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Do the pure martingale and joint normality hypotheses hold for futures contracts? Implications for the optimal hedge ratios

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Abstract

It is well known that the optimal hedge ratios derived based on the mean-variance approach, the expected utility maximizing approach, the mean extended-Gini approach, and the generalized semivariance approach will all converge to the minimum-variance hedge ratio if the futures price follows a pure martingale process and if the spot and futures returns are jointly normal. In this paper, we perform empirical tests to see if the pure martingale and joint normality hypotheses hold using 25 different futures contracts and five different hedging horizons. Our results indicate that the pure martingale hypothesis holds for all commodities and all hedging horizons except for three stock index futures contracts. As for joint normality, we propose two new tests based on the generalized method of moments, which allow for calculating multivariate test statistics that take account of the contemporaneous correlation across spot and futures returns. Our findings show that the joint normality hypothesis generally does not hold except for a few contracts and relatively long hedging horizons.

JEL classification: G13

Keywords: Hedge ratio; Pure martingale hypothesis; Joint normality hypothesis

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1. Introduction

Derivative securities such as futures contracts have been extensively used by practitioners in hedging risk exposure to commodity prices, exchange rates, interest rates, and prices of other financial securities. In the past, both academicians and practitioners have shown great interest on the issue of hedging with futures, which is evident from a large number of articles written in this area. One of the main issues in futures hedging involves the determination of the optimal hedge ratio. However, the optimal hedge ratio depends critically on the particular objective function to be optimized. Many different objective functions are currently being used. For example, one of the most widely used optimal hedge ratios is the so-called minimum-variance (MV) hedge ratio. This MV hedge ratio is derived by minimizing the variance of the hedged portfolio and it is quite simple to understand and estimate (e.g., see Johnson, 1960; Ederington, 1979; Myers & Thompson, 1989). However, the MV hedge ratio completely ignores the expected return of the hedged portfolio. Therefore, this strategy is in general inconsistent with the mean-variance framework unless the individuals are infinitely risk-averse or the futures price follows a pure martingale process (i.e., expected futures price change is zero).

Other objective functions used in the derivation of the optimal hedge ratio include some linear combinations of the expected return and risk (variance) of the hedged portfolio, where the expected return is maximized and at the same time the risk is minimized (e.g., see Howard & D'Antonio, 1984; Cecchetti, Cumby, & Figlewski, 1988; Hsin, Kuo, & Lee, 1994). These objective functions are consistent with the mean-variance framework. The optimal hedge ratios based on these objective functions can be referred to as the optimal mean-variance hedge ratios. It is important to note that if the futures price follows a pure martingale process, then the optimal mean-variance hedge ratio will be the same as the MV hedge ratio.

Another aspect of the mean-variance-based strategies is that even though they are an improvement over the MV strategy, for them to be consistent with the expected utility maximization principle, either the utility function needs to be quadratic or the returns should be jointly normal. If neither of these assumptions is valid, then the hedge ratio may not be optimal with respect to the expected utility maximization principle. Some researchers have solved this problem by deriving the optimal hedge ratio based on the maximization of the expected utility (e.g., see Cecchetti et al., 1988). However, this approach requires the use of a specific utility function and specific return distribution.

Attempts have been made to eliminate these specific assumptions regarding the utility function and return distributions. Some of them involve the minimization of the mean extended-Gini (MEG) coefficient, which is consistent with the concept of stochastic dominance (e.g., see Cheung, Kwan, & Yip, 1990; Kolb & Okunev, 1992, 1993; Lien & Luo, 1993; Shalit, 1995; Lien & Shaffer, 1999). Shalit (1995) shows that if the spot and futures returns are jointly normally distributed, then the MEG-based hedge ratio will be the same as the MV hedge ratio.

Hedge ratios based on the generalized semivariance (GSV) or lower partial moments have been recently proposed (e.g., see De Jong, De Roon, & Veld, 1997; Lien & Tse, 1998, 2000; Chen, Lee, & Shrestha, 2001). These hedge ratios are also consistent with the concept of stochastic dominance. Furthermore, these GSV-based hedge ratios have another attractive feature in that they measure portfolio risk by the GSV, which is consistent with the risk perceived by managers, because of its emphasis on the returns below the target return (see Crum, Laughhunn, & Payne, 1981; Lien & Tse, 2000). Lien and Tse (1998) show that if the spot and futures returns are jointly normally distributed, then the minimum-GSV hedge ratio will be equal to the MV hedge ratio.

It is clear from the above discussion that the optimal hedge ratios which are derived based on the mean-variance approach, the expected utility maximizing approach, the MEG approach, and the GSV approach will all converge to the MV hedge ratio if the futures price follows a pure martingale process and if the spot and futures returns are jointly normal. Because the MV hedge ratio is easy to understand, simple to compute, and most widely used, it is important to investigate if these two conditions hold. If these two conditions hold, then we do not have to compute various hedge ratios, because all of them will converge to the same MV hedge ratio.

In this paper, we perform empirical tests to see if the pure martingale and joint normality hypotheses hold using 25 different futures contracts. *T*-tests are used to test the pure martingale hypothesis that the expected return on futures is equal to zero. As for the joint normality of spot and futures returns, we develop two new tests based on the generalized method of moments (GMM). The GMM approach is proposed by Hansen (1982) and implemented by Richardson and Smith (1993) in their study of multivariate normality in stock returns. The two new tests developed in this study allow for calculating multivariate test statistics that take account of the contemporaneous correlation across spot and futures returns. To see if the results of our tests depend on the length of hedging horizon, we also perform tests using five different hedging horizons.

We find that the pure martingale hypothesis holds for all the 25 commodities except for SP500, TSE35, and FTSE100. This is true for all the five different hedging horizons. The results suggest that ignoring the expected return in the derivation of the optimal hedge ratio does not significantly change the optimal hedge ratio except for the three stock index futures. Therefore, with the exception of a few futures contracts, the mean-variance hedge ratio would be approximately the same as the MV hedge ratio. The empirical tests for the joint normality of spot and futures returns show that the joint normality hypothesis tends to be rejected for all the 25 commodities when the length of hedging horizon is relatively short. For longer hedging horizons, the joint normality hypothesis holds only for a few futures contracts according to our two tests based on the GMM approach. Our results suggest that the hedge ratios which are derived based on the expected utility maximizing approach, the MEG approach, and the GSV approach will not converge to the MV hedge ratio for most futures contracts and for shorter hedging horizons.

The remainder of this paper is organized as follows. In Section 2, alternative theories for deriving the optimal hedge ratios are reviewed. Section 3 develops two new tests for the joint normality of spot and futures returns. The empirical results are presented in Section 4. The paper concludes in the final section.

2. Alternative derivations of the optimal hedge ratio

Consider a hedged portfolio consisting of C_s units of a long spot position and C_f units of a short futures position.[1] Let S_t and F_t denote the spot and futures prices at time t, respectively. The return on the hedged portfolio, R_h, is then given by:

$$R_h = \frac{C_s S_t R_s - C_f F_t R_f}{C_s S_t} = R_s - h R_f, \tag{1}$$

where $(h = (C_f F_t / C_s S_t))$ is the so-called hedge ratio, and $(R_s = ((S_{t+1} - S_t)/S_t) = \Delta S_t/S_t)$ and $(R_f = ((F_{t+1} - F_t)/F_t) = \Delta F_t/F_t)$ are so-called one-period returns on the spot and futures positions, respectively. Sometimes, the hedge ratio is discussed in terms of price changes (profits)

[1] Without loss of generality, we assume that the size of the futures contract is one.

instead of returns. In this case, the profit on the hedged portfolio, ΔV_{H}, and the hedge ratio, H, are respectively, given by:

$$\Delta V_{\mathrm{H}} = C_{\mathrm{s}}\Delta S_t - C_{\mathrm{f}}\Delta F_t \text{ and } H = \frac{C_{\mathrm{f}}}{C_{\mathrm{s}}}. \tag{2}$$

The optimal hedge ratio (either h or H) will depend on a particular objective function to be optimized. In this section, we briefly discuss the optimal hedge ratios derived based on the MV approach, the mean-variance approach, the expected utility maximizing approach, the MEG approach, and the GSV approach.

2.1. Minimum-variance hedge ratio

The most common hedge ratio is the MV hedge ratio. Johnson (1960) derives this hedge ratio by minimizing the variance of changes in the value of the hedged portfolio as follows:

$$\mathrm{Var}(\Delta V_{\mathrm{H}}) = C_{\mathrm{s}}^2\mathrm{Var}(\Delta S) + C_{\mathrm{f}}^2\mathrm{Var}(\Delta F) - 2C_{\mathrm{s}}C_{\mathrm{f}}\mathrm{Cov}(\Delta S, \Delta F). \tag{3}$$

The MV hedge ratio, in this case, is given by:

$$H_{\mathrm{J}}^* = \frac{C_{\mathrm{f}}}{C_{\mathrm{s}}} = \frac{\mathrm{Cov}(\Delta S, \Delta F)}{\mathrm{Var}(\Delta F)}. \tag{4}$$

We can alternatively derive the MV hedge ratio by minimizing the variance of the return on the hedged portfolio ($\mathrm{Var}(R_{\mathrm{h}})$), which is given by:

$$\mathrm{Var}(R_{\mathrm{h}}) = \mathrm{Var}(R_{\mathrm{s}}) + h^2\mathrm{Var}(R_{\mathrm{f}}) - 2h\mathrm{Cov}(R_{\mathrm{s}}, R_{\mathrm{f}}). \tag{5}$$

In this case, the MV hedge ratio is given by:

$$h_{\mathrm{J}}^* = \frac{\mathrm{Cov}(R_{\mathrm{s}}, R_{\mathrm{f}})}{\mathrm{Var}(R_{\mathrm{f}})} = \rho\frac{\sigma_{\mathrm{s}}}{\sigma_{\mathrm{f}}} \tag{6}$$

where ρ is the correlation coefficient between R_{s} and R_{f}, and σ_{s} and σ_{f} are standard deviations of R_{s} and R_{f}, respectively.

The MV hedge ratio is easy to understand and simple to compute. However, in general the MV hedge ratio is not consistent with the mean-variance framework since it ignores the expected return on the hedged portfolio. The MV hedge ratio would be consistent with the mean-variance framework only if either the investors are infinitely risk-averse or the expected return on the futures contract is zero.

2.2. Optimum mean-variance hedge ratio

In order to make the hedge ratio consistent with the mean-variance framework, we need to explicitly include the expected return on the hedged portfolio in the objective function. For example, Hsin et al. (1994) derive the optimal hedge ratio that maximizes the following utility function:

$$\underset{C_{\mathrm{f}}}{\mathrm{Max}}\, V(E(R_{\mathrm{h}}), \sigma_{\mathrm{h}}; A) = E(R_{\mathrm{h}}) - 0.5A\sigma_{\mathrm{h}}^2, \tag{7}$$

where $\sigma_h^2 = \mathrm{Var}(R_h)$ and A represents the risk aversion parameter. In this case, the optimal hedge ratio is given by:

$$h_2 = -\frac{C_f^* F}{C_s S} = -\left[\frac{E(R_f)}{A\sigma_f^2} - \rho\frac{\sigma_s}{\sigma_f}\right]. \tag{8}$$

It can be seen from Eqs. (6) and (8) that if $A \to \infty$ or $E(R_f) = 0$, then h_2 would be equal to the MV hedge ratio h_J^*. Therefore, the MV hedge ratio would be the same as the optimal mean-variance hedge ratio if the expected return on the futures contracts is zero (i.e., futures prices follow a simple martingale process).

2.3. Sharpe hedge ratio

Another way of making the hedge ratio consistent with the mean-variance framework is incorporating the risk-return tradeoff (Sharpe measure) in the objective function. For example, Howard and D'Antonio (1984) consider the optimal level of futures contracts by maximizing the ratio of the portfolio's excess return to its volatility:

$$\underset{C_f}{\mathrm{Max}}\,\theta = \frac{E(R_h) - R_F}{\sigma_h}, \tag{9}$$

where R_F represents the risk-free interest rate. In this case, the optimal number of futures positions, C_f^*, is given by:

$$C_f^* = -C_s\frac{(S/F)(\sigma_s/\sigma_f)[(\sigma_s/\sigma_f)(E(R_f)/(E(R_s) - R_F)) - \rho]}{[1 - (\sigma_s/\sigma_f)(E(R_f)\rho/(E(R_s) - R_F))]}. \tag{10}$$

From the optimal futures position, we can obtain the following optimal hedge ratio:

$$h_3 = -\frac{(\sigma_s/\sigma_f)[(\sigma_s/\sigma_f)(E(R_f)/(E(R_s) - R_F)) - \rho]}{[1 - (\sigma_s/\sigma_f)(E(R_f)\rho/(E(R_s) - R_F))]}. \tag{11}$$

Again, if $E(R_f) = 0$, then h_3 reduces to:

$$h_3 = \left(\frac{\sigma_s}{\sigma_f}\right)\rho, \tag{12}$$

which is the same as the MV hedge ratio h_J^*.

2.4. Maximum expected utility hedge ratio

Another class of hedge ratios is based on the maximization of the expected utility derived from the hedged portfolio. For example, Cecchetti et al. (1988) derive the hedge ratio that maximizes the expected utility where the utility function is assumed to be the logarithm of terminal wealth. Specifically, they derive the optimal hedge ratio that maximizes the following expected utility function:

$$\int_{R_s}\int_{R_f} \log[1 + R_s - hR_f] f(R_s, R_f)\mathrm{d}R_s\mathrm{d}R_f, \tag{13}$$

where the density function $f(R_s, R_f)$ is assumed to be bivariate normal. If the returns on the spot and futures are jointly normally distributed, then the expected utility can be expressed in terms of the expected return and variance of return of the hedged portfolio. Therefore, if the futures prices also follow the pure martingale process, then the expected utility-based hedge ratio will be the same as the MV hedge ratio.

2.5. Minimum mean extended-Gini coefficient hedge ratio

Another approach to deriving the optimal hedge ratio is based on the mean extended-Gini coefficient, which can be shown to be consistent with the concept of stochastic dominance. For example, Cheung et al. (1990), Kolb and Okunev (1992), Lien and Luo (1993), Shalit (1995), and Lien and Shaffer (1999) all consider this approach. It minimizes the MEG coefficient $\Gamma_\nu(R_\mathrm{h})$ defined as follows:

$$\Gamma_\nu(R_\mathrm{h}) = -\nu \mathrm{Cov}(R_\mathrm{h}, (1 - G(R_\mathrm{h}))^{\nu-1}), \tag{14}$$

where G is the cumulative probability distribution and ν is the risk aversion parameter. Note that $0 \le \nu < 1$ implies risk seekers, $\nu = 1$ implies risk-neutral investors, and $\nu > 1$ implies risk-averse investors. Shalit (1995) shows that if the futures and spot returns are jointly normally distributed, then the minimum-MEG hedge ratio would be the same as the MV hedge ratio.

2.6. Optimum mean-MEG hedge ratio

Instead of minimizing the MEG coefficient, Kolb and Okunev (1993) alternatively consider maximizing the utility function defined as follows:

$$U(R_\mathrm{h}) = E(R_\mathrm{h}) - \Gamma_\nu(R_\mathrm{h}). \tag{15}$$

Unlike the minimum-MEG hedge ratio, this optimum mean-MEG hedge ratio incorporates the expected return on the hedged portfolio. Again, if the futures price follows a martingale process (i.e., $E(R_\mathrm{f}) = 0$) and if the futures and spot returns are jointly normally distributed, then the optimum mean-MEG hedge ratio would be the same as the MV hedge ratio.

2.7. Minimum generalized semivariance hedge ratio

The hedge ratio based on the generalized semivariance has been recently proposed (see De Jong et al., 1997; Lien & Tse, 1998, 2000; Chen et al., 2001). In this case, the optimal hedge ratio is obtained by minimizing the GSV given below:

$$V_{\delta,\alpha}(R_\mathrm{h}) = \int_{-\infty}^{\delta} (\delta - R_\mathrm{h})^\alpha \mathrm{d}G(R_\mathrm{h}), \quad \alpha > 0, \tag{16}$$

where $G(R_\mathrm{h})$ is the probability distribution function of the return on the hedged portfolio R_h. The parameters δ and α (which are both real numbers) represent the target return and risk aversion, respectively. The risk is defined in such a way that the investors consider only the returns below the target return (δ) to be risky. It can be shown (see Fishburn, 1977; Bawa, 1978) that $\alpha < 1$ represents a risk-seeking investor and $\alpha > 1$ represents a risk-averse investor.

Lien and Tse (1998) show that the GSV hedge ratio, which is obtained by minimizing the GSV, would be the same as the MV hedge ratio if the futures and spot returns are jointly normally distributed.

Table 1
Conditions for various hedge ratios to be equal to the minimum-variance (MV) hedge ratio

Hedge ratio	Required conditions
Optimum mean-variance hedge ratio	Pure martingale
Sharpe hedge ratio	Pure martingale
Maximum expected utility hedge ratio	Pure martingale and jointly normality
Minimum MEG coefficient hedge ratio	Joint normality
Optimum mean-MEG hedge ratio	Pure martingale and joint normality
Minimum GSV hedge ratio	Joint normality
Optimum mean-GSV hedge ratio	Pure martingale and joint normality

Note: This table summarizes specific conditions required for each hedge ratio to converge to the MV hedge ratio. The pure martingale condition refers to the condition that the futures price follows a pure or simple martingale process. The joint normality condition refers to the joint normality of returns on spot and futures positions.

2.8. Optimum mean-generalized semivariance hedge ratio

Chen et al. (2001) extend the GSV hedge ratio to a mean-GSV (M-GSV) hedge ratio by incorporating the mean return in the derivation of the optimal hedge ratio. The M-GSV hedge ratio is obtained by maximizing the following mean-risk utility function, which is similar to the conventional mean-variance based utility function (see Eq. (7)):

$$U(R_{\mathrm{h}}) = E(R_{\mathrm{h}}) - V_{\delta,\alpha}(R_{\mathrm{h}}). \tag{17}$$

Chen et al. (2001) show that the M-GSV hedge ratio would be the same as the MV hedge ratio if both the pure martingale and joint normality hypotheses hold.

From our discussion of various optimal hedge ratios above, it is clear that if the both the pure martingale and joint normality conditions hold, all the hedge ratios will be the same as the MV hedge ratio. Specific conditions required for each hedge ratio to converge to the MV hedge ratio are summarized in Table 1.

3. New tests for the joint normality of spot and futures returns

In this section, we develop new tests for the joint normality of spot and futures returns. Our new tests are based on the generalized method of moments approach, which is proposed by Hansen (1982) and implemented by Richardson and Smith (1993) in their study of multivariate normality in stock returns. Let R_1 and R_2, respectively, denote the return on spot and futures positions, σ_1 and σ_2, respectively, denote the standard deviations of R_1 and R_2, and ρ denote the correlation coefficient between R_1 and R_2. If the returns on the spot and futures are jointly normally distributed, then the following moment conditions must hold:

$$E[h(R, \theta)] = \underline{0}, \tag{18}$$

where

$$R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix},$$

$$h(R,\theta)=\begin{bmatrix}(R_1-\mu_1)\\(R_2-\mu_2)\\(R_1-\mu_1)^2-\sigma_1^2\\(R_2-\mu_2)^2-\sigma_2^2\\(R_1-\mu_1)(R_2-\mu_2)-\sigma_1\sigma_2\rho\\(R_1-\mu_1)^3\\(R_2-\mu_2)^3\\(R_1-\mu_1)^4-3\sigma_1^4\\(R_2-\mu_2)^4-3\sigma_2^4\\(R_1-\mu_1)^2(R_2-\mu_2)\\(R_1-\mu_1)(R_2-\mu_2)^2\\(R_1-\mu_1)^2(R_2-\mu_2)^2-\sigma_1^2\sigma_2^2(1+2\rho^2)\end{bmatrix},$$

and

$$\theta=\begin{bmatrix}\mu_1\\\mu_2\\\sigma_1^2\\\sigma_2^2\\\rho\end{bmatrix}.$$

Note that θ is the vector of the unknown parameters. In this case, we have twelve moment equations and five unknown parameters. The GMM involves the estimation of the five unknown parameters by setting the following five linear combinations of the moment equations to zero:

$$Ag_T(\theta)=\underline{0}, \tag{19}$$

where

$$A=\mathrm{d}^t S^{-1},$$

$$\mathrm{d}=E\left[\frac{\partial h(R,\theta)}{\partial\theta}\right],$$

$$S=E[h(R,\theta)h(R,\theta)^t],$$

$$g_T(\theta)=\frac{1}{T}\sum_{t=1}^{T}h(R_t,\theta),$$

and T denotes the sample size. Hansen (1982) provides the following asymptotic distribution for the estimators of parameters as well as moments:

$$\sqrt{T}(\widehat{\theta}-\theta)\overset{\text{asy}}{\sim}N(\underline{0},[\mathrm{d}^t S^{-1}\mathrm{d}]^{-1}), \tag{20}$$

$$\sqrt{T}g_T(\widehat{\theta})\overset{\text{asy}}{\sim}N(\underline{0},V), \tag{21}$$

where

$$V = S - \mathrm{d}[\mathrm{d}^t S^{-1}\mathrm{d}]^{-1}\mathrm{d}^t.$$

The expressions for the matrices d, S, A, and V are provided in the Appendix A.

We then discuss about the specific characteristics of the GMM applied to the current situation. For example, the value of the optimal matrix A is such that solving Eq. (19) for the unknown parameters is equivalent to solving the first five moment equations for the unknown parameters. This leads to the following estimators for the five unknown parameters:

$$\hat{\mu}_1 = \frac{1}{T}\sum_{t=1}^{T} R_{1t}, \qquad \hat{\mu}_2 = \frac{1}{T}\sum_{t=1}^{T} R_{2t}, \qquad \hat{\sigma}_1^2 = \frac{1}{T}\sum_{t=1}^{T}(R_{1t} - \hat{\mu}_1)^2,$$

$$\hat{\sigma}_2^2 = \frac{1}{T}\sum_{t=1}^{T}(R_{2t} - \hat{\mu}_2)^2, \quad \text{and } \widehat{\rho} = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)(R_{2t} - \widehat{\mu}_2)}{\sqrt{\widehat{\sigma}_1^2 \widehat{\sigma}_2^2}}.$$

These estimators are exactly the same as those obtained by Richardson and Smith (1993). Since we use the first five moment conditions to estimate the parameters, we can use the remaining seven moments to test for the joint normality hypothesis. As most of the empirical normality tests are performed using standardized moments, we use the following standardized sample moments, $g_T^*(\hat{\theta})$, instead of the moments given in Eq. (18):

$$g_T^*(\hat{\theta}) = [g_{1T}(\widehat{\theta}), g_{2T}(\widehat{\theta}), g_{3T}(\widehat{\theta}), g_{4T}(\widehat{\theta}), g_{5T}(\widehat{\theta}), g_{6T}(\widehat{\theta}), g_{7T}(\widehat{\theta})]^t,$$

$$g_{1T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)^3}{(\widehat{\sigma}_1^2)^{3/2}}, \quad g_{2T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{2t} - \widehat{\mu}_2)^3}{(\widehat{\sigma}_2^2)^{3/2}},$$

$$g_{3T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)^4}{(\widehat{\sigma}_1^2)^2} - 3, \quad g_{4T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{2t} - \widehat{\mu}_2)^4}{(\widehat{\sigma}_2^2)^2} - 3,$$

$$g_{5T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)^2(R_{2t} - \widehat{\mu}_2)}{\widehat{\sigma}_1^2(\widehat{\sigma}_2^2)^{1/2}},$$

$$g_{6T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)(R_{2t} - \widehat{\mu}_2)^2}{\widehat{\sigma}_2^2(\widehat{\sigma}_1^2)^{1/2}},$$

and

$$g_{7T}(\widehat{\theta}) = \frac{(1/T)\sum_{t=1}^{T}(R_{1t} - \widehat{\mu}_1)^2(R_{2t} - \widehat{\mu}_2)^2}{\widehat{\sigma}_1^2\widehat{\sigma}_2^2},$$

The first two sample standardized moments, g_{1T} and g_{2T}, are the sample skewness of R_1 and R_2, respectively. Furthermore, the third and fourth sample moments, g_{3T} and g_{4T}, are the standardized sample kurtosis of R_1 and R_2, respectively. It can be shown that the asymptotic distribution of the standardized sample moments is given by:

$$\sqrt{T} g_T^*(\widehat{\theta})^{\text{asy}} N(\underline{0}, V^*), \tag{22}$$

where

$$V^* = \begin{bmatrix} 6 & 6\rho^3 & 0 & 0 & 6\rho & 6\rho^2 & 0 \\ 6\rho^3 & 6 & 0 & 0 & 6\rho^2 & 6\rho & 0 \\ 0 & 0 & 24 & 24\rho^4 & 0 & 0 & 24\rho^2 \\ 0 & 0 & 24\rho^4 & 24 & 0 & 0 & 24\rho^2 \\ 6\rho & 6\rho^2 & 0 & 0 & 4\rho^2+2 & 4\rho+2\rho^3 & 0 \\ 6\rho^2 & 6\rho & 0 & 0 & 4\rho+2\rho^3 & 4\rho^2+2 & 0 \\ 0 & 0 & 24\rho^2 & 24\rho^2 & 0 & 0 & 4+16\rho^2+4\rho^4 \end{bmatrix}.$$

The joint normality test can then be performed based on the Wald statistic given below:

$$W^* = T(g_T^*)^t (V^*)^{-1} g_T^*, \tag{23}$$

where the Wald statistic has an asymptotic χ^2 distribution with seven degrees of freedom.

In addition to the above test for the joint normality, we can also perform the GMM test based only on the skewness and kurtosis. This amounts to using the following four moments, instead of seven moments, in the Wald test:

$$g_T^{**}(\hat{\theta}) = [g_{1T}(\widehat{\theta}), g_{2T}(\widehat{\theta}), g_{3T}(\widehat{\theta}), g_{4T}(\widehat{\theta})]^t.$$

In this case, the Wald statistic, which has an asymptotic χ^2 distribution with four degrees of freedom, is given by:[2]

$$W^{**} = T(g_T^{**})^t (V^{**})^{-1} g_T^{**}, \tag{24}$$

where

$$V^{**} = \begin{bmatrix} 6 & 6\rho^3 & 0 & 0 \\ 6\rho^3 & 6 & 0 & 0 \\ 0 & 0 & 24 & 24\rho^4 \\ 0 & 0 & 24\rho^4 & 24 \end{bmatrix}.$$

In this study, the test based on the Wald statistic given by Eq. (24) is referred to as the GMM test and that based on the Wald statistic given by Eq. (23) is referred to as the Extended GMM test. The relative performance of these two tests for various sample sizes and correlations will be analyzed based on the Monte Carlo simulation with 20,000 replications. There are at least two reasons why we examine two GMM-based tests even though (24) is the special case of (23). First, the GMM test is simpler than the Extended GMM tests. Therefore, if the performance of the GMM test is similar to that of the Extended GMM test, then we should use the simpler one, the

[2] If we assume that the two series are independent, then the W^{**} statistic is given by:

$$W^{**} = T\left[\frac{g_{1T}^2}{6} + \frac{g_{3T}^2}{24}\right] + T\left[\frac{g_{2T}^2}{6} + \frac{g_{4T}^2}{24}\right] \to \chi^2(2) + \chi^2(2) = \chi^2(4).$$

The Wald test is equivalent to the Jarque-Bera (1987) normality test when the two series are independent. Therefore, the Jarque-Bera normality test is a just special case of the Wald test.

Table 2
Summary of 25 futures contracts

	Commodity	Sample period	Sample size
1	SP500	June 1, 1982–December 31, 1997	4066
2	TSE35	March 1, 1991–December 31, 1997	1783
3	Nikkei 225	September 5, 1988–December 31, 1997	2432
4	TOPIX	September 5, 1988–December 31, 1997	2432
5	FTSE100	May 3, 1984–December 31, 1997	3564
6	CAC40	March 1, 1989–December 31, 1997	2305
7	All ordinary	January 3, 1984–December 31, 1997	3651
8	Soybean oil	January 2, 1979–December 31, 1997	4956
9	Soybean	January 2, 1979–December 31, 1997	4956
10	Soy meal	January 2, 1979–December 31, 1997	4956
11	Corn	January 2, 1979–December 31, 1997	4956
12	Wheat	March 30, 1982–December 31, 1997	4111
13	Cotton	January 3, 1980–December 31, 1997	4694
14	Cocoa	November 1, 1983–December 31, 1997	3696
15	Coffee	January 2, 1979–December 31, 1997	4956
16	Pork bellies	January 2, 1979–December 31, 1997	4956
17	Hogs	March 30, 1982–December 31, 1997	4111
18	Crude oil	April 4, 1983–December 31, 1997	3847
19	Silver	January 2, 1979–December 31, 1997	4956
20	Gold	January 2, 1979–December 31, 1997	4956
21	Japanese yen	January 2, 1986–December 31, 1997	3129
22	Deutsche mark	January 2, 1986–December 31, 1997	3129
23	Swiss franc	January 2, 1986–December 31, 1997	3129
24	British pound	January 2, 1986–December 31, 1997	3129
25	Canadian dollar	November 30, 1987–December 31, 1997	2632

Note: This table lists the commodities, sample periods, and sample sizes for the 25 different futures contracts used for empirical analyses in this study. The data are obtained from Datastream.

GMM test. Second, the Extended GMM test imposes more moment restrictions than the GMM test. This enables us to see if including more moment conditions improves the test performance.[3]

4. Empirical analysis

This article analyzes 25 different futures contracts where the futures prices are associated with nearest-to-maturity contracts. A list of the futures contracts, sample periods, and sample sizes are given in Table 2. The data are obtained from Datastream. To see if the results of our tests depend on the length of hedging horizon, we also perform tests using five different hedging horizons (1 day, 1 week, 4 week, 8 week, and 12 week).

We first test if the futures price follows a pure martingale process. *T*-tests are used to examine the pure martingale hypothesis that the expected return on futures is zero. Table 3 presents the results. We find that the pure martingale hypothesis holds for all the 25 futures contracts except for SP500, TSE35, and FTSE100. This is true for all the five different hedging horizons considered in

[3] The joint normality test imposes much more moment restrictions than those considered in this study. We can derive many different GMM tests that use various combinations of moment restrictions. Therefore, it is important to know if using more moment conditions improves the test statistic. In this paper, we partially answer this question. We would like to pursue this in our future research.

Table 3
Mean returns over various holding periods for different types of futures contracts

Commodity		Holding period				
		1 Day	1 Week	4 Week	8 Week	12 Week
SP500	Sample size	4066	813	203	101	67
	Mean return (%)	0.0535	0.2678	1.0548	2.1270	3.1631
	T-ratio	2.9051***	3.0611***	3.6122***	3.6652***	3.7557***
TSE35	Sample size	1783	356	89	44	29
	Mean return (%)	0.0354	0.1708	0.6832	1.3617	2.1386
	T-ratio	1.9734**	1.9629*	2.0725**	2.2758**	2.2400**
Nikkei 225	Sample size	2432	486	121	60	40
	Mean return (%)	−0.0245	−0.1291	−0.4544	−0.9070	−1.3604
	T-ratio	−0.8768	−0.8945	−0.7645	−0.9152	−0.6719
TOPIX	Sample size	2432	486	121	60	40
	Mean return (%)	−0.0238	−0.1261	−0.4712	−0.8664	−1.2995
	T-ratio	−0.9026	−0.9112	−0.8292	−0.9702	−0.6933
FTSE100	Sample size	3564	712	178	89	59
	Mean return (%)	0.0424	0.2082	0.8329	1.6659	2.4686
	T-ratio	2.3882**	2.3228**	2.3540**	2.3732**	2.2792**
CAC40	Sample size	2305	461	115	57	38
	Mean return (%)	0.0285	0.1426	0.5378	1.0494	1.5741
	T-ratio	1.1388	1.1909	1.1989	1.2520	1.1203
All ordinary	Sample size	3651	730	182	91	60
	Mean return (%)	0.0328	0.1624	0.6374	1.2748	2.0016
	T-ratio	1.3499	1.4353	1.3724	1.3238	1.4075
Soybean oil	Sample size	4956	991	247	123	82
	Mean return (%)	0.0001	0.0017	0.0063	0.0452	0.0678
	T-ratio	0.0043	0.0157	0.0148	0.0510	0.0555
Soybean	Sample size	4956	991	247	123	82
	Mean return (%)	−0.0005	−0.0010	0.0109	0.0565	0.0847
	T-ratio	−0.0276	−0.0106	0.0298	0.0768	0.0765
Soy meal	Sample size	4956	991	247	123	82
	Mean return (%)	0.0015	0.0094	0.0685	0.2015	0.3022
	T-Ratio	0.0750	0.0886	0.1788	0.2550	0.2566
Corn	Sample size	4956	991	247	123	82
	Mean return (%)	0.0027	0.0142	0.0791	0.1437	0.2155
	T-ratio	0.1428	0.1437	0.1904	0.1565	0.1619
Wheat	Sample size	4111	822	205	102	68
	Mean return (%)	−0.0030	−0.0143	−0.0350	−0.0625	−0.0937
	T-ratio	−0.1321	−0.1262	−0.0857	−0.0817	−0.0801
Cotton	Sample size	4694	938	234	117	78
	Mean return (%)	−0.0023	−0.0106	−0.0411	−0.0823	−0.1234
	T-ratio	−0.0875	−0.0819	−0.0776	−0.0753	−0.0837
Cocoa	Sample size	3696	739	184	92	61
	Mean return (%)	−0.0047	−0.0228	−0.0992	−0.1984	−0.2962
	T-ratio	−0.1594	−0.1540	−0.1807	−0.1859	−0.1937

Table 3 (*Continued*)

Commodity		Holding period				
		1 Day	1 Week	4 Week	8 Week	12 Week
	Sample size	4956	991	247	123	82
Coffee	Mean return (%)	0.0037	0.0201	0.1237	0.1190	0.1785
	T-ratio	0.1183	0.1263	0.1906	0.0874	0.0881
	Sample size	4956	991	247	123	82
Pork bellies	Mean return (%)	−0.0028	−0.0103	−0.0153	0.0566	0.0849
	T-ratio	−0.0736	−0.0522	−0.0187	0.0389	0.0409
	Sample size	4111	822	205	102	68
Hogs	Mean return (%)	−0.0041	−0.0204	−0.0607	−0.0830	−0.1244
	T-ratio	−0.1583	−0.1578	−0.1171	−0.0778	−0.0783
	Sample size	3847	769	192	96	64
Crude oil	Mean return (%)	−0.0133	−0.0668	−0.2471	−0.4941	−0.7412
	T-ratio	−0.3639	−0.3712	−0.3921	−0.3387	−0.3535
	Sample size	4956	991	247	123	82
Silver	Mean return (%)	−0.0003	0.0017	−0.0434	−0.1672	−0.2508
	T-ratio	−0.0099	0.0115	−0.0668	−0.1281	−0.1127
	Sample size	4956	991	247	123	82
Gold	Mean return (%)	0.0051	0.0266	0.0972	0.2654	0.3981
	T-ratio	0.2866	0.2922	0.2730	0.3554	0.3557
	Sample size	3129	625	156	78	52
Japanese yen	Mean return (%)	0.0136	0.0696	0.2847	0.5693	0.8540
	T-ratio	1.0721	1.1480	1.0934	1.0399	0.9939
	Sample size	3129	625	156	78	52
Deutsche mark	Mean return (%)	0.0095	0.0498	0.1987	0.3974	0.5962
	T-Ratio	0.7440	0.8280	0.7743	0.7151	0.6991
	Sample size	3129	625	156	78	52
Swiss franc	Mean return (%)	0.0107	0.0569	0.2264	0.4528	0.6792
	T-ratio	0.7536	0.8489	0.7900	0.7122	0.7358
	Sample size	3129	625	156	78	52
British pound	Mean return (%)	0.0041	0.0218	0.0855	0.1710	0.2564
	T-ratio	0.3225	0.3547	0.3342	0.3358	0.2830
	Sample size	2632	526	131	65	43
Canadian dollar	Mean return (%)	−0.0032	−0.0175	−0.0587	−0.1192	−0.1223
	T-ratio	−0.5701	−0.6035	−0.5281	−0.6014	−0.4088

Note: This table presents the results for the mean returns over various holding periods for each of the futures contracts listed in Table 2.

*** 1% significance level.

** 5% significance level.

* 10% significance level.

this study. The results suggest that with the exception of SP500, TSE35, and FTSE100, ignoring the expected return in the derivation of the optimal hedge ratio does not significantly change the optimal hedge ratio. Therefore, the mean-variance hedge ratio would be approximately the same as the MV hedge ratio for most futures contracts.

We now test the joint normality of spot and futures returns. As mentioned above, we use two tests for the joint normality hypothesis. The first test is the GMM test, which is

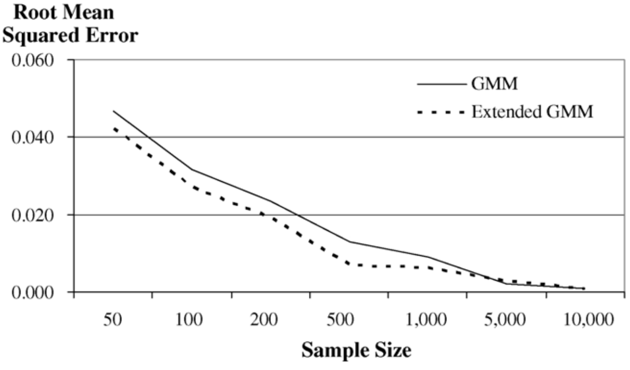

Fig. 1. Relationship between the root mean squared error and sample size for the correlation of 90% for the GMM and extended GMM tests.

based on the Wald statistic with four degrees of freedom (Eq. (24)). The second test is the Extended GMM test, which is based on the Wald statistic with seven degrees of freedom (Eq. (23)).

The empirical probabilities of rejecting a true null hypothesis of joint normal distribution (i.e., the empirical sizes) for four levels of critical value from the asymptotic distribution (nominal size of 1, 5, 10, and 20%) are given in Table 4. The empirical sizes are based on Monte Carlo simulations with 20,000 replications and are computed for various correlation coefficients between the two random variables (ranging from 0 to 98%) and for various sample sizes (ranging from 50 to 10,000). It is clear from Table 4 that for smaller sample sizes, the Extended GMM test performs better than the GMM tests.

Since Table 4 contains a large number of empirical sizes for different values of sample size and correlation, it would be worthwhile to summarize the results. We use the root mean squared error (RMSE) to summarize the deviation of the empirical size from the nominal size (i.e., the size distortion). The RMSE is computed as follows:

$$\mathrm{RMSE} = \sqrt{\frac{(e_{0.01} - 0.01)^2 + (e_{0.05} - 0.05)^2 + (e_{0.10} - 0.10)^2 + (e_{0.20} - 0.20)^2}{4}},$$

where e_α represents the empirical size at the level α. The root mean squared errors for various sample sizes and correlations are summarized in Table 5. The relationship between the RMSE and sample size for a correlation of 90% is shown in Fig. 1 for the GMM and Extended GMM test statistics.[4]

It is interesting to note from Table 5 and Fig. 1 that the RMSE (i.e., the size distortion) of the GMM and Extended GMM tests depends on the sample size and decreases as the sample size is increased. This is not surprising due to the fact that the GMM-based tests are asymptotic tests. It is also interesting to note that for smaller sample sizes, the Extended GMM test (the Wald test based on seven moments) performs better than the GMM test (the Wald test based on four

[4] We do not find any pattern in the relationship between RMSE and the correlation. Furthermore, the sample correlations between the futures and spot returns are very high (normally more than 90%). Therefore, we use a 90% correlation in Fig. 1.

Table 4
Empirical sizes of the GMM and extended GMM tests with 20,000 replications

N	ρ	GMM percentile				Extended GMM percentile			
		1%	5%	10%	20%	1%	5%	10%	20%
50	0.00	0.022	0.044	0.066	0.108	0.026	0.050	0.073	0.120
50	0.10	0.023	0.048	0.068	0.107	0.025	0.050	0.075	0.120
50	0.20	0.022	0.044	0.065	0.108	0.025	0.050	0.073	0.116
50	0.50	0.022	0.044	0.065	0.107	0.024	0.050	0.073	0.123
50	0.80	0.023	0.047	0.070	0.115	0.024	0.052	0.079	0.129
50	0.90	0.024	0.047	0.070	0.112	0.025	0.050	0.074	0.121
50	0.98	0.025	0.051	0.074	0.116	0.027	0.053	0.080	0.126
100	0.00	0.023	0.050	0.078	0.131	0.025	0.056	0.086	0.141
100	0.10	0.024	0.054	0.081	0.135	0.026	0.058	0.088	0.147
100	0.20	0.025	0.052	0.078	0.132	0.027	0.057	0.087	0.145
100	0.50	0.025	0.051	0.077	0.131	0.026	0.057	0.088	0.147
100	0.80	0.023	0.052	0.079	0.135	0.026	0.057	0.087	0.143
100	0.90	0.026	0.055	0.083	0.141	0.026	0.058	0.090	0.150
100	0.98	0.023	0.051	0.080	0.136	0.026	0.057	0.088	0.145
200	0.00	0.023	0.052	0.089	0.157	0.023	0.059	0.094	0.165
200	0.10	0.023	0.055	0.087	0.156	0.025	0.061	0.096	0.166
200	0.20	0.022	0.052	0.084	0.155	0.023	0.059	0.091	0.163
200	0.50	0.024	0.053	0.087	0.153	0.026	0.059	0.094	0.169
200	0.80	0.021	0.053	0.086	0.150	0.022	0.059	0.093	0.163
200	0.90	0.023	0.056	0.089	0.157	0.025	0.059	0.094	0.166
200	0.98	0.022	0.053	0.087	0.161	0.025	0.059	0.095	0.168
500	0.00	0.019	0.053	0.094	0.180	0.020	0.057	0.098	0.185
500	0.10	0.019	0.054	0.093	0.177	0.020	0.055	0.097	0.179
500	0.20	0.019	0.052	0.092	0.176	0.019	0.056	0.097	0.183
500	0.50	0.019	0.052	0.093	0.175	0.019	0.055	0.098	0.182
500	0.80	0.019	0.052	0.094	0.175	0.019	0.055	0.098	0.181
500	0.90	0.018	0.056	0.096	0.176	0.018	0.056	0.099	0.190
500	0.98	0.020	0.053	0.093	0.176	0.020	0.057	0.098	0.182
1000	0.00	0.017	0.052	0.097	0.187	0.016	0.054	0.099	0.187
1000	0.10	0.014	0.051	0.095	0.188	0.015	0.053	0.099	0.193
1000	0.20	0.015	0.049	0.092	0.182	0.016	0.053	0.098	0.188
1000	0.50	0.016	0.055	0.101	0.191	0.018	0.057	0.103	0.193
1000	0.80	0.015	0.051	0.095	0.186	0.016	0.052	0.095	0.187
1000	0.90	0.015	0.051	0.095	0.183	0.017	0.053	0.097	0.190
1000	0.98	0.015	0.052	0.097	0.187	0.015	0.055	0.099	0.192
5000	0.00	0.012	0.051	0.101	0.202	0.012	0.052	0.101	0.201
5000	0.10	0.012	0.052	0.101	0.198	0.012	0.051	0.101	0.200
5000	0.20	0.013	0.053	0.101	0.201	0.011	0.051	0.098	0.198
5000	0.50	0.011	0.049	0.100	0.196	0.012	0.053	0.101	0.200
5000	0.80	0.011	0.051	0.100	0.199	0.010	0.048	0.100	0.196
5000	0.90	0.010	0.049	0.097	0.197	0.011	0.049	0.096	0.196
5000	0.98	0.012	0.052	0.100	0.199	0.012	0.053	0.102	0.201
10000	0.00	0.010	0.051	0.099	0.199	0.010	0.050	0.099	0.197
10000	0.10	0.012	0.053	0.103	0.201	0.011	0.054	0.102	0.204
10000	0.20	0.011	0.052	0.100	0.195	0.010	0.051	0.104	0.200
10000	0.50	0.011	0.052	0.100	0.200	0.010	0.051	0.101	0.202
10000	0.80	0.011	0.051	0.100	0.195	0.010	0.051	0.102	0.199
10000	0.90	0.011	0.049	0.099	0.199	0.011	0.051	0.101	0.200
10000	0.98	0.011	0.050	0.101	0.200	0.012	0.052	0.099	0.198

Note: This table shows the empirical probabilities of rejecting a true null hypothesis of joint normal distribution (i.e., the empirical sizes) for four levels of critical value from the asymptotic distribution (nominal size of 1, 5, 10, and 20%). The empirical sizes are based on Monte Carlo simulations with 20,000 replications and are computed for various correlation coefficients between the two random variables (ranging from 0 to 98%) and for various sample sizes (ranging from 50 to 10,000). N denotes sample size and ρ denotes the correlation coefficient between the two simulated random variables.

Table 5
Root mean squared errors of the GMM and extended GMM tests

Sample size	Correlation	GMM test	Extended GMM test
50	0.00	0.050	0.043
50	0.10	0.049	0.042
50	0.20	0.050	0.045
50	0.50	0.050	0.041
50	0.80	0.046	0.038
50	0.90	0.047	0.042
50	0.98	0.044	0.039
100	0.00	0.037	0.031
100	0.10	0.035	0.029
100	0.20	0.036	0.030
100	0.50	0.037	0.029
100	0.80	0.035	0.030
100	0.90	0.032	0.027
100	0.98	0.034	0.029
200	0.00	0.023	0.020
200	0.10	0.024	0.020
200	0.20	0.024	0.021
200	0.50	0.025	0.018
200	0.80	0.027	0.020
200	0.90	0.023	0.019
200	0.98	0.022	0.018
500	0.00	0.011	0.010
500	0.10	0.013	0.012
500	0.20	0.013	0.010
500	0.50	0.014	0.010
500	0.80	0.014	0.011
500	0.90	0.013	0.007
500	0.98	0.013	0.011
1000	0.00	0.008	0.007
1000	0.10	0.007	0.005
1000	0.20	0.010	0.007
1000	0.50	0.006	0.007
1000	0.80	0.008	0.008
1000	0.90	0.009	0.006
1000	0.98	0.007	0.005
5000	0.00	0.001	0.002
5000	0.10	0.002	0.001
5000	0.20	0.002	0.002
5000	0.50	0.002	0.002
5000	0.80	0.001	0.002
5000	0.90	0.002	0.003
5000	0.98	0.001	0.002
10000	0.00	0.001	0.002
10000	0.10	0.002	0.003
10000	0.20	0.003	0.002
10000	0.50	0.001	0.001
10000	0.80	0.003	0.001
10000	0.90	0.001	0.001
10000	0.98	0.001	0.002

Note: This table summarizes the root mean squared errors of the GMM and Extended GMM tests for various sample sizes and correlations.

Table 6
The GMM and extended GMM tests of joint normality for different types of futures contracts

Commodity		Holding period				
		1 Day	1 Week	4 Week	8 Week	12 Week
SP500	Correlation (%)	93.96	97.33	98.75	99.39	99.73
	GMM test	7194823.86	258023.46	1190.90	319.98	155.83
	Extended GMM test	9100003.91	371834.42	1212.43	324.80	174.29
TSE35	Correlation (%)	87.13	95.88	96.57	98.95	98.44
	GMM test	2706.24	**5.97**	**1.90**	**0.99**	**7.45**
	Extended GMM test	6726.16	18.01	22.46	**4.58**	**12.75**
Nikkei 225	Correlation (%)	92.71	97.30	99.04	99.22	99.79
	GMM test	3362.80	225.63	14.03	**4.86**	22.76
	Extended GMM test	5479.85	522.86	22.28	**7.03**	46.96
TOPIX	Correlation (%)	88.09	96.53	99.15	99.40	99.79
	GMM test	4439.33	251.18	31.78	**0.28**	22.95
	Extended GMM test	16672.01	359.44	41.95	17.62	30.30
FTSE100	Correlation (%)	91.83	96.56	97.96	99.10	99.53
	GMM test	89098.73	18242.27	1186.55	196.12	11.70
	Extended GMM test	204594.66	19371.95	1195.95	196.84	19.13
CAC40	Correlation (%)	95.09	98.11	99.16	99.20	99.51
	GMM test	1417.93	26.04	13.80	17.24	14.89
	Extended GMM test	1854.57	27.82	15.76	17.63	16.86
All ordinary	Correlation (%)	29.09	87.26	96.06	95.88	98.54
	GMM test	10888707.70	81539.62	15772.48	3004.62	930.78
	Extended GMM test	11123966.19	81830.43	15782.74	3301.60	996.38
Soybean oil	Correlation (%)	80.97	91.47	95.55	97.09	97.13
	GMM test	1252.00	149.79	33.68	28.03	**5.23**
	Extended GMM test	3984.27	236.39	51.56	37.95	21.05
Soybean	Correlation (%)	85.72	91.87	93.48	94.99	95.33
	GMM test	4689.86	567.05	37.67	24.13	11.33
	Extended GMM test	21707.21	2659.71	54.37	34.62	28.70
Soy meal	Correlation (%)	79.16	88.04	88.69	88.69	91.17
	GMM test	12810.59	351.47	76.66	11.12	23.73
	Extended GMM test	29195.01	854.18	206.62	52.84	32.08
Corn	Correlation (%)	73.65	81.40	83.64	86.91	90.96
	GMM test	123853.48	2585.63	250.69	186.19	29.99
	Extended GMM test	293273.80	3460.75	523.97	398.92	36.42
Wheat	Correlation (%)	62.28	77.16	79.48	83.74	84.65
	GMM test	221619.84	2275.95	166.03	39.07	**1.16**
	Extended GMM test	424795.71	3808.78	308.97	47.24	**1.95**
Cotton	Correlation (%)	37.49	40.31	48.16	92.45	92.07
	GMM test	351397138.62	3110061.61	37520.48	2268.92	334.01
	Extended GMM test	407677749.96	3786817.86	54402.60	2288.36	373.03
Cocoa	Correlation (%)	71.66	89.40	91.44	90.95	89.04
	GMM test	5677.15	51.51	**5.31**	26.90	19.66
	Extended GMM test	9452.43	214.86	34.47	52.41	21.61

Table 6 (*Continued*)

Commodity		Holding period				
		1 Day	1 Week	4 Week	8 Week	12 Week
Coffee	Correlation (%)	57.65	69.95	76.62	86.18	91.70
	GMM test	513042.92	10148.36	287.26	30.44	11.57
	Extended GMM test	674689.61	15644.33	620.82	81.13	**13.65**
Pork bellies	Correlation (%)	25.57	48.49	67.00	66.49	61.03
	GMM test	813014.98	3069.79	36.64	**1.99**	12.02
	Extended GMM test	833588.14	3914.48	74.72	**13.33**	14.69
Hogs	Correlation (%)	14.03	35.71	52.93	52.50	65.22
	GMM test	77435.64	613.29	**5.52**	**1.71**	**0.99**
	Extended GMM test	77604.08	722.23	**9.54**	**4.86**	**3.09**
Crude oil	Correlation (%)	75.94	91.20	97.66	98.85	99.50
	GMM test	361983.66	4705.67	178.98	360.55	449.66
	Extended GMM test	403914.46	5258.44	231.13	360.82	457.62
Silver	Correlation (%)	52.63	81.11	95.85	98.09	99.71
	GMM test	8610770.32	20301.95	527.68	317.90	244.51
	Extended GMM test	8649436.52	49177.18	4322.66	2569.89	248.49
Gold	Correlation (%)	42.83	88.66	97.34	98.57	99.10
	GMM Test	53625.90	3326.17	861.15	559.55	91.41
	Extended GMM Test	56730.04	4034.65	887.98	792.72	96.86
Japanese yen	Correlation (%)	93.26	97.97	99.21	99.60	99.91
	GMM test	3351.01	105.11	**2.78**	**4.75**	11.34
	Extended GMM test	4670.46	331.67	35.92	**10.63**	73.44
Deutsche mark	Correlation (%)	94.21	98.42	99.36	99.67	99.89 D.
	GMM test	745.05	14.05	**3.71**	**1.72**	**4.42**
	Extended GMM test	765.41	42.53	**5.00**	**6.26**	14.90
Swiss franc	Correlation (%)	94.28	98.66	99.46	99.74	99.90
	GMM test	546.76	24.67	**1.60**	**2.33**	**1.70**
	Extended GMM test	572.31	91.32	**6.56**	**5.49**	**5.02**
British pound	Correlation (%)	92.23	95.85	99.12	99.48	99.85
	GMM test	1831.78	370.38	55.39	**4.36**	144.97
	Extended GMM test	4343.07	1049.84	110.27	17.68	327.63
Canadian dollar	Correlation (%)	89.70	94.62	97.16	98.38	99.38
	GMM test	1546.11	26.55	**7.09**	**6.56**	**1.47**
	Extended GMM test	3477.71	214.68	**10.61**	**11.30**	23.95

Note: This table presents the results of the joint normality tests over various holding periods for each of the futures contracts listed in Table 2. The GMM test statistics have a chi-square distribution with 4 degrees of freedom and their critical values at the 1, 5, and 10% levels are 13.277, 9.488, and 7.779, respectively. The Extended GMM test statistics have a chi-square distribution with 7 degrees of freedom and their critical values at the 1, 5, and 10% levels are 18.475, 14.067, and 12.017, respectively. The figures in bold face indicate that the joint normality hypothesis cannot be rejected at the 5% level.

moments). However, the difference in performance between the two tests disappears when the sample size is large (greater than 1000). This is an important result due to the fact that when developing GMM-based tests, we usually have an option in choosing the number of moments as well as specific moments to be included in the tests. The results indicate that for smaller sample sizes, it is better to choose more moments.

We now discuss the results of the joint normality tests for the 25 futures commodities listed in Table 2. Table 6 presents the results. Note that the GMM statistics have a chi-square distribution with four degrees of freedom. The critical values at the 1, 5, and 10% levels for the GMM test statistics are 13.277, 9.488, and 7.779, respectively. However, the Extended GMM test statistics have a chi-square distribution with seven degrees of freedom and their critical values at the 1, 5, and 10% levels are 18.475, 14.067, and 12.017, respectively. In order to see if the joint normality hypothesis holds for various data frequencies, we perform the GMM and Extended GMM tests for daily, 1-, 4-, 8-, and 12-week returns. It is important to note that we use non-overlapping returns here. This avoids the serial correlation problem caused by the use of overlapping returns.

Table 6 shows that when daily data is used, the joint normality of spot and futures returns is rejected for all the 25 commodities considered in this study. The results for one-week hedging periods are similar for all the futures contracts. The only exception is TSE 35 when the GMM test is employed to test for the joint normality. However, the joint normality hypothesis is rejected for TSE 35 when the Extended GMM test is used. Our findings suggest that the joint normality hypothesis tends to be rejected for all the futures contracts when the hedging horizon is relatively short.

Table 6 also shows that for hedging horizons equal to or longer than four weeks, both the GMM and Extended GMM test statistics are insignificant for the following commodities: TSE35 (8 and 12 week), Nikkei 225 (8 week), wheat (12 week), pork bellies (8 week), hogs (4, 8, and 12 week), Japanese yen (8 week), Deutsche mark (4 and 8 week), Swiss franc (4, 8, and 12 week), and Canadian dollar (4 and 8 week). That is, for those commodities, the joint normality of spot and futures returns is supported for some hedging horizons. Our findings suggest that, except for a few commodities and longer hedging horizons, the joint normality hypothesis generally does not hold. Therefore, the hedge ratios derived based on the expected utility maximizing approach, the MEG approach, and the GSV approach will not converge to the MV hedge ratio for most futures contracts and for relatively short hedging horizons.

5. Conclusions

The optimal hedge ratios which are derived based on the mean-variance approach, the expected utility maximizing approach, the mean extended-Gini approach, and the generalized semivariance approach will all converge to the minimum-variance hedge ratio if the futures price follows a pure martingale process and if the spot and futures returns are jointly normal. Because the minimum-variance hedge ratio is easy to understand, simple to compute, and most widely used, it is important to investigate if these two conditions hold. In this paper, we perform empirical tests to see if the pure martingale and joint normality hypotheses hold using 25 different futures contracts and five different hedging horizons.

In order to test for the joint normality of spot and futures returns, we develop two new tests, the GMM and Extended GMM tests, based on the generalized method of moments, which allow for calculating multivariate test statistics that take account of the contemporaneous correlation across spot and futures returns. The GMM test uses fewer moment conditions than the Extended GMM test. Since the relative performances of those two tests are not known, we analyze their relative performances based on Monte Carlo simulations with various correlation coefficients and sample sizes. For large sample sizes, we find no difference in performances between the two tests. However, for small sample sizes, the Extended GMM test is found to perform better than the GMM test in terms of size distortion. The results suggest that for smaller sample sizes, it is better to include more moment conditions.

Our tests of the pure martingale hypothesis show that the expected return on futures is generally zero except for a few futures contracts. This is true for all the five different hedging horizons. The results suggest that ignoring the expected return in the derivation of the optimal hedge ratio does not significantly change the optimal hedge ratio for most commodities. Therefore, with the exception of a few commodities, the mean-variance hedge ratio would be approximately the same as the minimum-variance hedge ratio. We also find that the joint normality hypothesis tends to be rejected for all the 25 commodities when the length of hedging period is short. For longer hedging horizons, the joint normality hypothesis holds only for a few commodities. Our results suggest that the hedge ratios which are derived based on the expected utility maximizing approach, the mean extended-Gini approach, and the generalized semivariance approach will not converge to the minimum-variance hedge ratio except for a few contracts and relatively long hedging horizons.

Finally, it is important to note that even though the test method suggested by Richardson and Smith (1993) and the one suggested in this paper are both based on the GMM method, these methods use different sets of moments in the implementation of normality tests. Furthermore, the results obtained by Richardson and Smith (1993) have implications for asset pricing whereas the results obtained in this paper have implications for the optimal hedge ratios under different approaches.

Appendix A

$$\mathrm{d}^t = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & -3\sigma_1^2 & 0 & 0 & 0 & -2\sigma_1\sigma_2\rho & -\sigma_2^2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -3\sigma_2^2 & 0 & 0 & -\sigma_1^2 & -2\sigma_1\sigma_2\rho & 0 \\ 0 & 0 & -1 & 0 & -\dfrac{\sigma_2}{2\rho}\sigma_1 & 0 & 0 & -6\sigma_1^2 & 0 & 0 & 0 & -\sigma_2^2(1+2\rho^2) \\ 0 & 0 & 0 & -1 & -\dfrac{\sigma_1}{2\sigma_2}\rho & 0 & 0 & 0 & -6\sigma_2^2 & 0 & 0 & -\sigma_1^2(1+2\rho^2) \\ 0 & 0 & 0 & 0 & -\sigma_1\sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 & -4\sigma_1^2\sigma_2^2\rho \end{bmatrix}$$

$$S = [S_1, S_2]$$

$$S_1 = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho & 0 & 0 & 0 & 3\sigma_1 & 3\sigma_1\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 & 0 & 0 & 0 & 3\sigma_2\rho & 3\sigma_2 \\ 0 & 0 & 2\sigma_1^4 & 2\sigma_1^2\sigma_2^2\rho^2 & 2\sigma_1^3\sigma_2\rho & 0 & 0 \\ 0 & 0 & 2\sigma_1^2\sigma_2^2\rho^2 & 2\sigma_2^4 & 2\sigma_1\sigma_2^3\rho & 0 & 0 \\ 0 & 0 & 2\sigma_1^3\sigma_2\rho & 2\sigma_1\sigma_2^3\rho & \sigma_1^2\sigma_2^2(1+\rho^2) & 0 & 0 \\ 3\sigma_1 & 3\sigma_2\rho & 0 & 0 & 0 & 15 & \rho(9+6\rho^2) \\ 3\sigma_1\rho & 3\sigma_2 & 0 & 0 & 0 & \rho(9+6\rho^2) & 15 \\ 0 & 0 & 12\sigma_1^2 & 12\sigma_2^2\rho^2 & 12\sigma_1\sigma_2\rho & 0 & 0 \\ 0 & 0 & 12\sigma_1^2\rho^2 & 12\sigma_2^2 & 12\sigma_1\sigma_2\rho & 0 & 0 \\ 3\sigma_1\rho & \sigma_2(1+2\rho^2) & 0 & 0 & 0 & 15\rho & 3(1+4\rho^2) \\ \sigma_1(1+2\rho^2) & 3\sigma_2\rho & 0 & 0 & 0 & 3(1+4\rho^2) & 15\rho \\ 0 & 0 & 2\sigma_1^2(1+5\rho^2) & 2\sigma_2^2(1+5\rho^2) & \sigma_1\sigma_2\rho(8+4\rho^2) & 0 & 0 \end{bmatrix}$$

$$
S_2 = \begin{bmatrix}
0 & 0 & 3\sigma_1\sigma_2\rho & \sigma_1(1+2\rho^2) & 0 \\
0 & 0 & \sigma_2(1+2\rho^2) & 3\sigma_2\rho & 0 \\
12\sigma_1^2 & 12\sigma_1^2\rho^2 & 0 & 0 & 2\sigma_1^2(1+5\rho^2) \\
12\sigma_2^2\rho^2 & 12\sigma_2^2 & 0 & 0 & 2\sigma_2^2(1+5\rho^2) \\
12\sigma_1\sigma_2\rho & 12\sigma_1\sigma_2\rho & 0 & 0 & \sigma_1\sigma_2\rho(8+4\rho^2) \\
0 & 0 & 15\rho & 3(1+4\rho^2) & 0 \\
0 & 0 & 3(1+4\rho^2) & 15\rho & 0 \\
96 & \rho^2(72+24\rho^2) & 0 & 0 & 12+84\rho^2 \\
\rho^2(72+24\rho^2) & 96 & 0 & 0 & 12+84\rho^2 \\
0 & 0 & 3(1+4\rho^2) & \rho(9+6\rho^2) & 0 \\
0 & 0 & \rho(9+6\rho^2) & 3(1+4\rho^2) & 0 \\
12+84\rho^2 & 12+84\rho^2 & 0 & 0 & 8+68\rho^2+20\rho^4
\end{bmatrix}
$$

$$
A = \begin{bmatrix}
\frac{1}{\sigma_1^2(\rho^2-1)} & -\frac{\rho}{\sigma_1\sigma_2(\rho^2-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\rho}{\sigma_1\sigma_2(\rho^2-1)} & \frac{1}{\sigma_2^2(\rho^2-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2\sigma_1^4(\rho^2-1)} & 0 & -\frac{\rho}{2\sigma_1^3\sigma_2(\rho^2-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2\sigma_2^4(\rho^2-1)} & -\frac{\rho}{2\sigma_1\sigma_2^3(\rho^2-1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\rho}{\sigma_1^2(1-2\rho^2+\rho^4)} & \frac{\rho}{\sigma_2^2(1-2\rho^2+\rho^4)} & -\frac{1+\rho^2}{\sigma_1\sigma_2(1-2\rho^2+\rho^4)} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
V = \begin{bmatrix}
\underset{5\times 5}{0} & \underset{5\times 7}{0} \\
\underset{7\times 5}{0} & \underset{7\times 7}{V_1}
\end{bmatrix}
$$

$$
V_1 = \begin{bmatrix}
6\sigma_1^6 & 6\sigma_1^3\sigma_2^3\rho^3 & 0 & 0 & 6\sigma_1^5\sigma_2\rho & 6\sigma_1^4\sigma_2^2\rho^2 & 0 \\
6\sigma_1^3\sigma_2^3\rho^3 & 6\sigma_2^6 & 0 & 0 & 6\sigma_1^2\sigma_2^4\rho^2 & 6\sigma_1\sigma_2^5\rho & 0 \\
0 & 0 & 24\sigma_1^8 & 24\sigma_1^4\sigma_2^4\rho^4 & 0 & 0 & 24\sigma_1^6\sigma_2^2\rho^2 \\
0 & 0 & 24\sigma_1^4\sigma_2^4\rho^4 & 24\sigma_2^8 & 0 & 0 & 24\sigma_1^2\sigma_2^6\rho^2 \\
6\sigma_1^5\sigma_2\rho & 6\sigma_1^2\sigma_2^4\rho^2 & 0 & 0 & 4\sigma_1^4\sigma_2^2\rho^2+2\sigma_1^4\sigma_2^2 & 4\sigma_1^3\sigma_2^3\rho+2\sigma_1^3\sigma_2^3\rho^3 & 0 \\
6\sigma_1^4\sigma_2^3\rho^2 & 6\sigma_1\sigma_2^5\rho & 0 & 0 & 4\sigma_1^3\sigma_2^3\rho+2\sigma_1^3\sigma_2^3\rho^3 & 2\sigma_1^2\sigma_2^4+4\sigma_1^2\sigma_2^4\rho^2 & 0 \\
0 & 0 & 24\sigma_1^6\sigma_2^2\rho^2 & 24\sigma_1^2\sigma_2^6\rho^2 & 0 & 0 & 4\sigma_1^4\sigma_2^4+16\sigma_1^4\sigma_2^4\rho^2+4\sigma_1^4\sigma_2^4\rho^4
\end{bmatrix}
$$

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