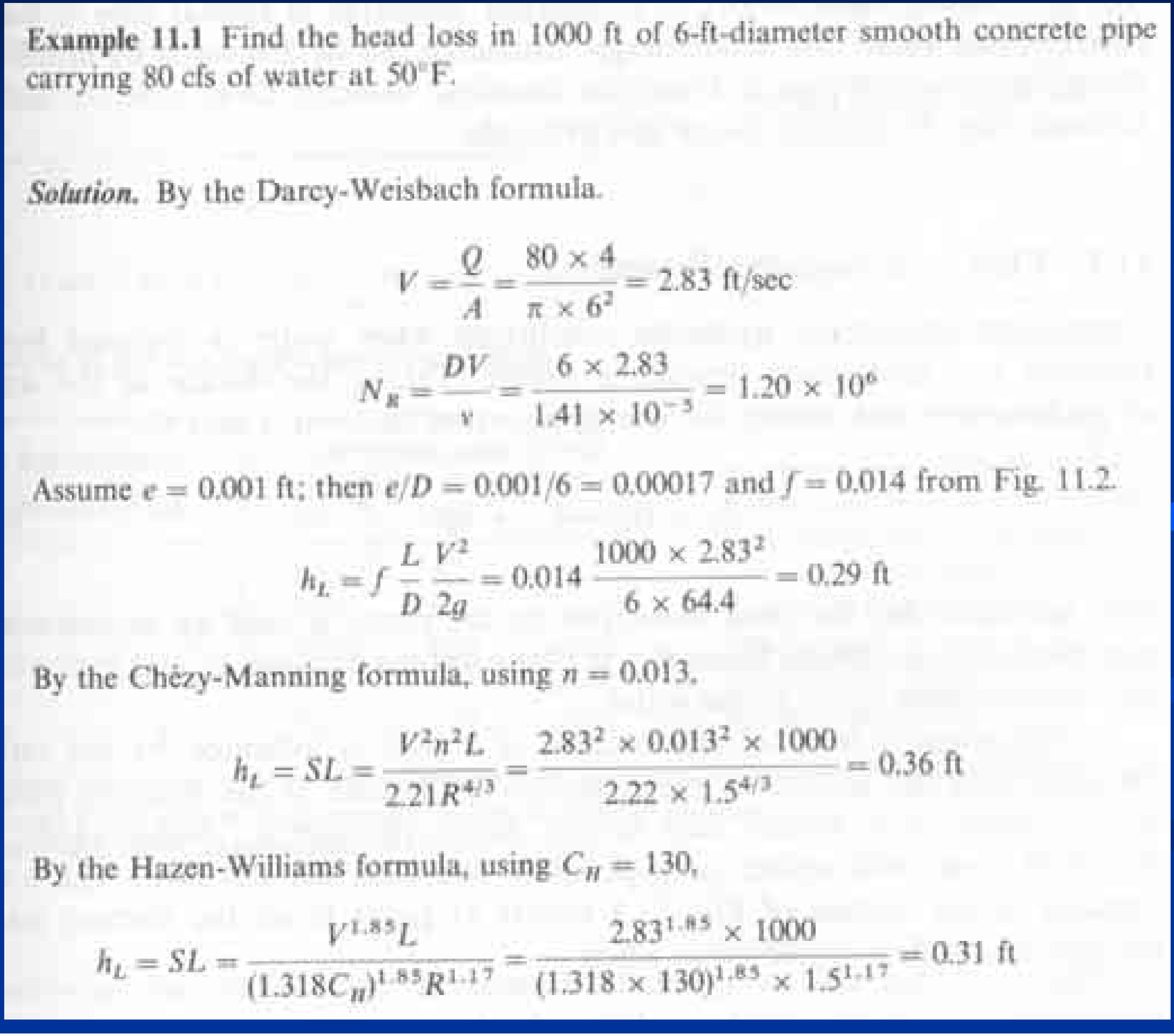


TABLE 11.1
Values of Hazen-Williams coefficient C_H

Pipe material	C_H
Brick sewers	100
Cast iron	
New	130
5 yr old	120
20 yr old	100
Concrete (regardless of age)	130
Plastic (PVC)	140
Riveted steel, new	110
Vitrified clay	110
Welded steel, new	120
Wood stave (regardless of age)	120

Example 11.1



Example 11.1 Find the head loss in 1000 ft of 6-ft-diameter smooth concrete pipe carrying 80 cfs of water at 50°F.

Solution. By the Darcy-Weisbach formula.

$$V = \frac{Q}{A} = \frac{80 \times 4}{\pi \times 6^2} = 2.83 \text{ ft/sec}$$

$$N_R = \frac{DV}{\nu} = \frac{6 \times 2.83}{1.41 \times 10^{-5}} = 1.20 \times 10^6$$

Assume $e = 0.001$ ft; then $e/D = 0.001/6 = 0.00017$ and $f = 0.014$ from Fig. 11.2.

$$h_L = f \frac{L}{D} \frac{V^2}{2g} = 0.014 \frac{1000 \times 2.83^2}{6 \times 64.4} = 0.29 \text{ ft}$$

By the Chézy-Manning formula, using $n = 0.013$,

$$h_L = SL = \frac{V^2 n^2 L}{2.21 R^{4/3}} = \frac{2.83^2 \times 0.013^2 \times 1000}{2.22 \times 1.5^{4/3}} = 0.36 \text{ ft}$$

By the Hazen-Williams formula, using $C_H = 130$,

$$h_L = SL = \frac{V^{1.85} L}{(1.318 C_H)^{1.85} R^{1.17}} = \frac{2.83^{1.85} \times 1000}{(1.318 \times 130)^{1.85} \times 1.5^{1.17}} = 0.31 \text{ ft}$$

11.2 Minor Losses in Pipelines

- 管線中的次要損失是由水流幾何性質的突變所造成，這類突變包括管徑的改變、彎管、閥及各種配件所引起的。
- 在長管線中，這些次要損失常予以忽略而不會有嚴重的誤差，但它們在短管中卻十分重要。
- 次要損失一般在水流減速處比在加速處大些，此因從渠道邊界之流動分離會生成漩渦所致。
- 亂流中的次要損失約略隨著速度的平方而變，且通常表示為速度水頭的函數(表11.2)。

11.2 Minor Losses in Pipelines

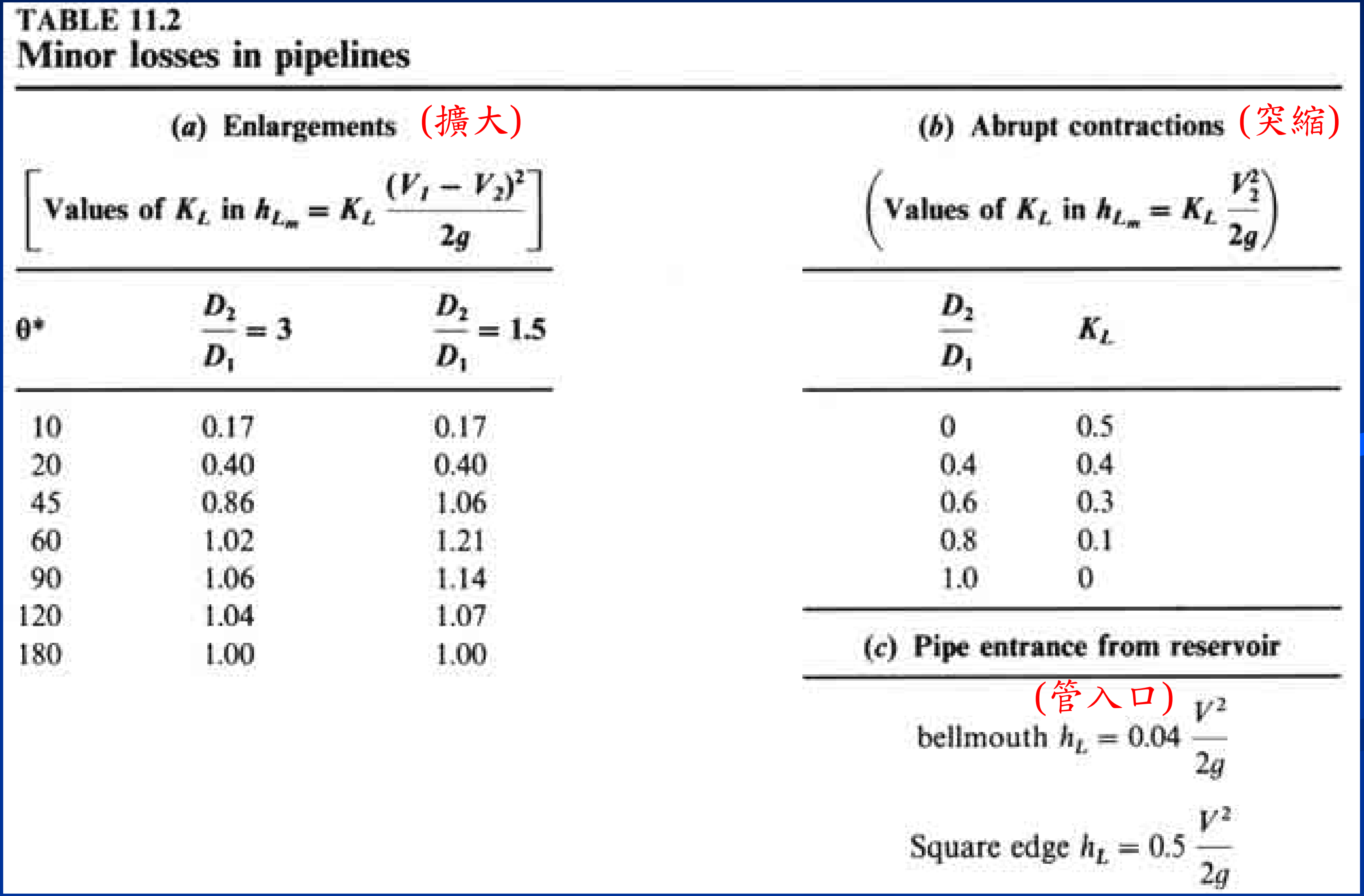


TABLE 11.2
Minor losses in pipelines

(*a*) **Enlargements** (擴大)

$$\left[\text{Values of } K_L \text{ in } h_{L_m} = K_L \frac{(V_1 - V_2)^2}{2g}\right]$$

θ*	$\frac{D_2}{D_1} = 3$	$\frac{D_2}{D_1} = 1.5$
10	0.17	0.17
20	0.40	0.40
45	0.86	1.06
60	1.02	1.21
90	1.06	1.14
120	1.04	1.07
180	1.00	1.00

(*b*) **Abrupt contractions** (突縮)

$$\left(\text{Values of } K_L \text{ in } h_{L_m} = K_L \frac{V_2^2}{2g}\right)$$

$\frac{D_2}{D_1}$	K_L
0	0.5
0.4	0.4
0.6	0.3
0.8	0.1
1.0	0

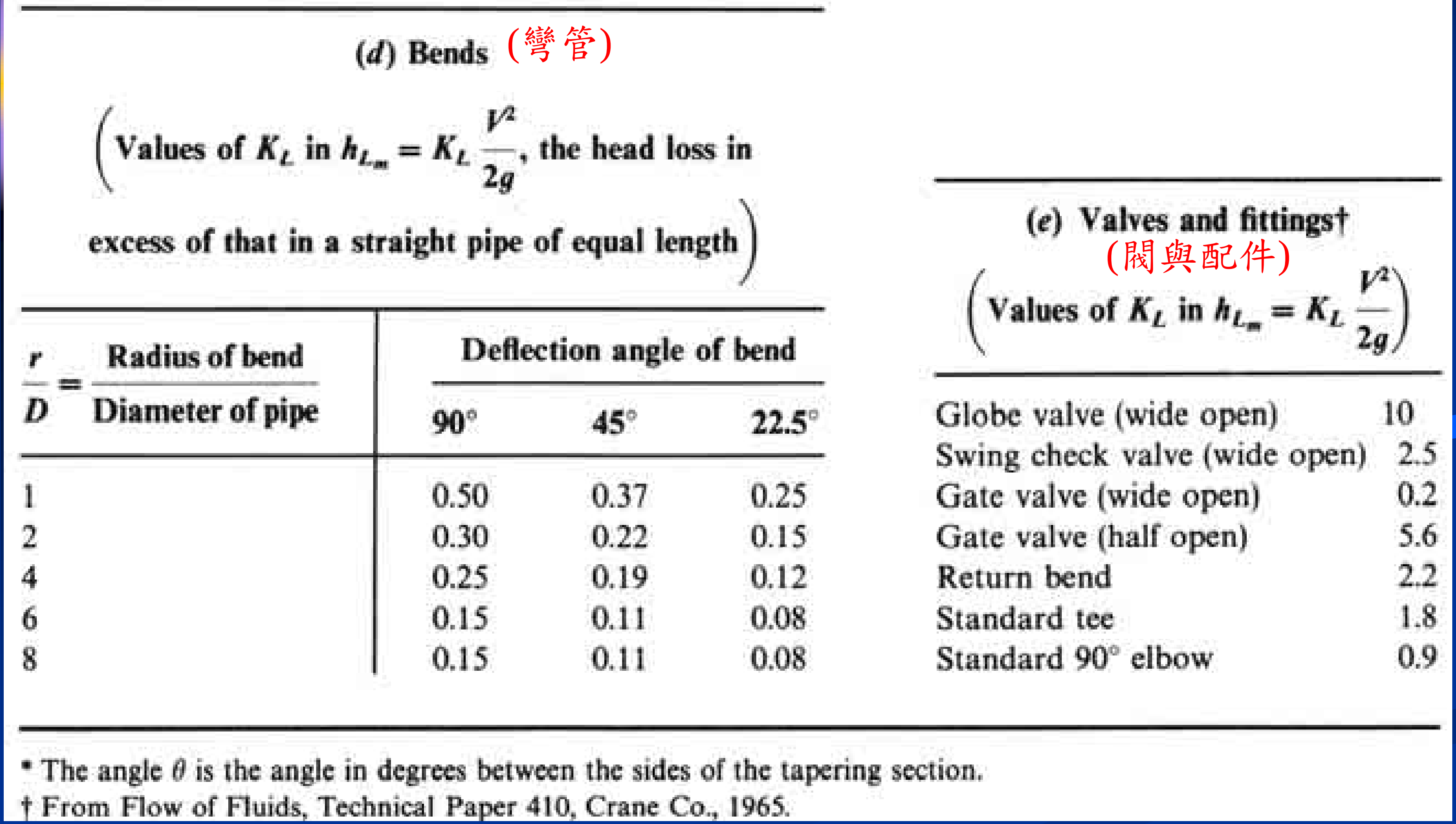
(*c*) **Pipe entrance from reservoir**

(管入口)

bellmouth $h_L = 0.04 \frac{V^2}{2g}$

Square edge $h_L = 0.5 \frac{V^2}{2g}$

11.2 Minor Losses in Pipelines



(*d*) Bends (彎管)

(Values of K_L in $h_{L_m} = K_L \frac{V^2}{2g}$, the head loss in excess of that in a straight pipe of equal length)

$\frac{r}{D} = \frac{\text{Radius of bend}}{\text{Diameter of pipe}}$	Deflection angle of bend 90°	45°	22.5°
1	0.50	0.37	0.25
2	0.30	0.22	0.15
4	0.25	0.19	0.12
6	0.15	0.11	0.08
8	0.15	0.11	0.08

(*e*) Valves and fittings† (閥與配件)

(Values of K_L in $h_{L_m} = K_L \frac{V^2}{2g}$)	
Globe valve (wide open)	10
Swing check valve (wide open)	2.5
Gate valve (wide open)	0.2
Gate valve (half open)	5.6
Return bend	2.2
Standard tee	1.8
Standard 90° elbow	0.9

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* The angle θ is the angle in degrees between the sides of the tapering section.

† From Flow of Fluids, Technical Paper 410, Crane Co., 1965.

11.3 Flow with Negative Pressure

- 圖11.3表示從一水庫抽水並經由噴嘴流出的水力坡降圖。
- 取水庫表面為基準面，並寫出A和B之間的能量方程式

$$h_P = h_L + h_{Lm} + Z_B + \frac{V_B^2}{2g}$$

式中 h_p：抽水機加給水的能量水頭

h_L：管路摩擦損失

h_{Lm}：次要損失

Z_B：將水提昇距離

$\frac{V_B^2}{2g}$：速度水頭

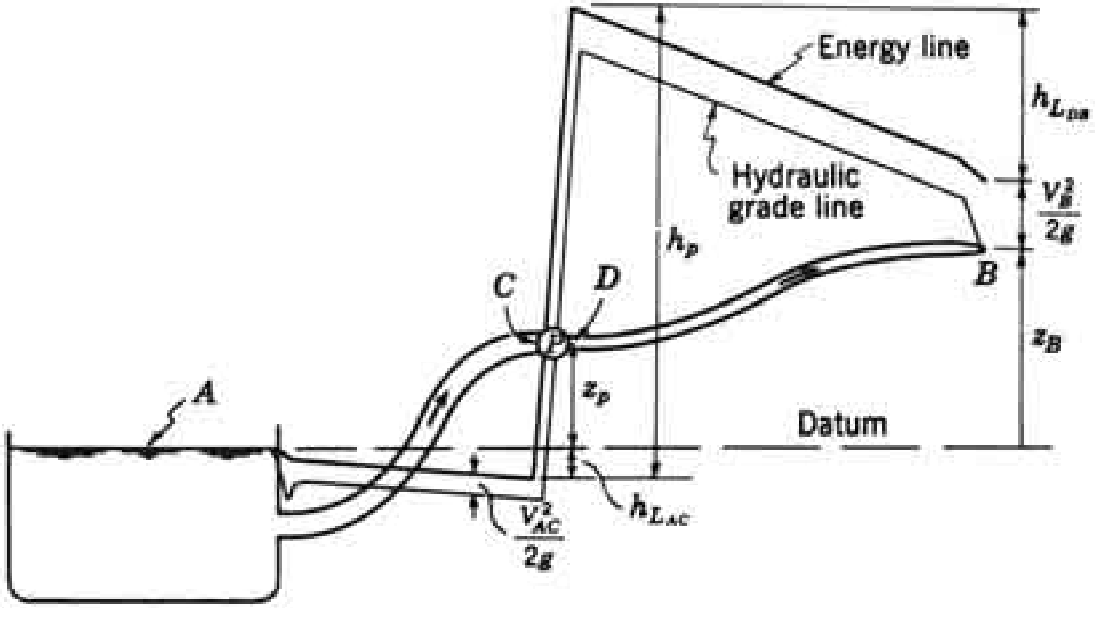


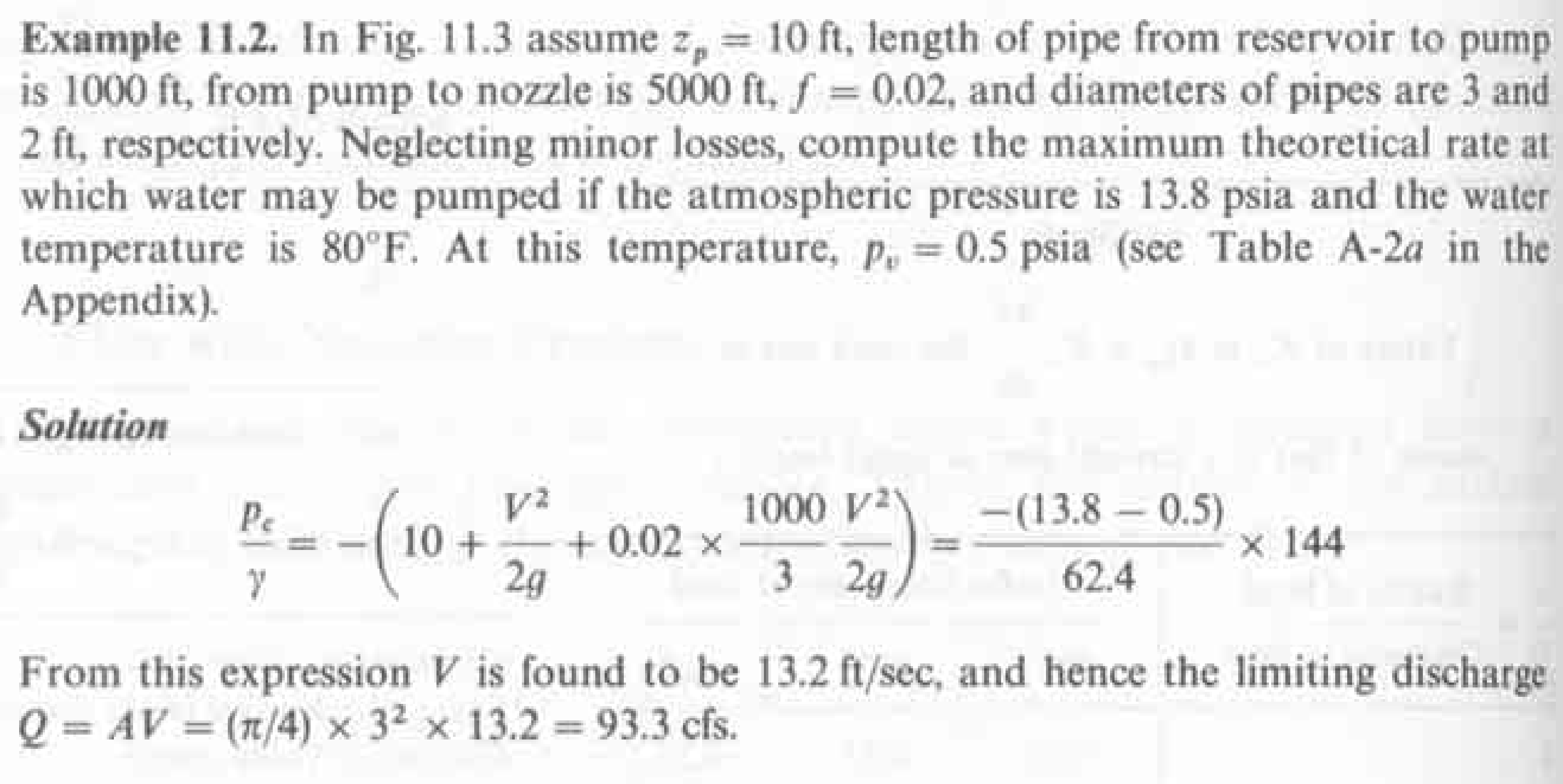
FIGURE 11.3
Conditions of flow with negative pressure.

11.3 Flow with Negative Pressure

- 管線上任一斷面的壓力水頭是由該斷面至水力坡降線的垂直距離所表示。
- 若在一系統中任一點的絕對壓力降至水汽壓力 P_v 以下，則水蒸氣與水中溶解的氣體將集中在高處而阻礙水流。
- 圖11.3之系統中最低壓力發生在抽水機吸取側的C點，而且水流將會中斷當

$$\frac{P_C}{\gamma}=-(Z_P+\frac{V_{AC}^2}{2g}+h_{LAC})=-\frac{P_{atm}-P_v}{\gamma}$$

Example 11.2



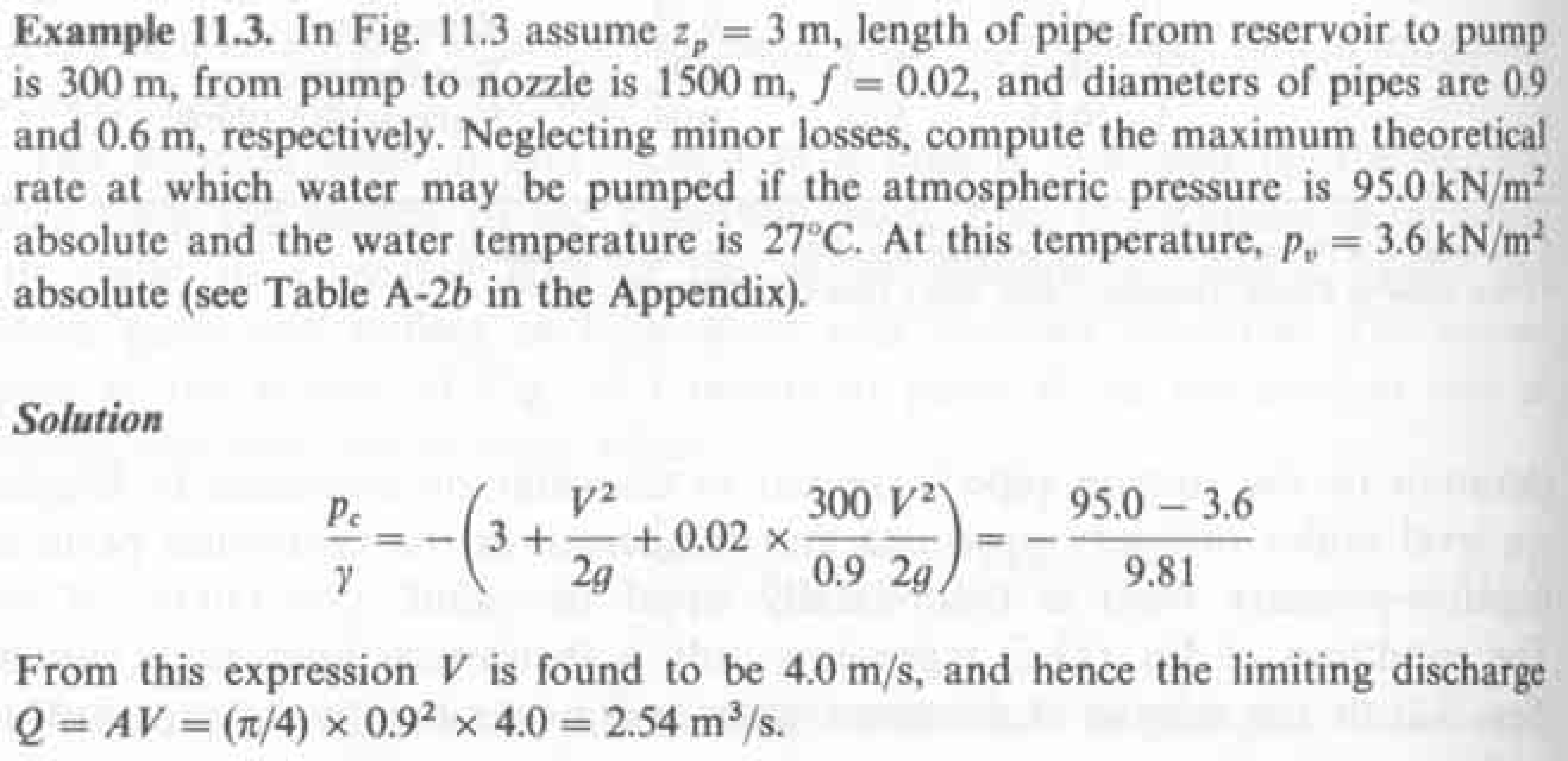
Example 11.2. In Fig. 11.3 assume $z_p = 10$ ft, length of pipe from reservoir to pump is 1000 ft, from pump to nozzle is 5000 ft, $f = 0.02$, and diameters of pipes are 3 and 2 ft, respectively. Neglecting minor losses, compute the maximum theoretical rate at which water may be pumped if the atmospheric pressure is 13.8 psia and the water temperature is 80°F. At this temperature, $p_v = 0.5$ psia (see Table A-2*a* in the Appendix).

Solution

$$\frac{p_c}{\gamma} = -\left(10 + \frac{V^2}{2g} + 0.02 \times \frac{1000}{3}\frac{V^2}{2g}\right) = \frac{-(13.8 - 0.5)}{62.4} \times 144$$

From this expression V is found to be 13.2 ft/sec, and hence the limiting discharge $Q = AV = (\pi/4) \times 3^2 \times 13.2 = 93.3$ cfs.

Example 11.3



Example 11.3. In Fig. 11.3 assume $z_p = 3$ m, length of pipe from reservoir to pump is 300 m, from pump to nozzle is 1500 m, $f = 0.02$, and diameters of pipes are 0.9 and 0.6 m, respectively. Neglecting minor losses, compute the maximum theoretical rate at which water may be pumped if the atmospheric pressure is 95.0 kN/m^2 absolute and the water temperature is 27°C. At this temperature, $p_v = 3.6$ kN/m^2 absolute (see Table A-2*b* in the Appendix).

Solution

$$\frac{p_c}{\gamma} = -\left(3 + \frac{V^2}{2g} + 0.02 \times \frac{300}{0.9}\frac{V^2}{2g}\right) = -\frac{95.0 - 3.6}{9.81}$$

From this expression V is found to be 4.0 m/s, and hence the limiting discharge $Q = AV = (\pi/4) \times 0.9^2 \times 4.0 = 2.54$ m^3/s.

11.4 Flow in Branching Pipes

- 圖11.4中，水流將從水庫A與B流到C。如果能量線在交點D處高於水庫B的水面高程，則水流將流入水庫B。流量分配可由下面所寫出的能量和連續方程式所求出：

$$Z_A = Z_D + \frac{P_D}{\gamma} + f_A \frac{L_A}{D_A}\frac{V_A^2}{2g}$$

$$Z_B = Z_D + \frac{P_D}{\gamma} + f_B \frac{L_B}{D_B}\frac{V_B^2}{2g}$$

$$Z_C = Z_D + \frac{P_D}{\gamma} - f_C \frac{L_C}{D_C}\frac{V_C^2}{2g}$$

$$\frac{\pi D_A^2 V_A}{4} + \frac{\pi D_B^2 V_B}{4} = \frac{\pi D_C^2 V_C}{4}$$

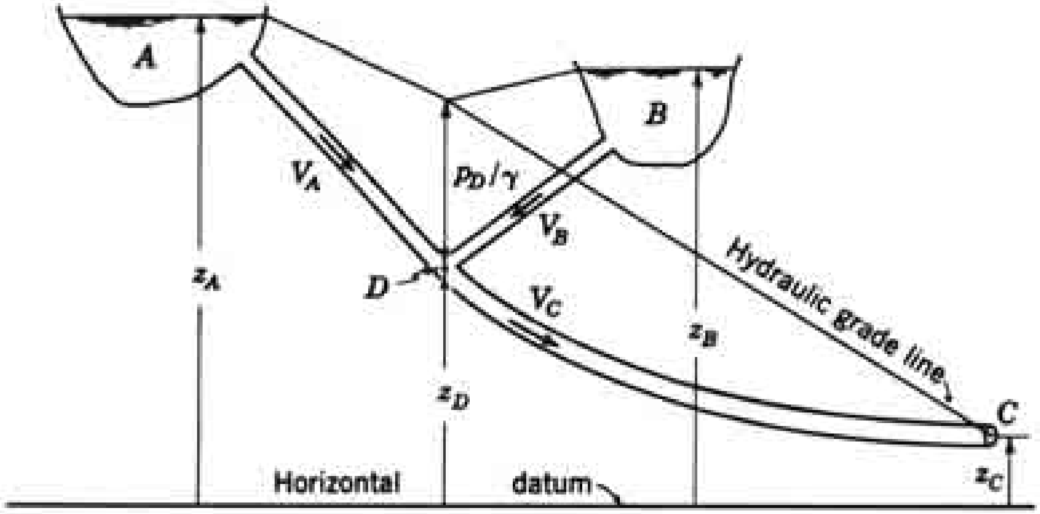


FIGURE 11.4
Definition sketch for branching pipes.

以上四個方程式可聯立求解出 V_A、V_B、V_C 及 P_D。

11.5 Flow in Pipe Systems

- 若兩管並聯(圖11.5)，則經由任何一管的水頭損失必定相同。
- 若已知管的特性，則管中的流量分配可由令水頭損失相等及連續方程式計算出。
- 若有很多根管並聯或串聯，或是兩者的組合，則以下式表示管中的水頭損失較為方便：

$$h_L = KQ^x$$

上式中K乃視管的配備、長度、直徑與糙度及流體性質而定。

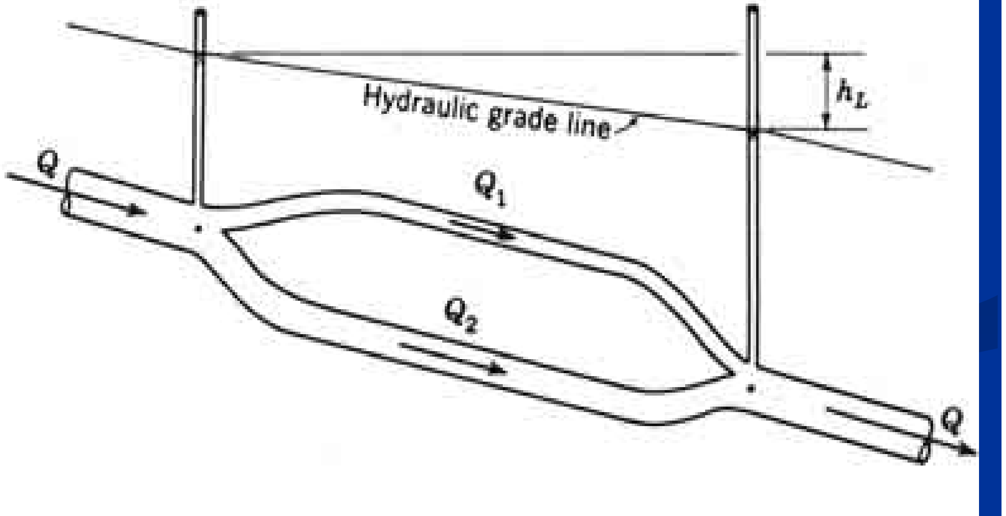


FIGURE 11.5
Parallel pipes.

11.5 Flow in Pipe Systems

- 管中的水頭損失 $h_L = KQ^x$ 式中
  - 根據曼寧方程式(Manning equation)

    指數 X 將為2

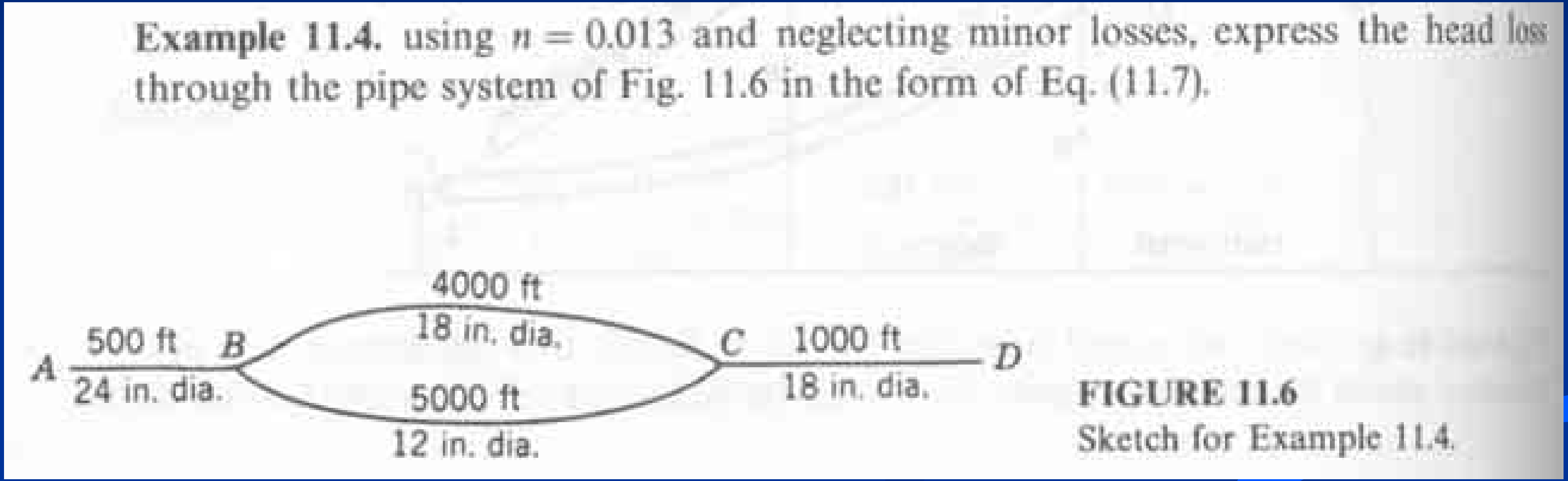
  - 根據海森-威廉方程式(Hazen-Williams equation)

    指數 X 將為1.85

  - 根據達西-威士巴公式(Darcy-Weisbach formula)

    指數 X 從光滑管的1.75變化到粗糙管的2.0

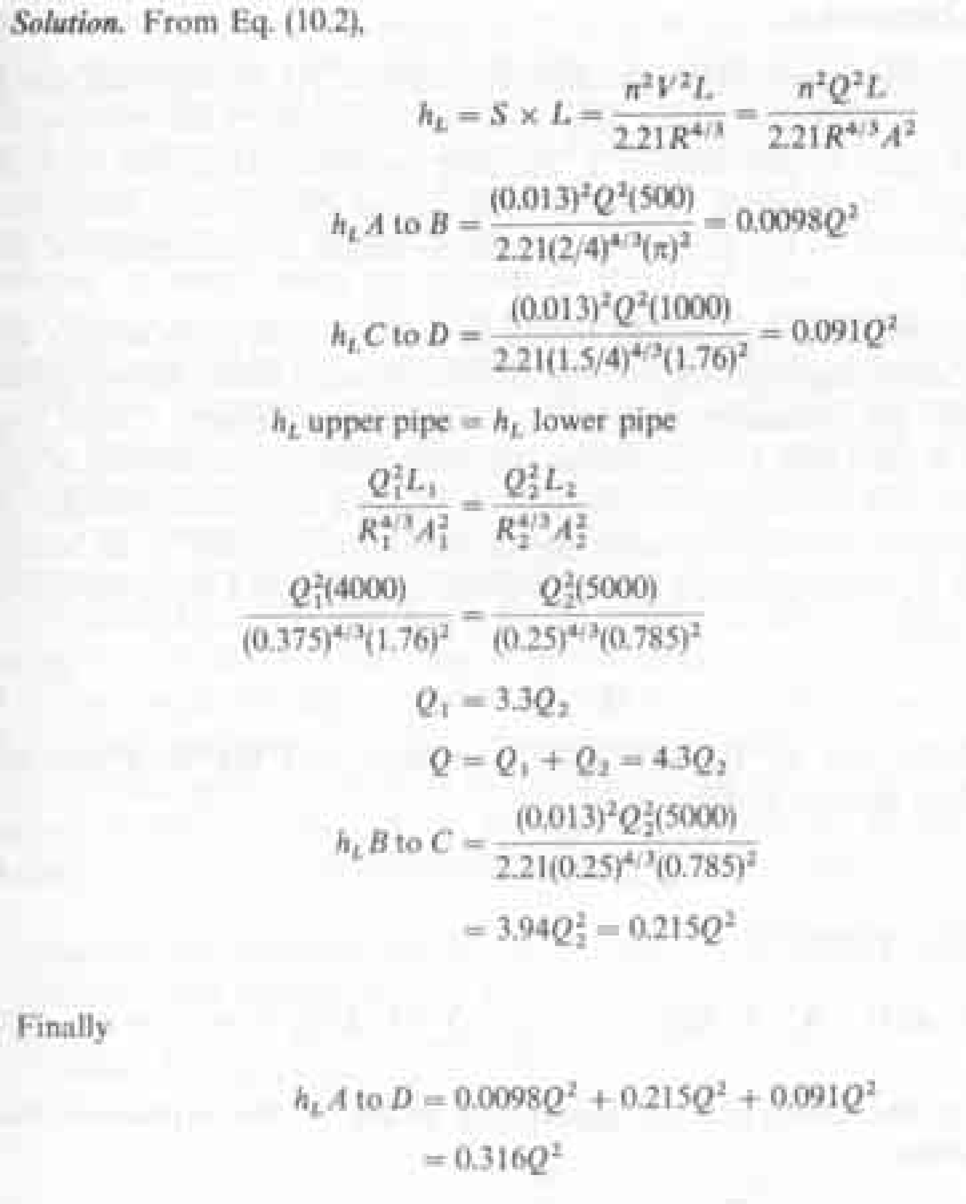
Example 11.4(1/2)



Example 11.4. using $n = 0.013$ and neglecting minor losses, express the head loss through the pipe system of Fig. 11.6 in the form of Eq. (11.7).

FIGURE 11.6
Sketch for Example 11.4.

Example 11.4(2/2)



Solution. From Eq. (10.2),

$$h_L = S \times L = \frac{n^2V^2L}{2.21R^{4/3}} = \frac{n^2Q^2L}{2.21R^{4/3}A^2}$$

$$h_L\ A \text{ to } B = \frac{(0.013)^2Q^2(500)}{2.21(2/4)^{4/3}(\pi)^2} = 0.0098Q^2$$

$$h_L\ C \text{ to } D = \frac{(0.013)^2Q^2(1000)}{2.21(1.5/4)^{4/3}(1.76)^2} = 0.091Q^2$$

$$h_L \text{ upper pipe} = h_L \text{ lower pipe}$$

$$\frac{Q_1^2L_1}{R_1^{4/3}A_1^2} = \frac{Q_2^2L_2}{R_2^{4/3}A_2^2}$$

$$\frac{Q_1^2(4000)}{(0.375)^{4/3}(1.76)^2} = \frac{Q_2^2(5000)}{(0.25)^{4/3}(0.785)^2}$$

$$Q_1 = 3.3Q_2$$

$$Q = Q_1 + Q_2 = 4.3Q_2$$

$$h_L\ B \text{ to } C = \frac{(0.013)^2Q_2^2(5000)}{2.21(0.25)^{4/3}(0.785)^2}$$

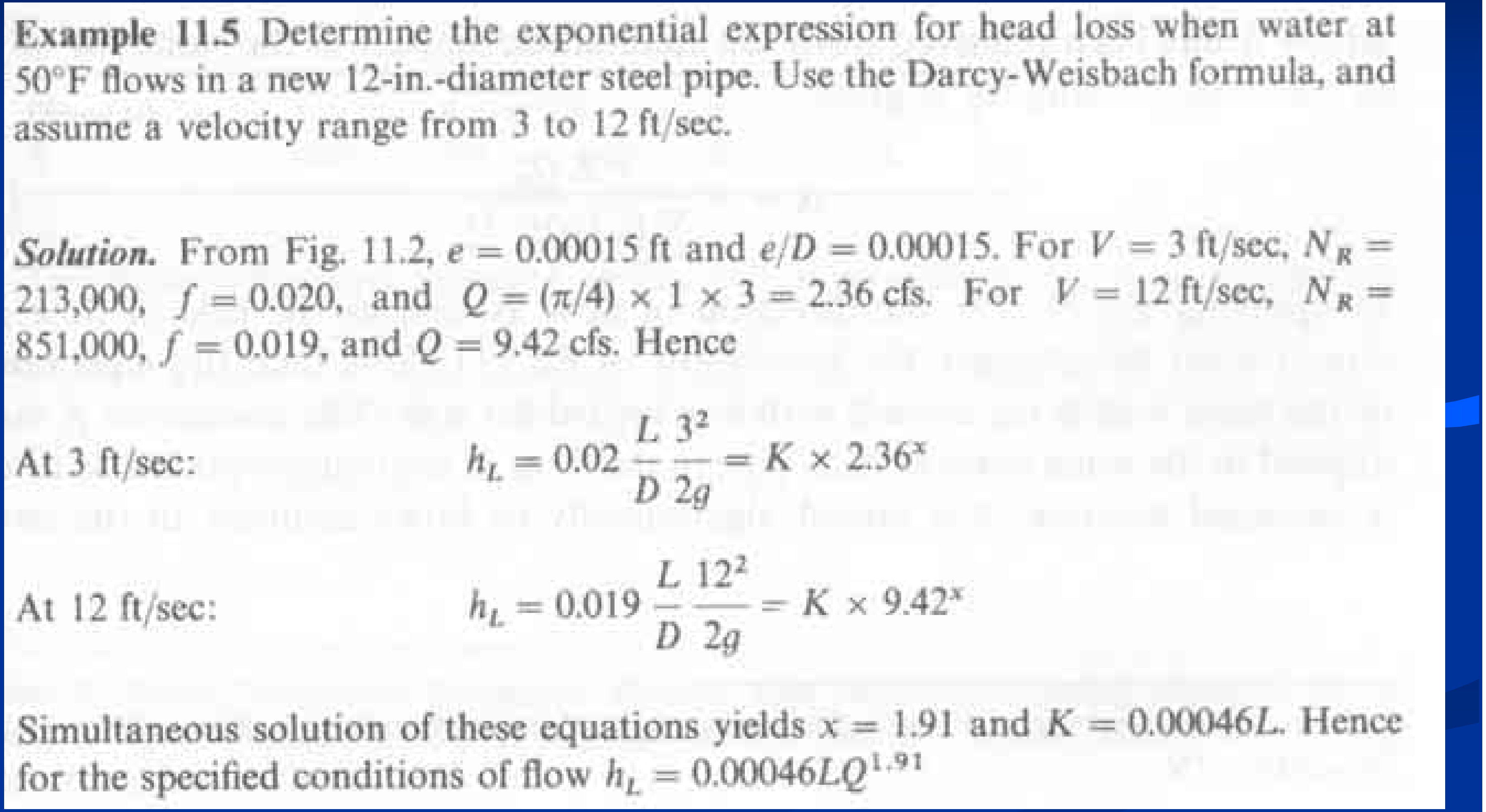
$$= 3.94Q_2^2 = 0.215Q^2$$

Finally

$$h_L\ A \text{ to } D = 0.0098Q^2 + 0.215Q^2 + 0.091Q^2$$

$$= 0.316Q^2$$

Example 11.5



Example 11.5 Determine the exponential expression for head loss when water at 50°F flows in a new 12-in.-diameter steel pipe. Use the Darcy-Weisbach formula, and assume a velocity range from 3 to 12 ft/sec.

Solution. From Fig. 11.2, $e = 0.00015$ ft and $e/D = 0.00015$. For $V = 3$ ft/sec, $N_R = 213{,}000$, $f = 0.020$, and $Q = (\pi/4) \times 1 \times 3 = 2.36$ cfs. For $V = 12$ ft/sec, $N_R = 851{,}000$, $f = 0.019$, and $Q = 9.42$ cfs. Hence

At 3 ft/sec: $$h_L = 0.02 \frac{L}{D} \frac{3^2}{2g} = K \times 2.36^x$$

At 12 ft/sec: $$h_L = 0.019 \frac{L}{D} \frac{12^2}{2g} = K \times 9.42^x$$

Simultaneous solution of these equations yields $x = 1.91$ and $K = 0.00046L$. Hence for the specified conditions of flow $h_L = 0.00046LQ^{1.91}$

11.6 Pipe Networks

- 任何管網必須滿足兩個條件
  - 環繞任何封閉迴路之壓力降落(pressure drops)的代數和必定為零。
  - 進入每一接點(junction)的流量必須等於離開該接點的流量。

11.6 Pipe Networks

- 計算管網中各管的流量—哈第克羅斯(Hardy Cross)法
  - ◆ 若 Q_a 為假設的流量，而 Q 為管中真正的流量，則修正量 Δ 為 $Q-Q_a$，即

$$Q = Q_a + \Delta$$

以 $h_L = KQ^x$ 式表示水頭損失，則環繞任何封閉迴路的水頭損失為零的條件給出

$$\sum K(Q_a+\Delta)^x = 0$$

展開此一累加式

$$\sum KQ_a^x + \sum xK\Delta Q_a^{x-1} + \frac{x-1}{2}\sum xK\Delta^2 Q_a^{x-2} + \cdots = 0$$

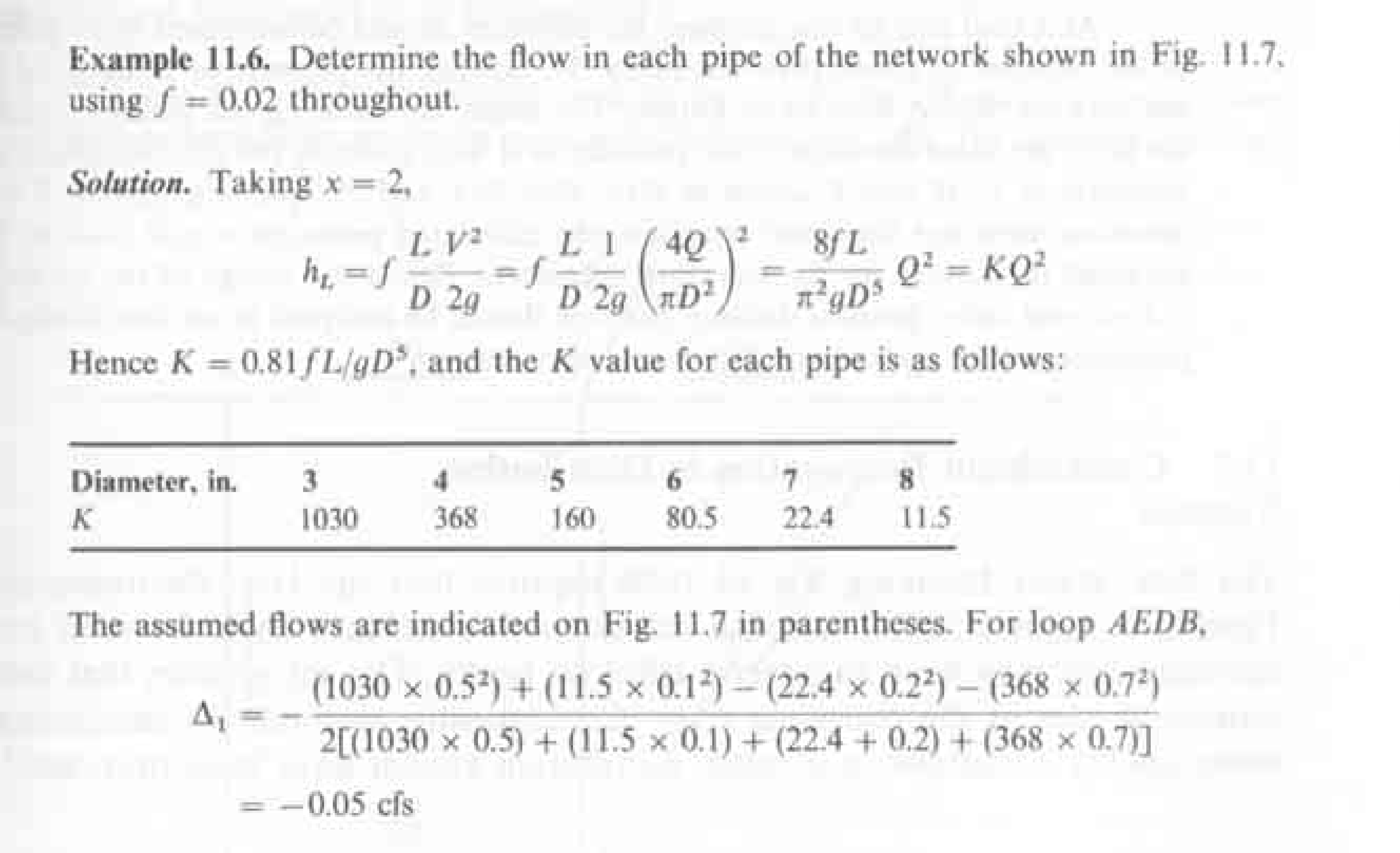
若 Δ 很小，則此展開式的第三項與所有後續項均可忽略，故

$$\sum KQ_a^x + \sum xK\Delta Q_a^{x-1} = 0$$

則

$$\Delta = -\frac{\sum KQ_a^x}{\sum \left| xKQ_a^{x-1} \right|}$$

Example 11.6(1/2)



Example 11.6. Determine the flow in each pipe of the network shown in Fig. 11.7, using $f = 0.02$ throughout.

Solution. Taking $x = 2$,

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{8fL}{\pi^2 g D^5}Q^2 = KQ^2$$

Hence $K = 0.81 fL/gD^5$, and the K value for each pipe is as follows:

Diameter, in.	**3**	**4**	**5**	**6**	**7**	**8**
K	1030	368	160	80.5	22.4	11.5

The assumed flows are indicated on Fig. 11.7 in parentheses. For loop $AEDB$,

$$\Delta_1 = -\frac{(1030 \times 0.5^2) + (11.5 \times 0.1^2) - (22.4 \times 0.2^2) - (368 \times 0.7^2)}{2[(1030 \times 0.5) + (11.5 \times 0.1) + (22.4 + 0.2) + (368 \times 0.7)]}$$

$$= -0.05 \text{ cfs}$$

Example 11.6(2/2)

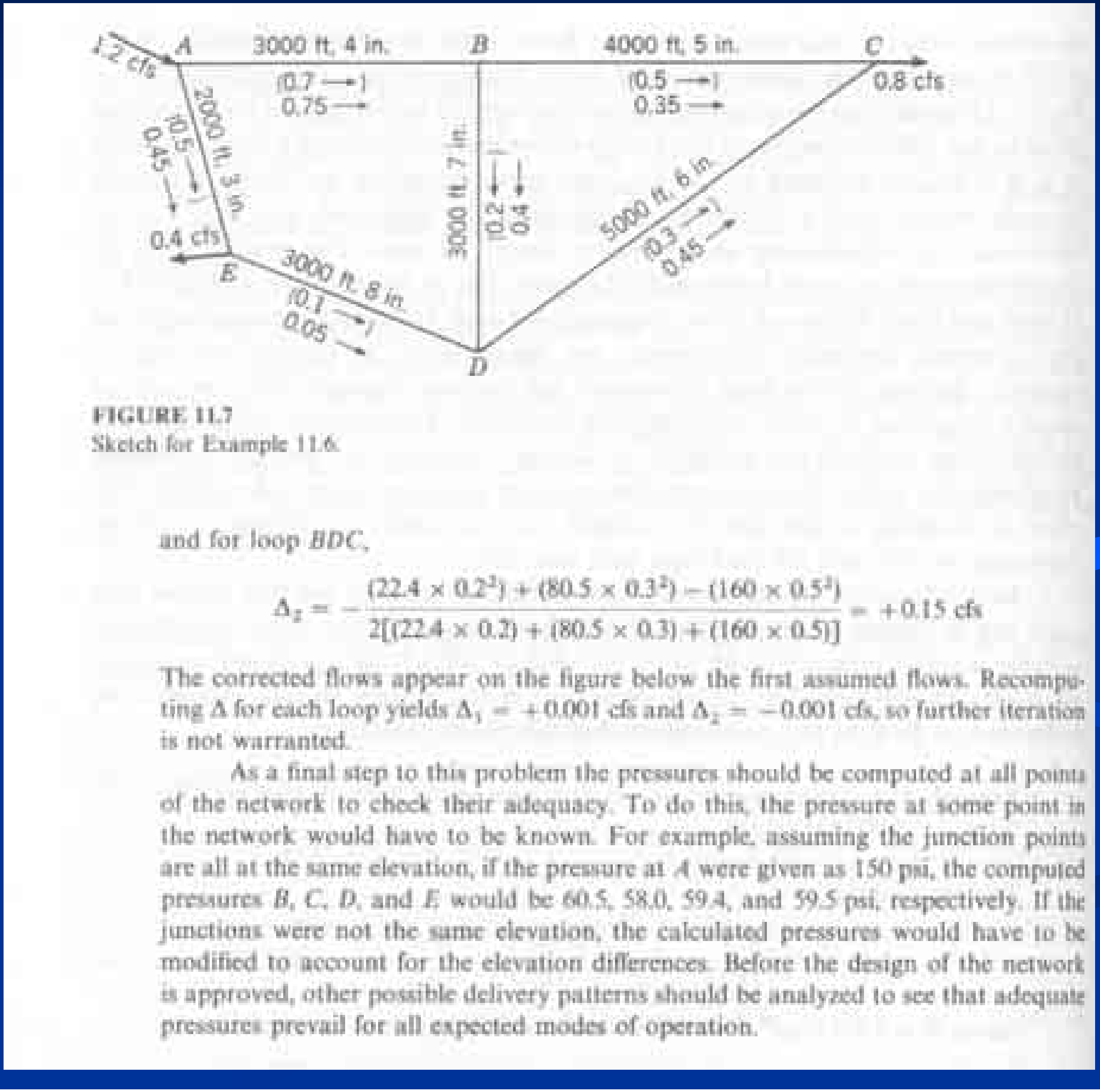


FIGURE 11.7
Sketch for Example 11.6.

and for loop *BDC*,

$$\Delta_2 = -\frac{(22.4 \times 0.2^2) + (80.5 \times 0.3^2) - (160 \times 0.5^2)}{2[(22.4 \times 0.2) + (80.5 \times 0.3) + (160 \times 0.5)]} = +0.15 \text{ cfs}$$

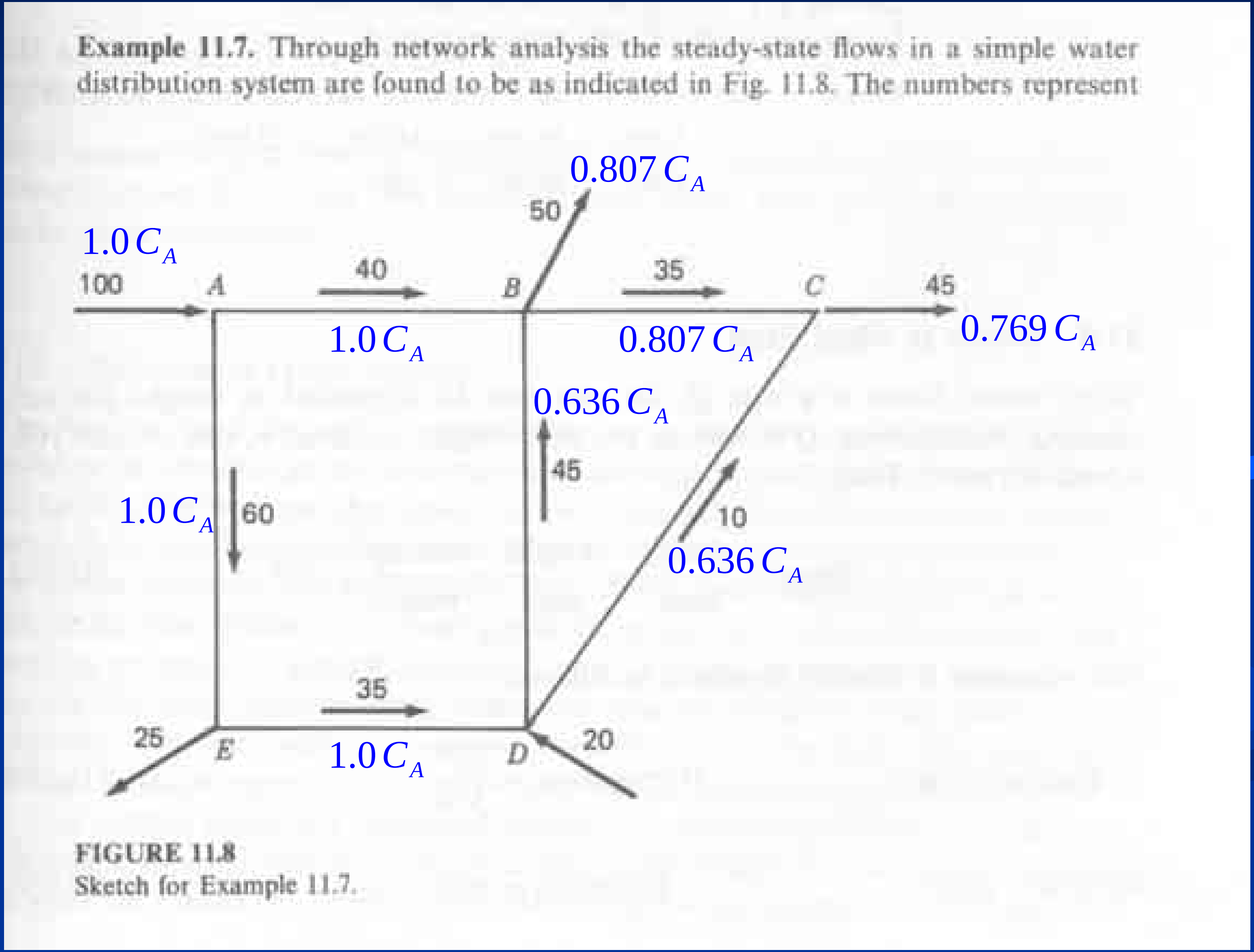
The corrected flows appear on the figure below the first assumed flows. Recomputing Δ for each loop yields $\Delta_1 = +0.001$ cfs and $\Delta_2 = -0.001$ cfs, so further iteration is not warranted.

As a final step to this problem the pressures should be computed at all points of the network to check their adequacy. To do this, the pressure at some point in the network would have to be known. For example, assuming the junction points are all at the same elevation, if the pressure at *A* were given as 150 psi, the computed pressures *B*, *C*, *D*, and *E* would be 60.5, 58.0, 59.4, and 59.5 psi, respectively. If the junctions were not the same elevation, the calculated pressures would have to be modified to account for the elevation differences. Before the design of the network is approved, other possible delivery patterns should be analyzed to see that adequate pressures prevail for all expected modes of operation.

11.7 Contaminant Propagation in Distribution Systems

- 1974年安全飲用法案要求美國環保機構制定各種污染物的最大污染標準，並且警告這些污染物將危害人體健康。特別說明的是這個標準是以消費者家的水龍頭所測得水質為主，因此建議很多方法來決定各水分佈系統中不同的水質。
- 簡單污染物污染的例子在例題11.7中說明。

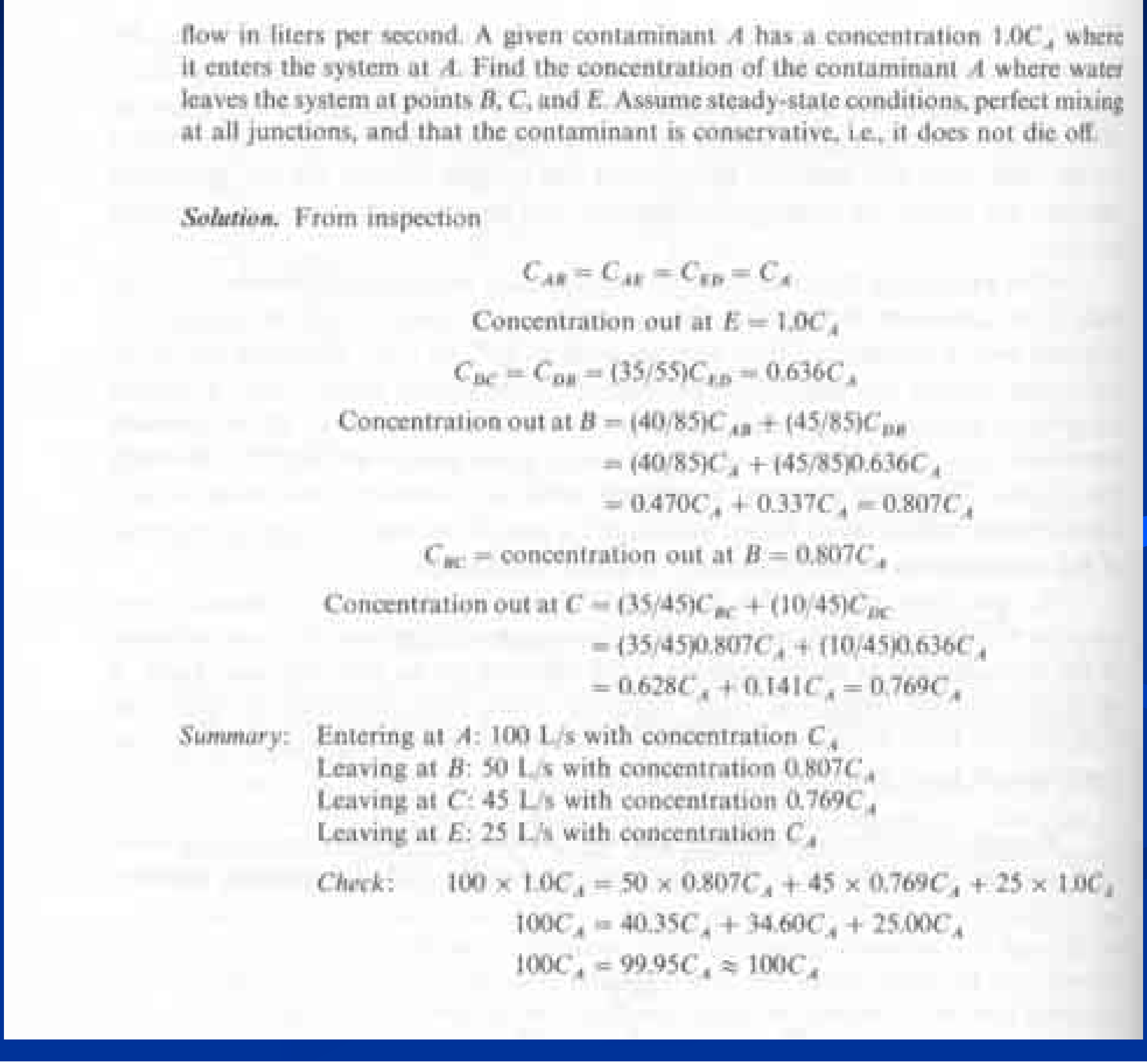
Example 11.7(1/2)



Example 11.7. Through network analysis the steady-state flows in a simple water distribution system are found to be as indicated in Fig. 11.8. The numbers represent

FIGURE 11.8
Sketch for Example 11.7.

Example 11.7(2/2)



flow in liters per second. A given contaminant A has a concentration $1.0C_A$ where it enters the system at A. Find the concentration of the contaminant A where water leaves the system at points B, C, and E. Assume steady-state conditions, perfect mixing at all junctions, and that the contaminant is conservative, i.e., it does not die off.

Solution. From inspection

$$C_{AB} = C_{AE} = C_{ED} = C_A$$

$$\text{Concentration out at } E = 1.0C_A$$

$$C_{DC} = C_{DB} = (35/55)C_{ED} = 0.636C_A$$

$$\begin{aligned}\text{Concentration out at } B &= (40/85)C_{AB} + (45/85)C_{DB}\\ &= (40/85)C_A + (45/85)0.636C_A\\ &= 0.470C_A + 0.337C_A = 0.807C_A\end{aligned}$$

$$C_{BC} = \text{concentration out at } B = 0.807C_A$$

$$\begin{aligned}\text{Concentration out at } C &= (35/45)C_{BC} + (10/45)C_{DC}\\ &= (35/45)0.807C_A + (10/45)0.636C_A\\ &= 0.628C_A + 0.141C_A = 0.769C_A\end{aligned}$$

Summary:
Entering at A: 100 L/s with concentration C_A
Leaving at B: 50 L/s with concentration $0.807C_A$
Leaving at C: 45 L/s with concentration $0.769C_A$
Leaving at E: 25 L/s with concentration C_A

Check:

$$\begin{aligned}100 \times 1.0C_A &= 50 \times 0.807C_A + 45 \times 0.769C_A + 25 \times 1.0C_A\\ 100C_A &= 40.35C_A + 34.60C_A + 25.00C_A\\ 100C_A &= 99.95C_A \approx 100C_A\end{aligned}$$

11.8 Power in Fluid Flow

- 當水以流率 Q 流動時，此流率可表示為每單位時間通過的重量 γQ。γQ 乘以每單位重量的能量即水頭 h，則得到 γQh ，乃為一功率的單位，故

$$功率=\frac{能量}{時間}=\frac{重量}{時間}\times\frac{能量}{重量}=\gamma Qh$$

上式通常表示如下：

英制單位： $馬力(hp)=\dfrac{\gamma Qh}{550}$

公制單位： $仟瓦(kW)=\gamma Qh$

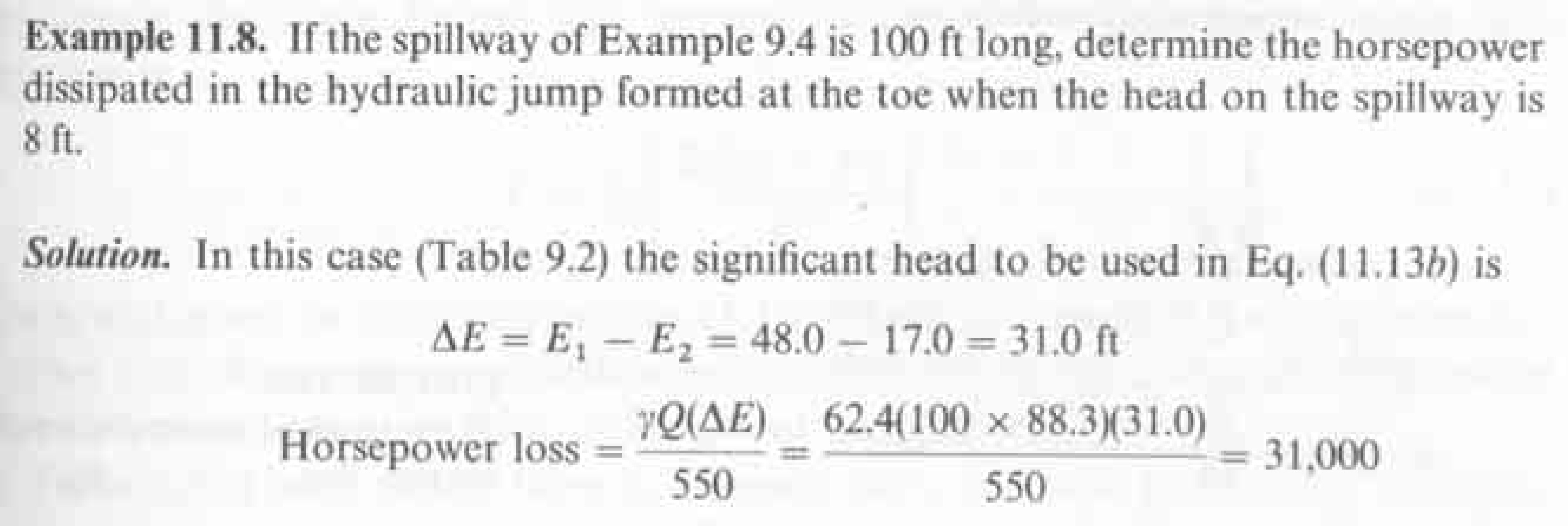
式中 γ ：流體的單位重量，lb/ft^3 (SI公制單位為 kN/m^3)

Q ：流率， cfs (SI公制單位為 $m^3/\sec$)

h ：能量水頭， f (SI公制單位為 m)

註：$1hp=550\,ft\cdot lb/\sec=0.746kW$

Example 11.8



Example 11.8. If the spillway of Example 9.4 is 100 ft long, determine the horsepower dissipated in the hydraulic jump formed at the toe when the head on the spillway is 8 ft.

Solution. In this case (Table 9.2) the significant head to be used in Eq. (11.13*b*) is

$$\Delta E = E_1 - E_2 = 48.0 - 17.0 = 31.0 \text{ ft}$$

$$\text{Horsepower loss} = \frac{\gamma Q(\Delta E)}{550} = \frac{62.4(100 \times 88.3)(31.0)}{550} = 31{,}000$$

Measurement of Flow in Pressure Conduits

Outline— Measurement of Flow In Pressure Conduits

- 11.9 Differential Head Meters
- 11.10 Mechanical Meters
- 11.11 Other Methods of Flow Measurement

11.9 Differential Head Meters

- 流體流經壓力管路中的束縮會導致束縮處的壓力降低。未擾動水流與束縮之間的壓力水頭降落是流率的函數。文氏流量計、噴嘴流量計及孔口流量計都是利用這個原理的束縮流量計。

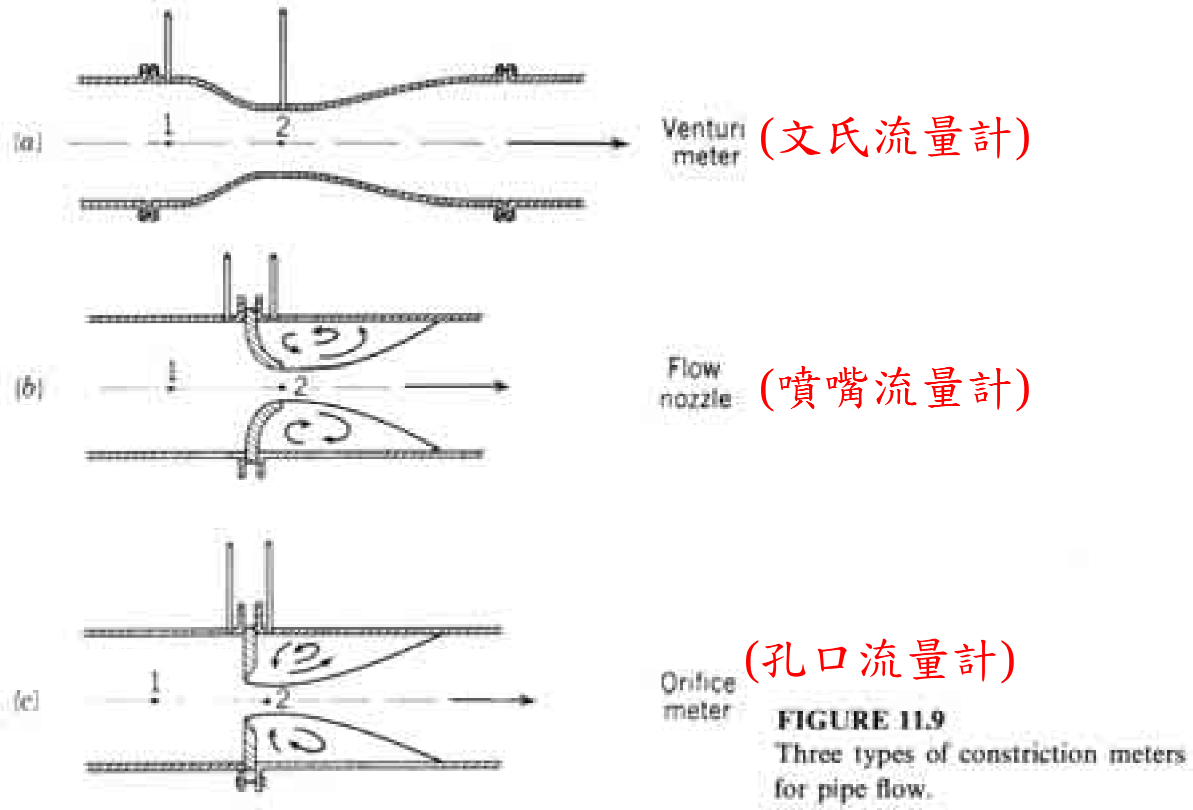


FIGURE 11.9 Three types of constriction meters for pipe flow.

11.9 Differential Head Meters

- 應用柏努力方程式(Bernoulli equation)於圖11.9的斷面1與斷面2之間，忽略水頭損失，以 Q 表示 V 且解出 Q 為

$$Q_{\text{理論值}} = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{1/2}}\left[2g\left(Z_1+\frac{P_1}{\gamma}-Z_2-\frac{P_2}{\gamma}\right)\right]^{1/2}$$

這方程式可修正為

$$Q = C_d A_2\left[2g\Delta\left(Z+\frac{P}{\gamma}\right)\right]^{1/2} \qquad \text{式中}\ C_d = \frac{1}{\left[1-(A_2/A_1)^2\right]^{1/2}}$$

流量係數 C_d 包含斷面1與2之間的水頭損失之修正且考率到流量計的幾何特性。

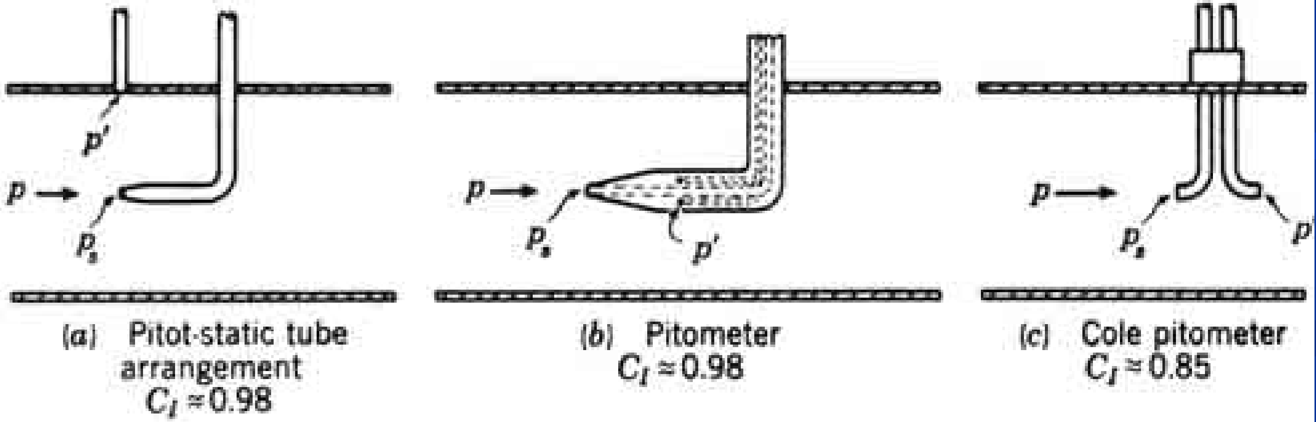
11.9 Differential Head Meters

- 另一種水頭差流量計為彎管流量計(bend meter)，乃是利用彎管內側與外側的壓力差(h)所製成，其流量計方程式為

$$Q = C_d A(2gh)^{1/2}$$

- 靜力皮托管與皮托計(圖11.10)也可歸類於水頭差流量計。其通用方程式為

$$V = C_I \left[\frac{2g(P_s - P')}{\gamma} \right]^{1/2}$$



(a) Pitot-static tube arrangement $C_I \approx 0.98$

(b) Pitometer $C_I \approx 0.98$

(c) Cole pitometer $C_I \approx 0.85$

FIGURE 11.10
Various types of pitot-tube flow-measuring devices.

11.10 Mechanical Meters

- 普通使用的兩種機械水錶為變位式水錶(displacement meter)與推測式水錶(inferential meter)。
- 簡單變位式水錶含有隨著通過的流體作往復運動的活塞，有個計數器記錄活塞的衝程數，而流過的體積即為衝程數與每衝程之位移的乘積。
- 推測式水錶含有螺旋葉，旋葉轉速為流速的函數。迴轉數以一計數器求出，再經檢定而與流量相關。

11.11 Other Methods of Flow Measurement

- 管路中測量流量的其他技術如下：
  - 化學追蹤劑—如追蹤劑稀釋法(Tracer-dilution method)
  - 磁性與音響的流量測量計(Magnetic and acoustic flowmeters)
  - 熱風力計(Thermal anenometer)
  - 雷射-都卜勒風力計(Laser-Doppler anenometer)

Forces Acting on Pipes

Outline— Forces Acting on Pipes

- 11.12 Internal Pressures
- 11.13 Water Hammer
- 11.14 Forces at Bends and Changes in Cross Section
- 11.15 Temperature Stresses
- 11.16 Flexural Stresses
- 11.17 External Loads on Buried Pipes
- 11.18 Crushing Strength of Pipe

11.12 Internal Pressures

- 管路中的內壓力是由靜壓力與水錘(wtaer hammer)所造成。內壓力造成管壁中的周向張力，可以下式估計

$$\sigma = \frac{pr}{t}$$

σ ：張應力

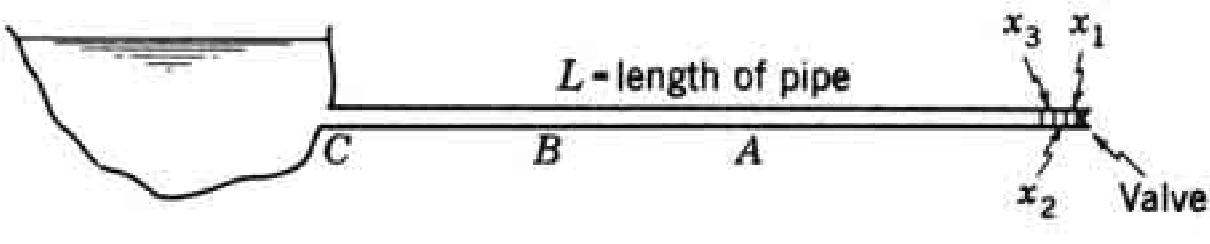
p ：壓力(靜水壓加水錘)

r ：管的內徑

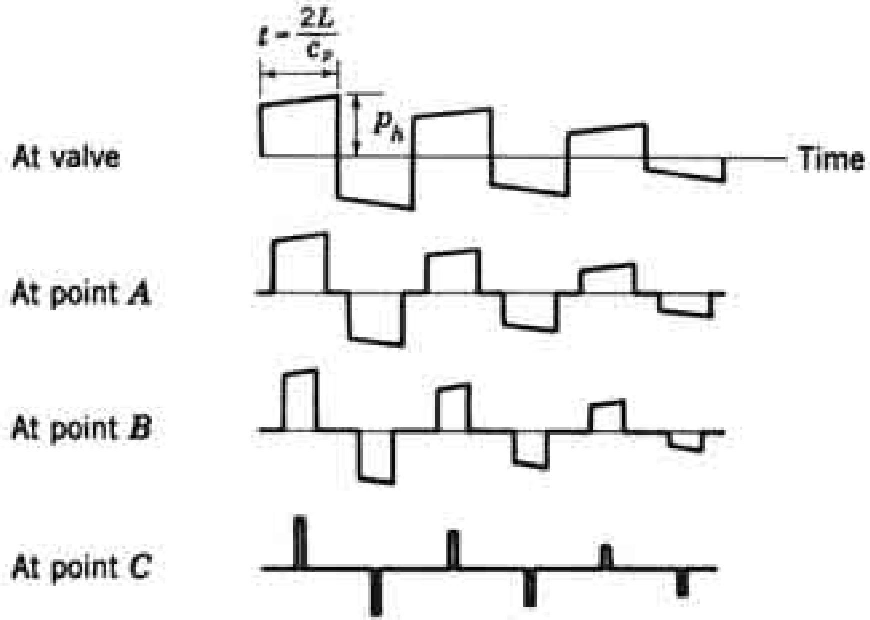
t ：管壁厚度

11.13 Water Hammer

- 當在管線中流動的液體受到閥門的關閉而突然停止時，動能即轉化為彈性能，且有一連串的正與負壓力波在管中往復移動直到因摩擦衰減而消失為止，這種現象稱為水錘(water hammer)。



(a) Valve at end of pipeline



(b) Water hammer pressures at selected points for the case of abrupt valve closure

FIGURE 11.11
The nature of water hammer.

11.13 Water Hammer

- 一壓力波在任何介質中的速度 c (波速celerity)與聲音在該介質中的速度相同，且由下式給出

$$c = \left(\frac{E}{\rho}\right)^{1/2}$$

式中 E 為該介質的彈性模數而 ρ 為密度

若阻止管的縱向伸長讓其周向伸長自由發生，則壓力波的速度 c_p 為

$$c_p = \left(\frac{E}{\rho}\right)^{1/2}\left(\frac{1}{1+ED/E_p t^*}\right)^{1/2}$$

式中
E_p：管壁的彈性模數
D：管的直徑
t^*：管壁厚度

11.13 Water Hammer

- 若圖11.12的閥門瞬間關閉，則有一壓力波以速度c_p沿著管傳播。在一短時距 dt 內，使長度為 $c_p dt$ 的水元素靜止。應用牛頓第二定律並忽略摩擦

$$F\ dt = M\ dV$$

$$-A\ dp\ dt = \rho\ A\, c_p\ dt\ dV$$

$$-dp = \rho\ c_p\ dV$$

由於速度減低至零，$dV = -V$ 且 dp等於水錘所造成的壓力P_h。於是

$$P_h = \rho\ c_p\ V$$

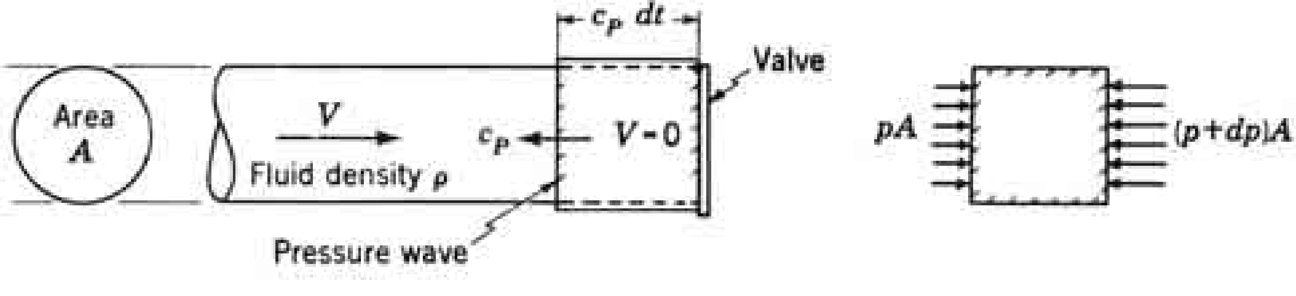
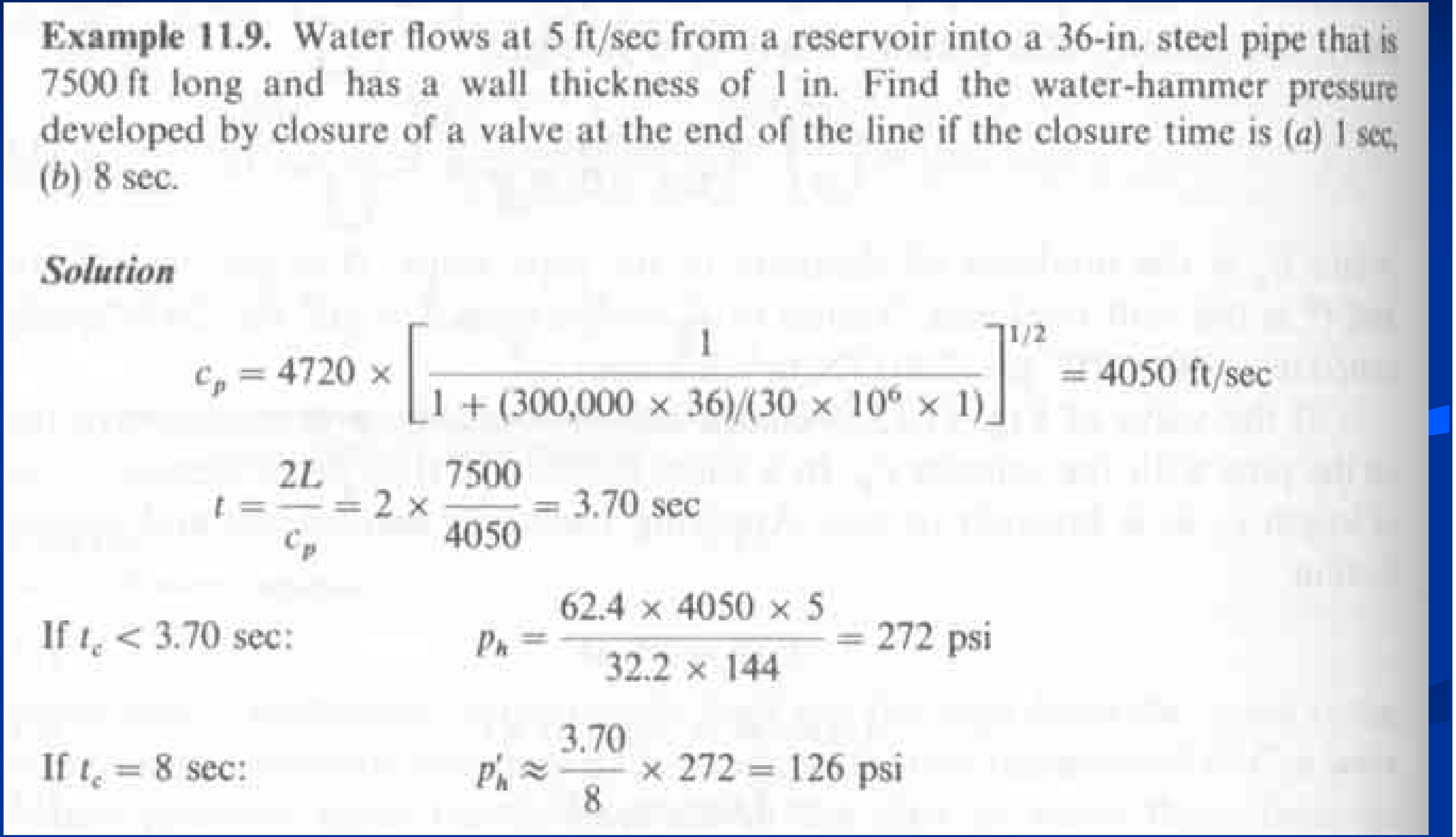


FIGURE 11.12
Definition sketch for analysis of water hammer in pipes.

11.13 Water Hammer

- 閥門處的總壓力在剛關閉後為 $P_h + P$，其中 P 為管中的靜水壓。
- 若管長為 L、壓力波在時間 $t = 2L/c_p$ 內從閥門傳到水庫再傳回來、 t_c 為閥門關閉時間，則
  - 當 $t_c \le t$ 時，水錘壓力 $P_h' = P_h = \rho\, c_p\, V$
  - 當 $t_c > t$ 時，水錘壓力 $P_h' = \dfrac{t}{t_c} P_h = \dfrac{2LP_h}{c_p t_c} = \dfrac{2LV\rho}{t_c}$
- 水錘壓力可藉著使用慢關閥門(slow-closing valves)、自動減壓閥(automatic relief valves)、氣室(air chambers)及平壓塔(surge tanks)而大量降低。

Example 11.9



Example 11.9. Water flows at 5 ft/sec from a reservoir into a 36-in. steel pipe that is 7500 ft long and has a wall thickness of 1 in. Find the water-hammer pressure developed by closure of a valve at the end of the line if the closure time is (*a*) 1 sec, (*b*) 8 sec.

Solution

$$c_p = 4720 \times \left[\frac{1}{1 + (300{,}000 \times 36)/(30 \times 10^6 \times 1)}\right]^{1/2} = 4050 \text{ ft/sec}$$

$$t = \frac{2L}{c_p} = 2 \times \frac{7500}{4050} = 3.70 \text{ sec}$$

If $t_c < 3.70$ sec: $$p_h = \frac{62.4 \times 4050 \times 5}{32.2 \times 144} = 272 \text{ psi}$$

If $t_c = 8$ sec: $$p'_h \approx \frac{3.70}{8} \times 272 = 126 \text{ psi}$$

11.14 Forces at Bends and Changes in Cross Section

- 圖11.13(b)所示為作用在水平彎管內水流之各力的自由體圖。應用衝量-動量原理得出

$$P_1A_1 - F_x - P_2A_2 \cos\theta = \rho Q(V_2 \cos\theta - V_1)$$

$$F_y - P_2A_2 \sin\theta = \rho QV_2 \sin\theta$$

- 一類似的分析可應用到作用在一管束縮區之水的力(圖11.14)。

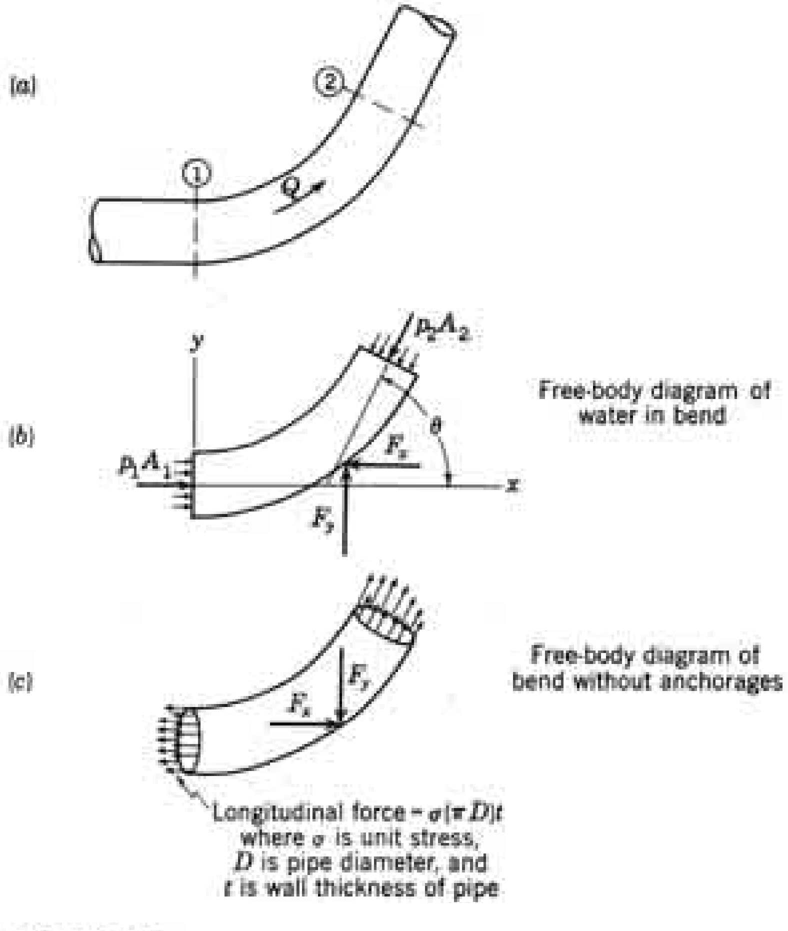


FIGURE 11.13
Forces at a horizontal pipe bend.

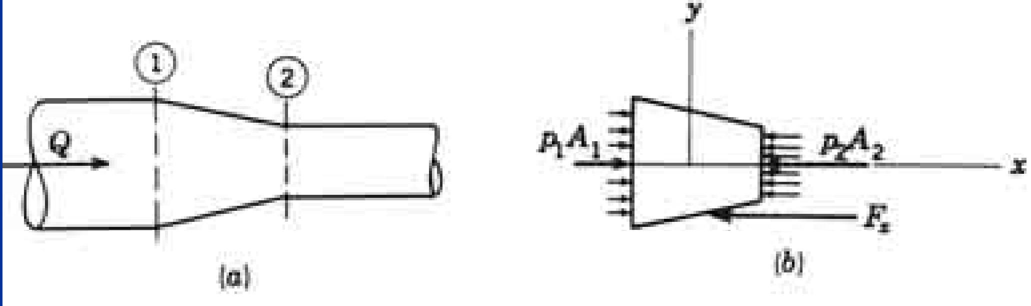


FIGURE 11.14
Forces at a contraction in a pipeline.

11.15 Temperature Stresses

- 管路若暴露在溫度有大變化之處，管中會產生相當大的縱向應力。當受到 ΔT 的溫度變化時，長度為 L 之管路的長度變化量 δ 為

$$\delta = \alpha\ L\ \Delta T$$

式中 α 為管材的熱澎脹係數

若阻止此一長度變化，則將生成縱向應力

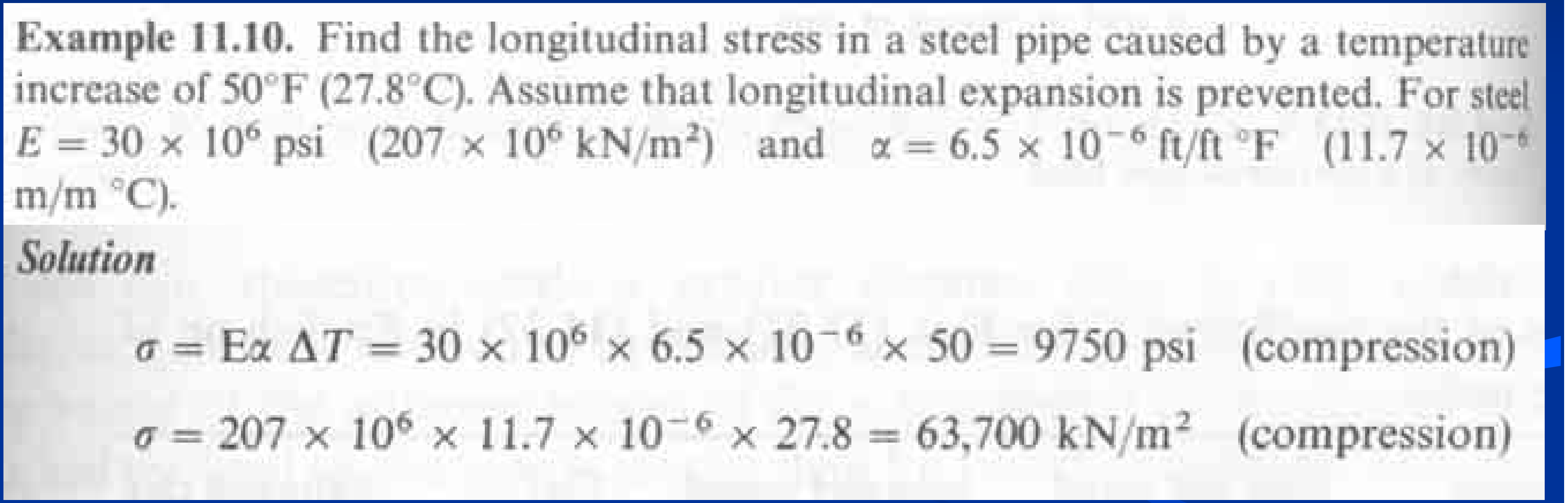
$$\sigma = E\varepsilon = E\frac{\delta}{L}$$

式中
ε：單位應變(每單位長度的伸長量)
E：彈性模數
σ：生成的單位應力

合併上二式，則可得

$$\sigma = E\alpha\ \Delta T$$

Example 11.10



Example 11.10. Find the longitudinal stress in a steel pipe caused by a temperature increase of 50°F (27.8°C). Assume that longitudinal expansion is prevented. For steel $E = 30 \times 10^6$ psi $(207 \times 10^6\ \text{kN/m}^2)$ and $\alpha = 6.5 \times 10^{-6}$ ft/ft °F $(11.7 \times 10^{-6}$ m/m °C).

Solution

$$\sigma = E\alpha\, \Delta T = 30 \times 10^6 \times 6.5 \times 10^{-6} \times 50 = 9750\ \text{psi} \quad \text{(compression)}$$

$$\sigma = 207 \times 10^6 \times 11.7 \times 10^{-6} \times 27.8 = 63{,}700\ \text{kN/m}^2 \quad \text{(compression)}$$

11.16 Flexural Stresses

- 一未受支承的管子，其作用如同一樑，所受載重為管重、管中水重及任何添加的載重。
- 由樑作用所生成的應力可用一般分析樑的方法求出。
- 管子是頗有效率的樑斷面，故除了長跨度或有很大的添加載重時，通常由樑作用所產生的應力都予以忽略。
- 對由內壓力、外載重、溫度變化及樑作用所產生聯合應力的嚴格分析包含有彈性原理的應用。

11.17 External Loads on Buried Pipes

- 埋設管上的外載重大小乃視管子的剛性、墊床及填料的特性而定。
- 對在窄溝內的剛性管(rigid pipe)而言，以每呎管長所受磅數表示的載重 w 經求得為

$$w = C\gamma B^2$$

式中 B：管頂部的溝寬度
γ：填料的比重量
C：填料及覆蓋厚度對溝寬度之比值的特性係數(表11.3)

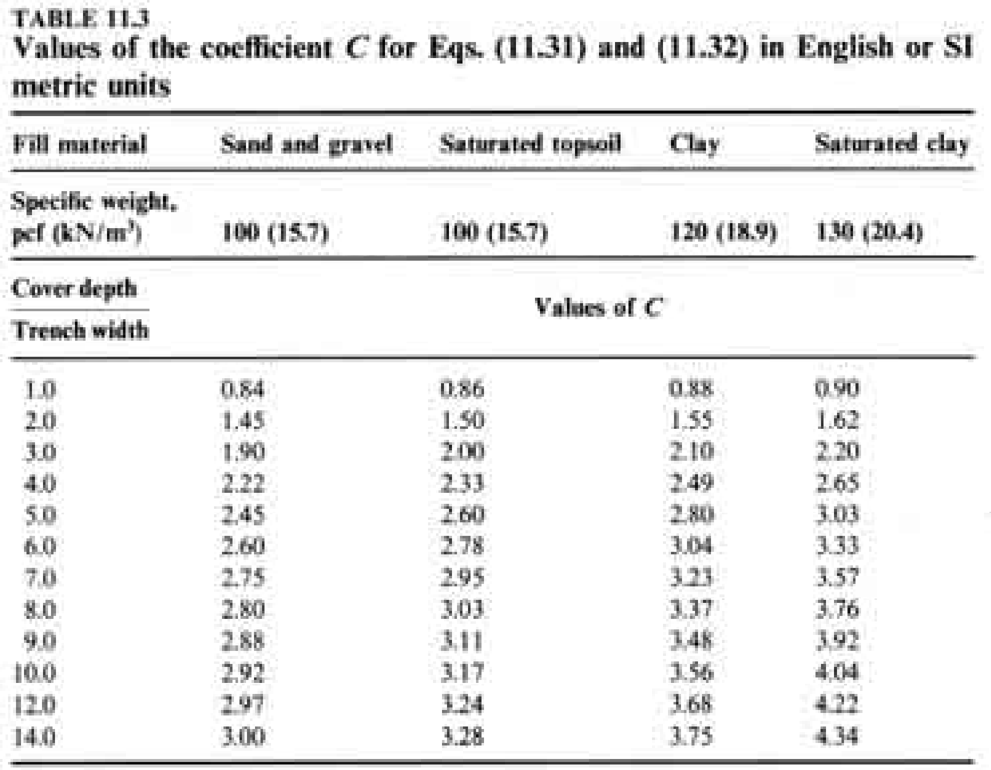


TABLE 11.3
Values of the coefficient *C* for Eqs. (11.31) and (11.32) in English or SI metric units

Fill material	Sand and gravel	Saturated topsoil	Clay	Saturated clay
Specific weight, pcf (kN/m³)	100 (15.7)	100 (15.7)	120 (18.9)	130 (20.4)
Cover depth / Trench width	Values of *C*			
1.0	0.84	0.86	0.88	0.90
2.0	1.45	1.50	1.55	1.62
3.0	1.90	2.00	2.10	2.20
4.0	2.22	2.33	2.49	2.65
5.0	2.45	2.60	2.80	3.03
6.0	2.60	2.78	3.04	3.33
7.0	2.75	2.95	3.23	3.57
8.0	2.80	3.03	3.37	3.76
9.0	2.88	3.11	3.48	3.92
10.0	2.92	3.17	3.56	4.04
12.0	2.97	3.24	3.68	4.22
14.0	3.00	3.28	3.75	4.34

11.17 External Loads on Buried Pipes

- 窄溝內(圖11.15(a))埋設之柔性管(flexible)上的載重經驗公式為

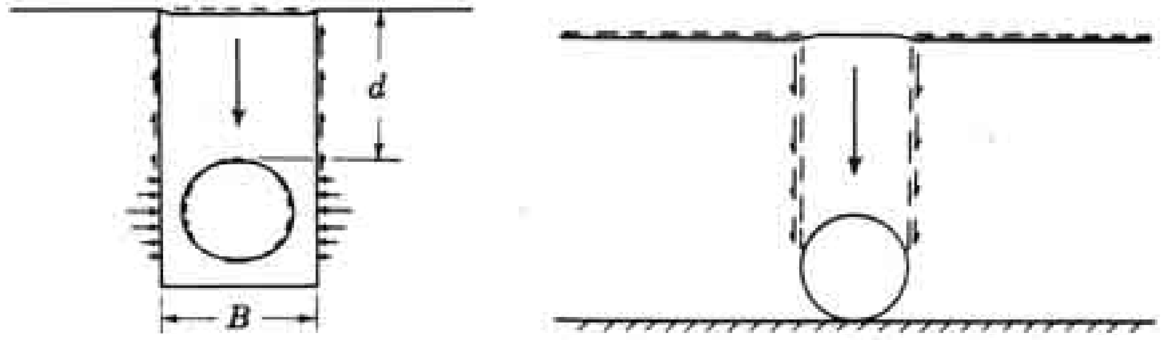
$$w = C\gamma BD$$

式中 D 為管的外直徑

- 在填土情況下(圖11.15(b))，埋管上載重的方程式為

$$w = C_p \gamma D^2$$

式中 C_p 值依管的型式及基礎與回填的特性而定(表11.4)



(a) Flexible pipe buried in a trench

(b) Rigid pipe on solid ground under fill

FIGURE 11.15
Loads on a buried pipe

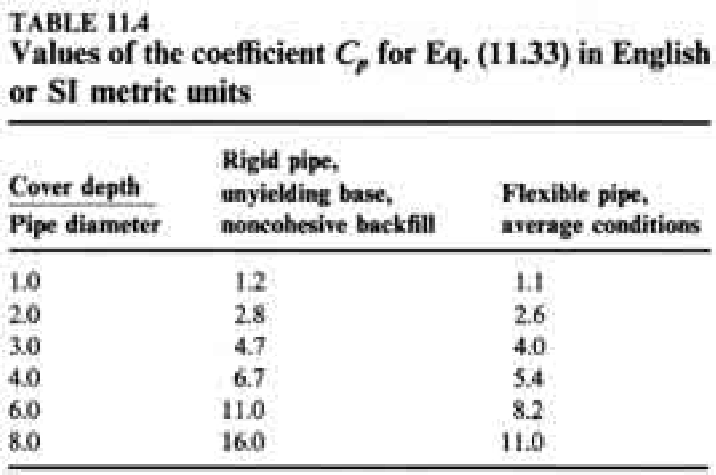


TABLE 11.4
Values of the coefficient C_p for Eq. (11.33) in English or SI metric units

Cover depth / Pipe diameter	Rigid pipe, unyielding base, noncohesive backfill	Flexible pipe, average conditions
1.0	1.2	1.1
2.0	2.8	2.6
3.0	4.7	4.0
4.0	6.7	5.4
6.0	11.0	8.2
8.0	16.0	11.0

11.17 External Loads on Buried Pipes

- 疊加載重傳遞至埋管的部份可應用彈性固體內應力分佈之包新內克方程式(Boussinesq’s equation)求出近似值。假設填土表面係水平的，此方程式為

$$p = \frac{3d^3P}{2\pi Z^5}$$

式中 p為填土中在深度d處任意點的單位壓力，Z為至載重P的傾斜高度。

- 圖11.16中單位長度管路上的總載重可藉上式對管子的投影面積積分而得。

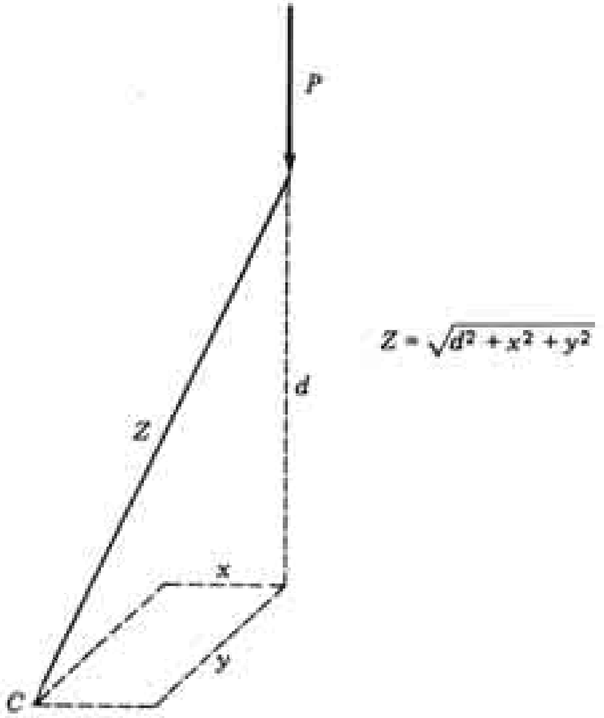
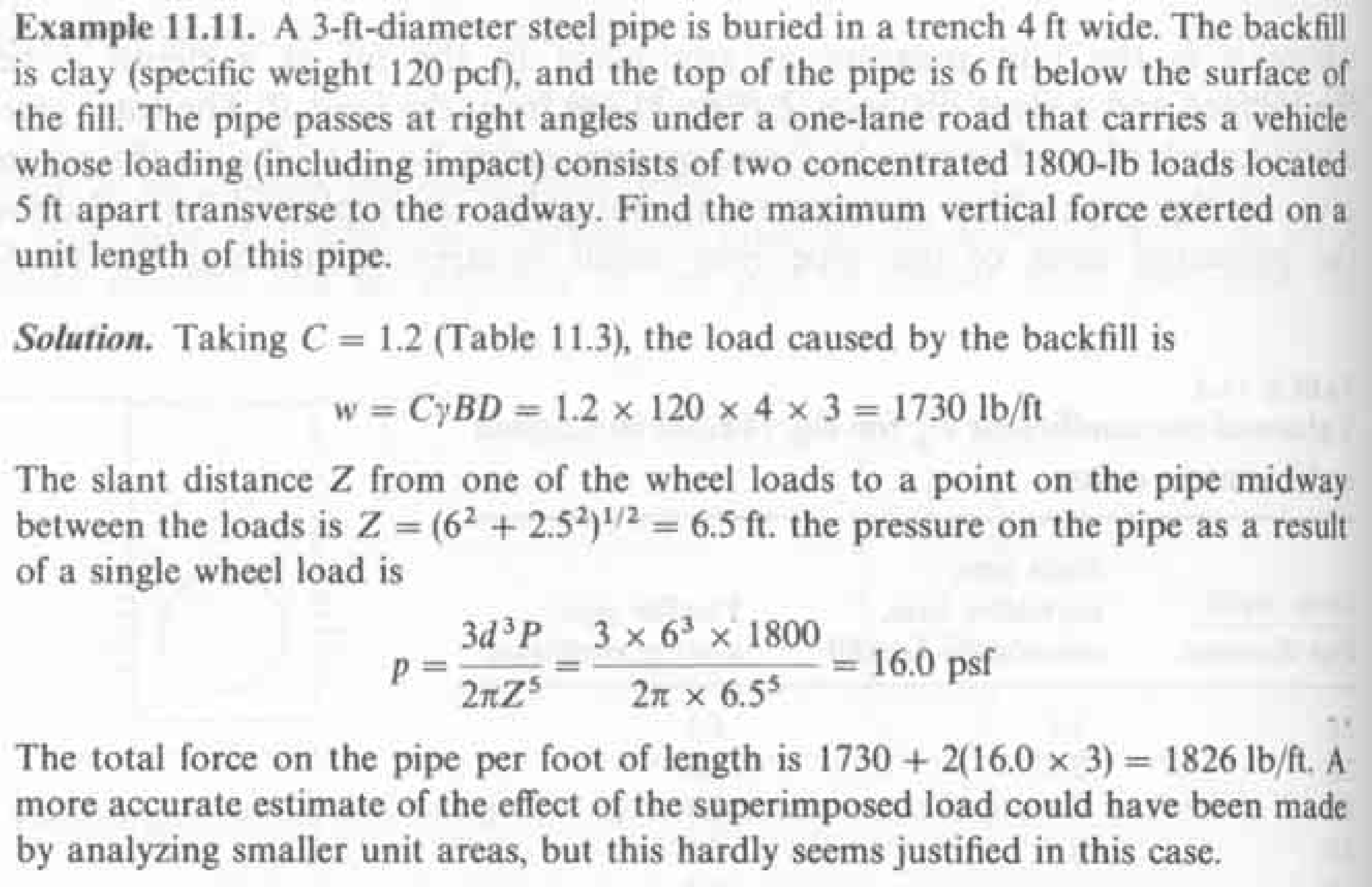


FIGURE 11.16
Slant distance Z from superimposed load P to center C of a vertically projected area of an element of a pipe.

Example 11.11



Example 11.11. A 3-ft-diameter steel pipe is buried in a trench 4 ft wide. The backfill is clay (specific weight 120 pcf), and the top of the pipe is 6 ft below the surface of the fill. The pipe passes at right angles under a one-lane road that carries a vehicle whose loading (including impact) consists of two concentrated 1800-lb loads located 5 ft apart transverse to the roadway. Find the maximum vertical force exerted on a unit length of this pipe.

Solution. Taking $C = 1.2$ (Table 11.3), the load caused by the backfill is

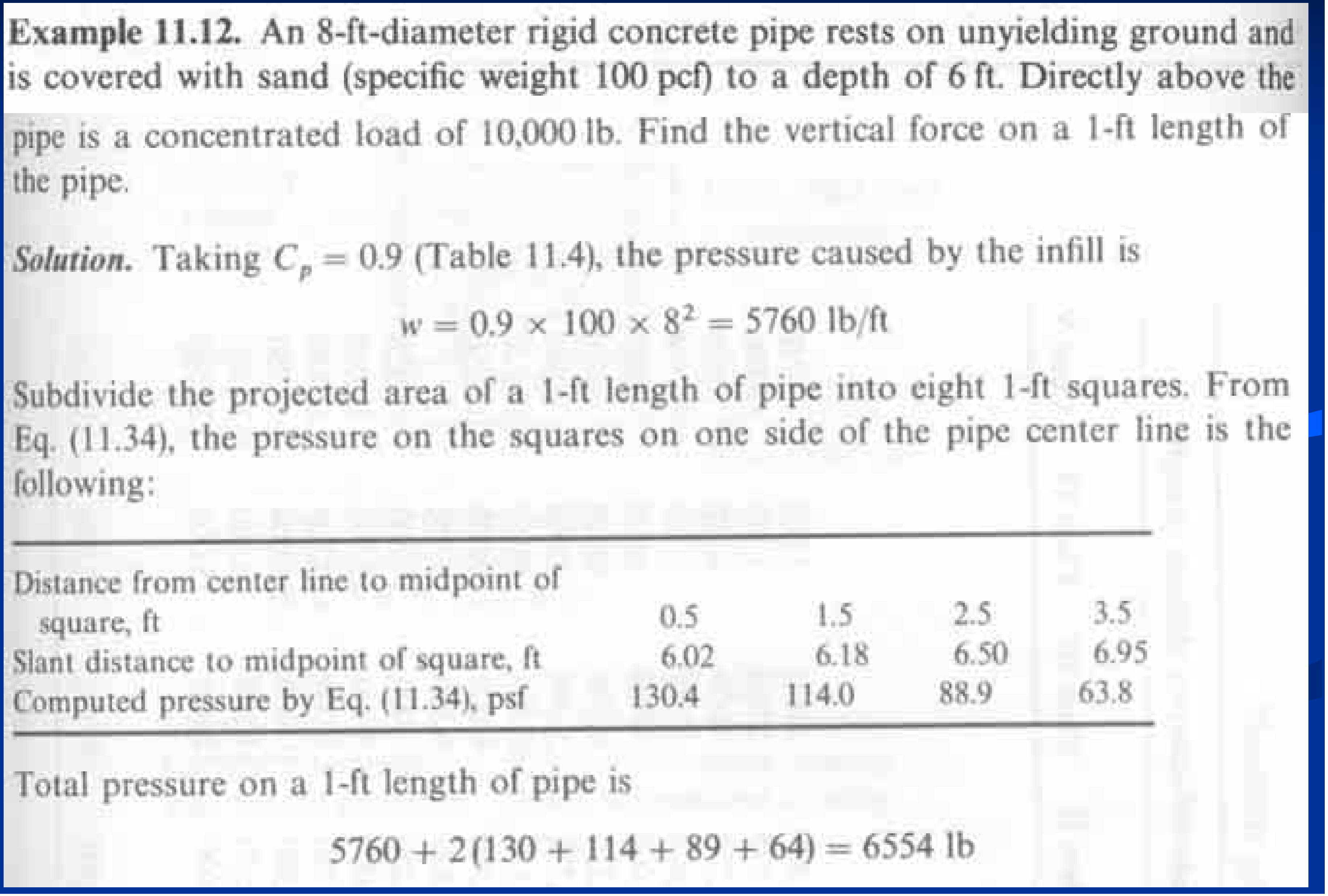
$$w = C\gamma BD = 1.2 \times 120 \times 4 \times 3 = 1730 \text{ lb/ft}$$

The slant distance Z from one of the wheel loads to a point on the pipe midway between the loads is $Z = (6^2 + 2.5^2)^{1/2} = 6.5$ ft. the pressure on the pipe as a result of a single wheel load is

$$p = \frac{3d^3P}{2\pi Z^5} = \frac{3 \times 6^3 \times 1800}{2\pi \times 6.5^5} = 16.0 \text{ psf}$$

The total force on the pipe per foot of length is $1730 + 2(16.0 \times 3) = 1826$ lb/ft. A more accurate estimate of the effect of the superimposed load could have been made by analyzing smaller unit areas, but this hardly seems justified in this case.

Example 11.12



Example 11.12. An 8-ft-diameter rigid concrete pipe rests on unyielding ground and is covered with sand (specific weight 100 pcf) to a depth of 6 ft. Directly above the pipe is a concentrated load of 10,000 lb. Find the vertical force on a 1-ft length of the pipe.

Solution. Taking $C_p = 0.9$ (Table 11.4), the pressure caused by the infill is

$$w = 0.9 \times 100 \times 8^2 = 5760 \text{ lb/ft}$$

Subdivide the projected area of a 1-ft length of pipe into eight 1-ft squares. From Eq. (11.34), the pressure on the squares on one side of the pipe center line is the following:

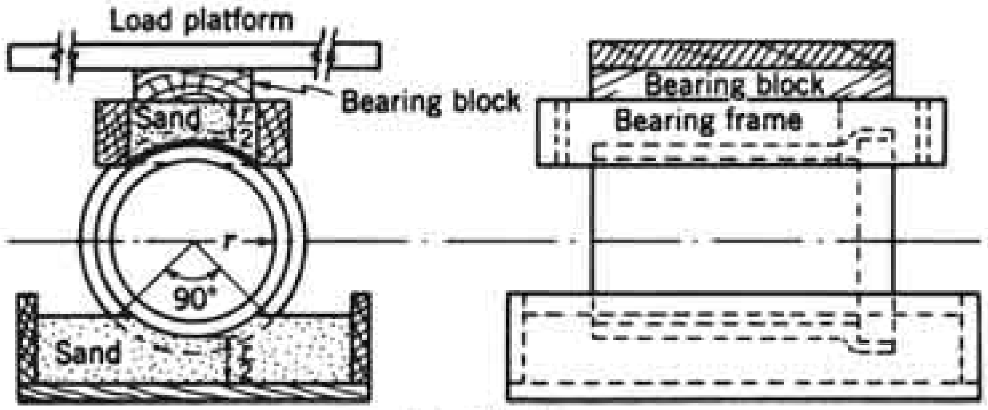
Distance from center line to midpoint of square, ft	0.5	1.5	2.5	3.5
Slant distance to midpoint of square, ft	6.02	6.18	6.50	6.95
Computed pressure by Eq. (11.34), psf	130.4	114.0	88.9	63.8

Total pressure on a 1-ft length of pipe is

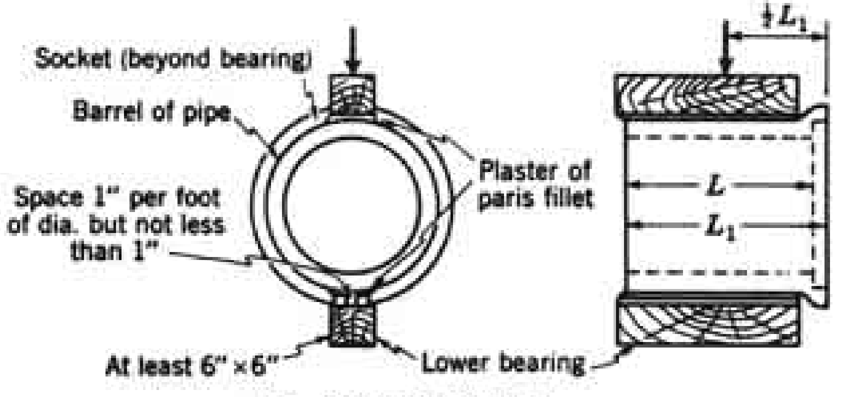
$$5760 + 2(130 + 114 + 89 + 64) = 6554 \text{ lb}$$

11.18 Crushing Strength of Pipe

- 由於管中組合應力的性質複雜，故除非是大且重要的結構，通常很少詳細地分析此應力。



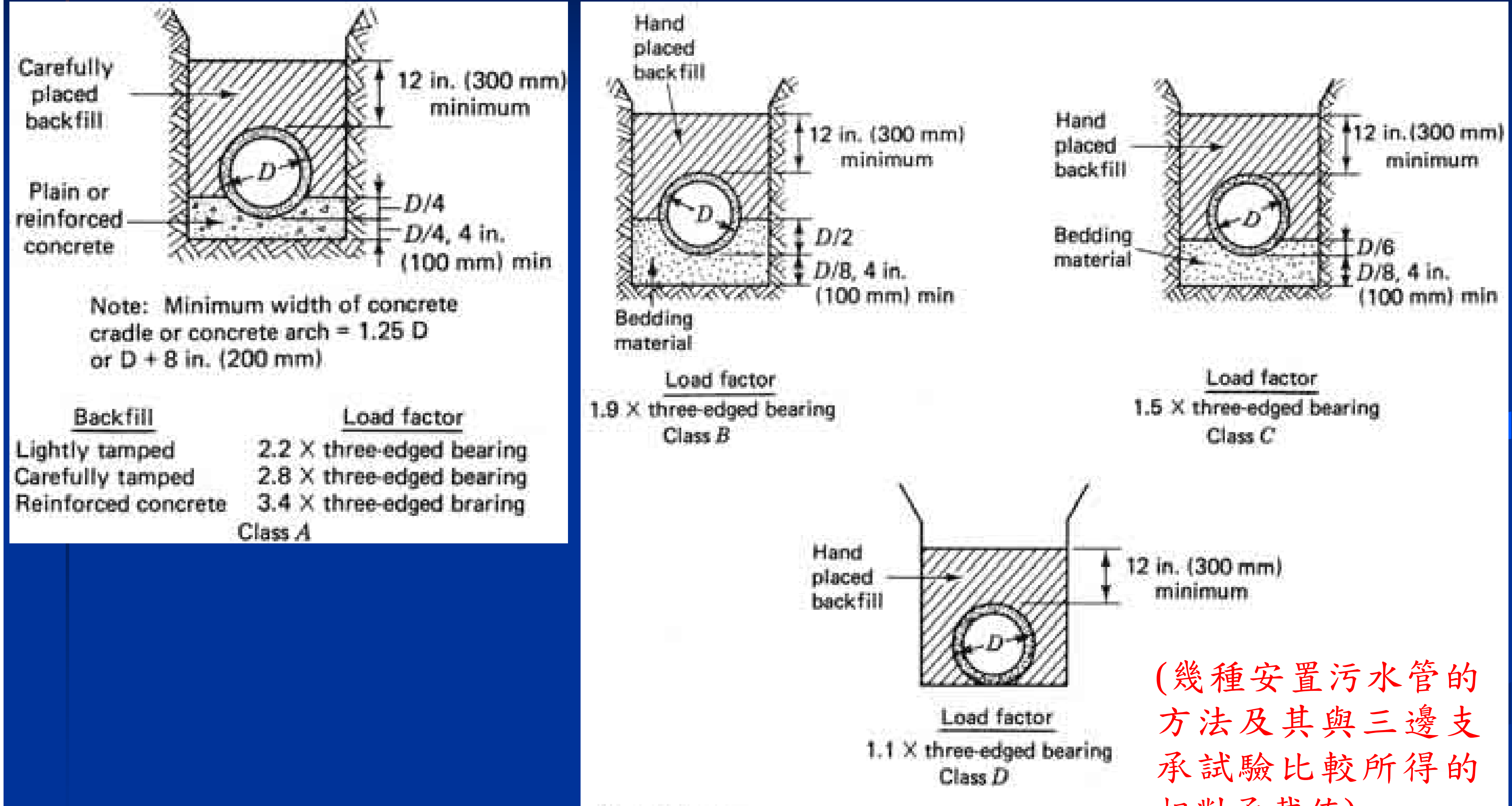
(a) Sand bearing



(b) Three-edge bearing

FIGURE 11.17 (決定管之壓碎強度的方法)
Methods for determining the crushing strength of pipe. (*ASTM Specification C14*)

11.18 Crushing Strength of Pipe



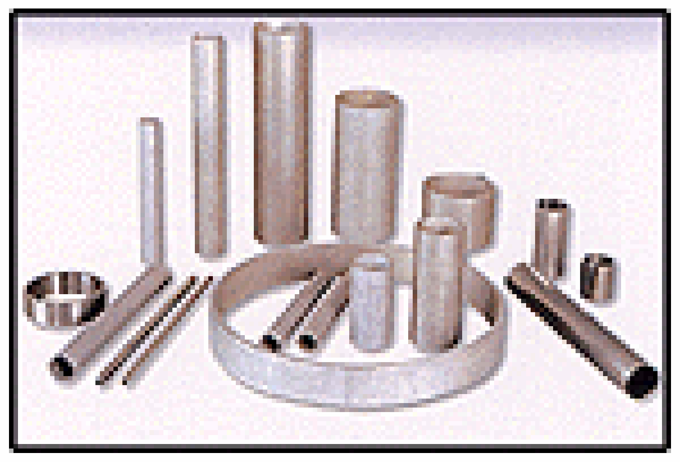
(幾種安置污水管的方法及其與三邊支承試驗比較所得的相對承載值)

FIGURE 11.18

Some methods of laying sewer pipe and the relative bearing values developed as compared to the three-edge bearing tests. (*ASTM Specification C12*)

Materials for Pressure Conduits

- 壓力管路主要的管材有鋼(steel)、鑄鐵(cast iron)、混凝土(concrete)、釉陶(vitrified clay)及各種塑膠(plastics)。
- 11.19 Steel Pipe(鋼管)



各種鋼管

各種配件

Materials for Pressure Conduits

- 11.20 Ductile-Iron Pipe(延展鐵管) and Cast-Iron Pipe(鑄鐵管)
  - 延展鐵管與鑄鐵管不同處在於管上的游離碳

延展鐵管

平頭式鑄鐵管

Materials for Pressure Conduits

- 11.21 Corrosion of Metal Pipes(金屬管的腐蝕)
  - 金屬管會受到化學腐蝕。最簡單的形式發生於鐵進入溶液中形成正離子，並與水中負離子結合生成氫氧化亞鐵。
- 11.22 Concrete Pipe (混凝土管)

混凝土管

金屬管線腐蝕
產生許多小洞

Materials for Pressure Conduits

- 11.23 Vitrified-Clay Pipe(釉陶管)
- 11.24 Plastic Pipe(塑膠管)

釉陶管

PVC硬質塑膠管

Materials for Pressure Conduits

- 11.25 Miscellaneous Types of Pipe(其他種管)

金屬軟管

建築用給水銅管

Appurtenances for Pressure Conduits

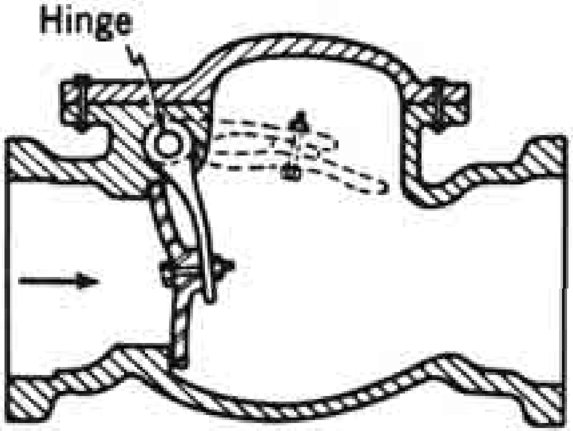
Outline— Appurtenances for Pressure Conduits

- 11.26 Gates and Valves
- 11.27 Surge Tanks
- 11.28 Inverted Siphons

11.26 Gates and Valves

- 管線的適當運作需要很多不同類的閥，種類如下：
  - ◆ 閘閥(gate valve)：用來調節管線中的水流。
  - ◆ 止回閥(check valve)：僅容許一個方向的水流。
  - ◆ 排水閥(drain valve)或吹洩閥(blow-off valve)：放空管中的水以便檢修。

FIGURE 11.24 (閘閥)
Outside-screw type, cast-iron gate valve. (*Iowa Valve Company*)



(止回閥)
FIGURE 11.25
Details of a check valve.

11.26 Gates and Valves

- 減壓閥 (pressure-relief valve)：消減管中的水錘壓力。
- 進氣閥 (air-inlet valve)：當一管中的壓力降低至某一預定值時會自動打開，讓空氣進入管內。
- 調壓閥 (pressure-regulating valve)：容許從高壓系統至低壓系統的水流僅發生在當低壓側的壓力太不大時。

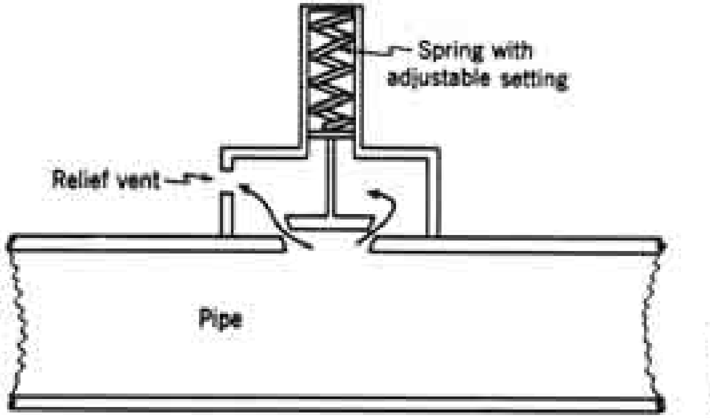


FIGURE 11.26 (減壓閥)
Pressure-relief valve in the open position.

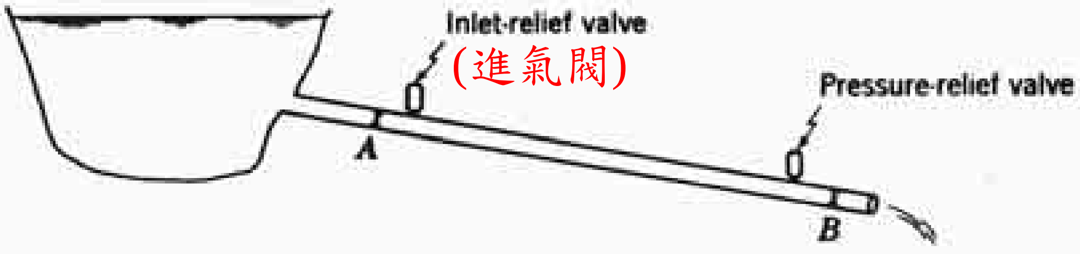
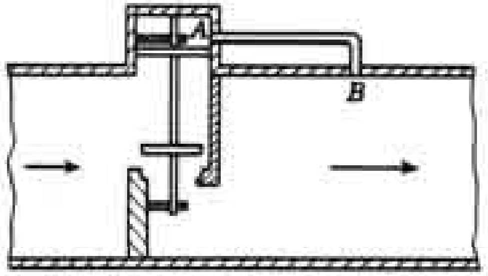


FIGURE 11.27
Air-relief valves installed on a pipeline.



Note: An increased pressure in chamber A will close the valve

FIGURE 11.28 (調壓閥)
Schematic diagram of a pressure-regulating valve.

11.27 Surge Tanks

- 平壓塔係設置在大管線上用來消滅由水錘所造成的過度壓力，且在閥門突然開啟時能提供水量以減少負壓力。

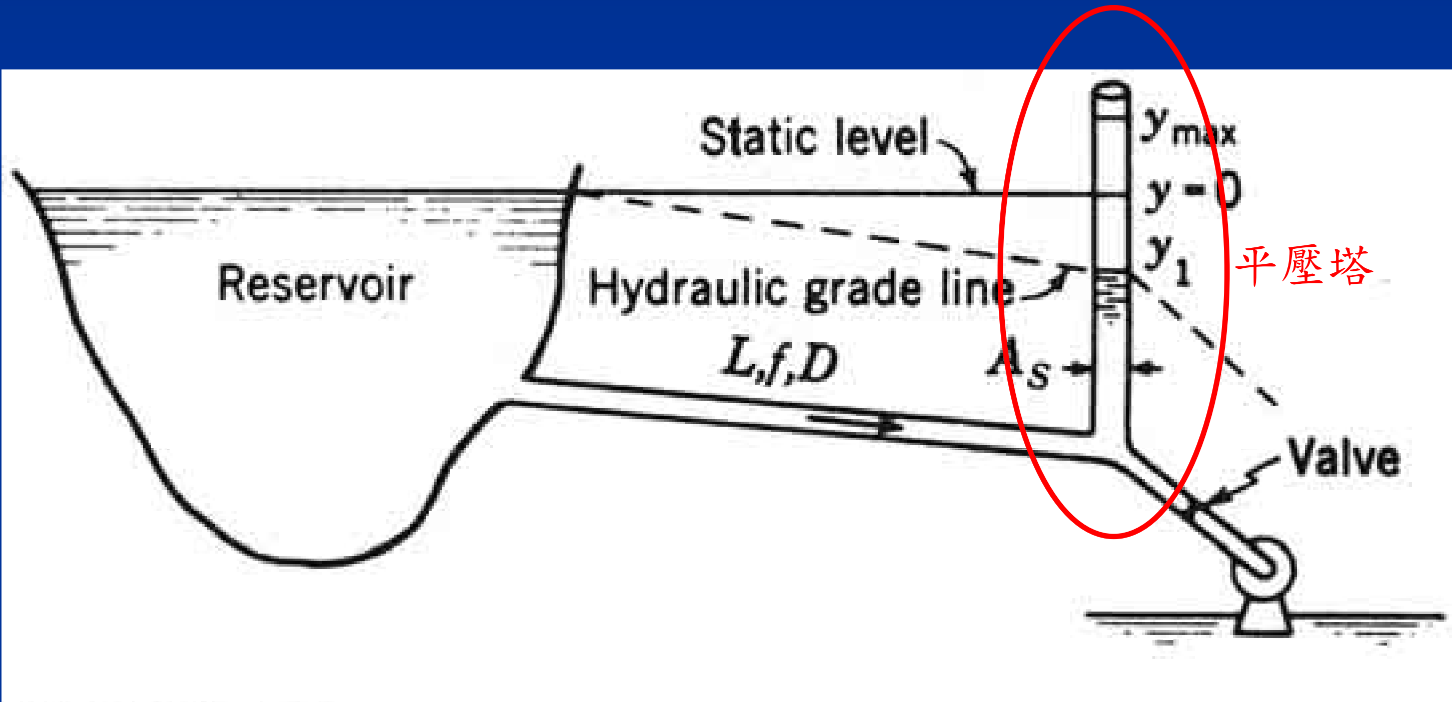


FIGURE 11.29
Definition sketch for a surge-tank analysis.

11.27 Surge Tanks

- 忽略流體摩擦、平壓塔中的速度水頭及在管與平壓塔入口處的損失後，可寫出變量流的能量方程式為

$$y + f\frac{L}{D}\frac{V^2}{2g} + \frac{L}{g}\frac{dV}{dy}\frac{dy}{dt} = 0$$

與連續方程式為

$$AV = A_s\frac{dy}{dt}$$

式中
y ：平壓塔中的水位
L、f、D ：管路的特徵量
C ：平壓塔的斷面積

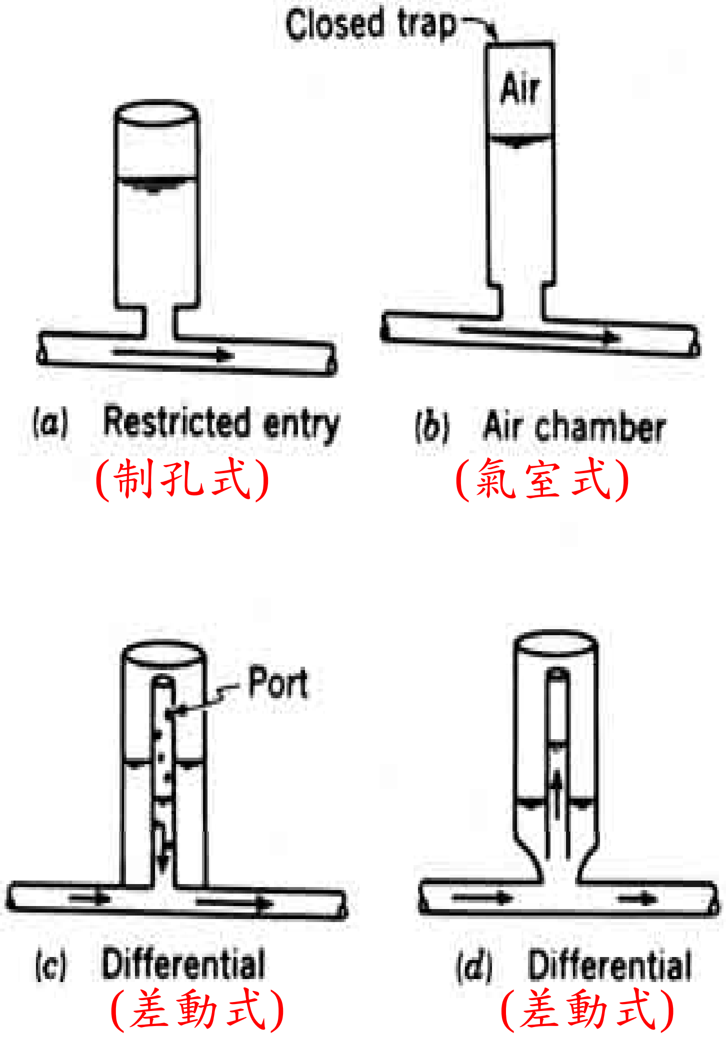
合併上二式，積分後解出 V 為

$$V^2 = \frac{2gAD^2}{LA_s f^2}\left(1 - \frac{fA_s}{AD}y\right) - Ce^{-(fA_s/AD)y}$$

此式表示出從閥關閉到第一個湧浪頂部之時距內，管中流速與塔中水位的關係。

11.27 Surge Tanks

- 平壓塔的種類如圖11.30所示。



(a) Restricted entry (制孔式)

(b) Air chamber (氣室式)

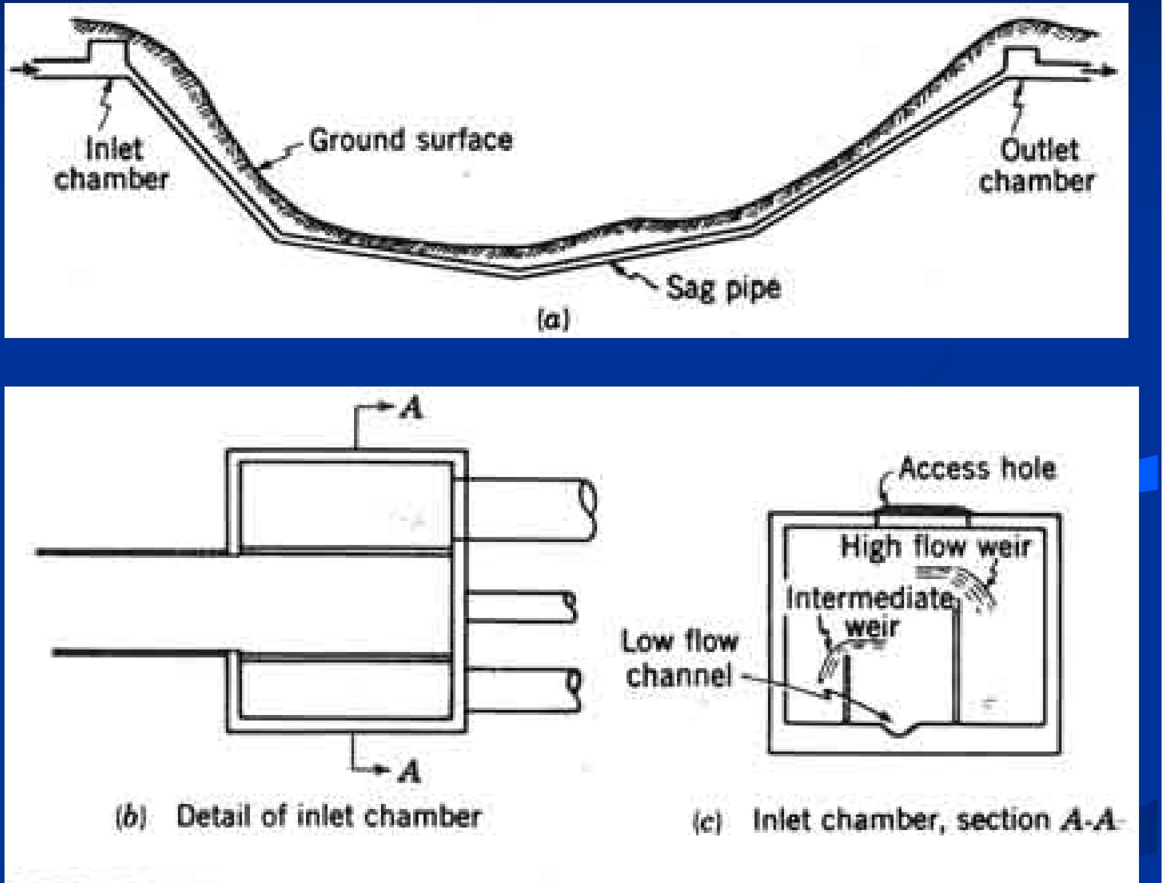
(c) Differential (差動式)

(d) Differential (差動式)

FIGURE 11.30
Types of surge tanks.

11.28 Inverted Siphons

- 倒虹吸管(inverted siphon)係指輸送渠道或污水管裏的水流越過窪地的一段壓力管路，實際上不含有虹吸作用，而下垂管(sag pipe)或窪陷污水管(depressed sewer)將是兩個更有描述力與更正確的術語。



(b) Detail of inlet chamber

(c) Inlet chamber, section A-A

FIGURE 11.31
A multiple-pipe inverted siphon, or sag pipe.