行政院國家科學委員會補助專題研究計畫成果報告

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Mean-Preserving-Spread and Demand for Market Insurance as well as Self Insurance

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Abstract

This paper intends to investigate whether a risk averse individual demands more market insurance as well as self insurance when there is a mean-preserved-spread on his loss distribution. This paper proves that, when either only market insurance or only self-insurance is available, the necessary and sufficient condition which can generate clear predictions is a special case of central risk dominance derived by Gollier (1995) when $\gamma = 1$. Moreover, the paper proves that, when both market insurance and self-insurance are available, a mean-preserved-spread in risk has no impact on self insurance and the necessary and sufficient condition which can generate clear predictions for market insurance is the same condition as that when only market insurance is available. The paper also proves that the above finding is indifferent to whether the action of self insurance is observable by the insurer.

Introduction

Rothschild and Stiglitz (1970, 1971) pioneered to study whether a risk averse individual demands more risky assets when facing an increase in risk. Since then, some researchers [Dreze and Modigliani (1972), Diamond and Stiglitz (1974), Dionne and Eeckhoudt (1987), Briys, Eeckhoudt, and Dionne (1989)] have found conditions on the utility functions which can generate determinant comparative statics with a mean preserving increase in risk. Others [Meyer and Ormiston (1983, 1985), Black and Bulkley(1989), Eeckhoudt and Hasen (1980, 1983)] have found the constraints on the increase in risk which can provide clear prediction. Gollier (1995) first defined the concept of central risk and showed that "greater central riskness" dominance is the necessary and sufficient condition for unambiguous comparative statics of risk increases. Most papers of this literature focused on a desirable risk but insurance indeed copes with an unfavorable risk. Without further modification, the results of those papers may not be directly applied to insurance purchasing behavior.

Although this branch of research has made tremendous contribution on application of investments, relatively few research [Eeckhoudt, Gollier and Schlesinger (1991), Dionne and Gollier (1992), Meyer and Ormiston (1995)] have extended to study whether a risk averse individual demands more insurance when facing an increase of an undesirable risk. Moreover, when examining the impact of an increase in risk, these papers did not consider the interaction between self insurance and market insurance suggested by Ehrlich and Becker (1972). This paper intends to investigate whether a risk averse individual demands more market insurance as well as self insurance when there is a mean-preserved-spread on his loss distribution.

Model for Market Insurance

In this section, it is assumed that only market insurance is available. Assume that the insured with an initial wealth faces a random loss $x \in [0, L]$ which follows a distribution $f(x)$. Let P_f and Q denote the insurance price and insurance amount respectively. The insured can pay a premium $P_f Q$ for an indemnity $(\frac{Q}{L})x$. Assume that P_f is actuarially fair price with expense loading λ. Thus, the final wealth of the insured, Z, is $\quad -x + \frac{Q}{L}x - P_f Q$. Assume that the insured chooses an optimal insurance amount Q to maximize his expected utility $E[u(Z)]$, where $u'(.) > 0$ and $u''(.) < 0$. The model can be written as:

$$\underset{Q}{\text{MAX}}\ H(Q;u,f,Z)=\int_0^L u(W-x+\frac{Q}{L}x-P_fQ)f(x)dx,$$

$$s.t.\quad P_f=\frac{1+\lambda}{L}\int_0^L xf(x)dx. \tag{1}$$

The optimal insurance amount can be determined by the following first order condition of Equation (1):

$$H'(Q^*;u,f,Z)=\int_0^L(\frac{x}{L}-P_f)u'(W-x+\frac{Q^*}{L}x-P_fQ^*)f(x)dx=0. \tag{2}$$

After an integration by parts, Equation (2) can be rewritten as:

$$H'(Q^*;u,f,Z)=\int_0^L(1-\frac{Q^*}{L})u''(W-x+\frac{Q^*}{L}x-P_fQ^*)T_M(x,Q^*,f,Z)dx$$
$$+u'(W-L+Q^*-P_fQ^*)T_M(L,Q^*,f,Z)=0, \tag{3}$$

$$\text{where}\quad T_M(x,Q^*,f,Z)=\int_0^x(\frac{t}{L}-P_f)f(t)dt. \tag{4}$$

Further, we identify situation which can generate clear comparative statics.

Theorem 1

Given that the price of insurance is actuarially fair and that the increase in risk is a mean preserving spread, the insured purchases more insurance if and only if $T_M(x,Q,g,Z)\le T_M(x,Q,f,Z),\forall x\in[0,L)$.

It is very important to recognize that the necessary and sufficient condition in Theorem 1 which can generate clear predictions is a special case of central risk dominance derived by Gollier (1995) when $\gamma=1$.

Model for Self Insurance

In this section, it is assumed that only self insurance is available. Assume that an individual with an initial wealth faces a random loss $x\in[0,L]$ which follows a distribution $f(x)$. Assume that the individual can make an effort with a cost, C, to reduce the loss by $v(C)$ proportion. Assume that $v'(C)>0$. Thus, the final wealth of the individual, Z, is $-(1-v(C))x-C$. Assume that the individual chooses a self insurance C to maximize his expected utility $E[u(Z)]$, where $u'(.)>0$ and $u''(.)<0$. The model can be written as:

$$\underset{C}{\text{MAX}}\ H(C;u,f,Z)=\int_0^L u(W-(1-v(C))x-C)f(x)dx. \quad (5)$$

The optimal amount of self insurance can be determined by the following first order conditions of Equation (5):

$$\frac{\partial H}{\partial C}=\int_0^L [v'(C)x-1]u'(W-(1-v(C))x-C)f(x)dx=0. \quad (6)$$

After an integration by parts, Equation (6) can be rewritten as:

$$H'(C^*;u,f,Z)=\int_0^L (1-v(C^*))u''(W-(1-v(C^*))x-C^*)T_S(x,C^*,f,Z)dx$$
$$+u'(W-(1-v(C^*))L-C^*)T_S(L,C^*,f,Z)=0, \quad (7)$$

where $T_S(x,C^*,f,Z)=\int_0^x (v'(C)t-1)f(t)dt.$ (8)

Theorem 2

Given that the increase in risk is a mean preserving spread, an individual spends more self insurance if and only if $T_S(x,C,g,Z)\le T_S(x,C,f,Z), \forall x\in[0,L)$.

Like that in Theorem 1, the necessary and sufficient condition in Theorem 2 which can generate clear predictions is a special case of central risk dominance derived by Gollier (1995) when $\gamma=1$.

Model for Market and Self Insurance
When the Self-Insurance is Unobservable by the Insurer

In this section, both market insurance and self insurance are assumed to be available. Assume that the insured with an initial wealth faces a random loss $x\in[0,L]$ which follows a distribution $f(x)$. Let P_f and Q denote the insurance price and insurance amount respectively. The insured can pay a premium P_fQ for an indemnity $(\frac{Q}{L})x$. Assume that P_f is actuarially fair price with expense loading λ. Further assume that the insured can make an effort with a cost, C, to reduce the loss by $v(C)$ proportion. Assume that $v'(C)>0$. However, the self insurance is unobservable for the insurer. Thus, the final wealth of the insured, Z, is $-(1-v(C))x+\frac{Q}{L}x-P_fQ-C$. Assume that the insured chooses an optimal insurance amount Q and self insurance C to maximize his expected utility

$E[u(Z)]$, where $u'(.) > 0$ and $u''(.) < 0$. The model can be written as:

$$\underset{Q,C}{\text{MAX}}\ H(Q,C;u,f,Z) = \int_0^L u(W-(1-v(C))x+\frac{Q}{L}x-P_fQ-C)f(x)dx.$$

$$s.t.\quad P_f = \frac{(1+\lambda)}{L}\int_0^L xf(x)dx. \tag{9}$$

The optimal insurance and self insurance amount can be determined by the following first order conditions of Equation (9):

$$\frac{\partial H}{\partial Q} = \int_0^L [\frac{x}{L}-P_f]u'(W-(1-v(C))x+\frac{Q}{L}x-P_fQ-C)f(x)dx = 0, \tag{10}$$

and

$$\frac{\partial H}{\partial C} = \int_0^L [v'(C)x-1]u'(W-(1-v(C))x+\frac{Q}{L}x-P_fQ-C)f(x)dx = 0. \tag{11}$$

From Equations (10) and (11), it is obvious that

$$v'(C) = \frac{1}{LP_f}. \tag{12}$$

After an integration by parts, Equation (10) can be rewritten as:

$$\frac{\partial H}{\partial Q} = \int_0^L [1-v(C)-\frac{Q}{L}]u''(W-(1-v(C))x+\frac{Q}{L}x-P_fQ-C)T_{MS}(x,f)dx$$
$$+u'(W-(1-v(C))L+Q-P_fQ-C)T_{MS}(L,f) = 0, \tag{13}$$

where $T_{MS}(x,f) = \int_0^x (\frac{t}{L}-P_f)f(t)dt. \quad (14)$

Theorem 3 $C_g^* = C_f^*$.

Theorem 4 $Q_g^* > Q_f^*$ ***if and only if*** $T_{MS}(x,g) > T_{MS}(x,f), \forall x \in [0,L]$.

It is very surprise to find that Theorem 4 is nothing but Theorem 2. Even more striking finding is that mean-preserved spread in risk has no impact on self insurance when both self insurance and market insurance are available.

Model for Market and Self Insurance When the Self Insurance is Observable by the Insurer

In this section, both market insurance and self insurance are assumed to be available. Assume that the insured with an initial wealth faces a random loss $x \in [0, L]$ which follows a distribution $f(x)$. Let $\hat{P}_f$ and $\hat{Q}$ denote the insurance price and insurance amount respectively. The insured can pay a premium $\hat{P}_f \hat{Q}$ for an indemnity $(\frac{\hat{Q}}{L})x$. Assume that $\hat{P}_f$ is actuarially fair price with expense loading λ. Further assume that the insured can make an effort with a cost, $\hat{C}$, to reduce the loss by $v(\hat{C})$ proportion. Assume that $v'(\hat{C}) > 0$. However, the self insurance is unobservable for the insurer. Thus, the final wealth of the insured, Z, is $-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C}$. Assume that the insured chooses an optimal insurance amount $\hat{Q}$ and self insurance $\hat{C}$ to maximize his expected utility $E[u(Z)]$, where $u'(.) > 0$ and $u''(.) < 0$. The model can be written as:

$$\underset{\hat{Q},\hat{C}}{\text{MAX}} \ \hat{H}(\hat{Q},\hat{C};u,f,Z) = \int_0^L u(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C})f(x)\,dx.$$

$$s.t. \quad \hat{P}_f = \frac{(1+\lambda)(1-v(\hat{C}))}{L}\int_0^L xf(x)\,dx. \tag{15}$$

The optimal insurance and self insurance amount can be determined by the following first order conditions of Equation (15):

$$\frac{\partial \hat{H}}{\partial \hat{Q}} = \int_0^L [\frac{1-v(\hat{C})}{L}x]u'(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C})f(x)\,dx$$
$$-\int_0^L \hat{P}_f u'(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C})f(x)\,dx = 0, \tag{16}$$

and

$$\frac{\partial \hat{H}}{\partial \hat{C}} = \int_0^L [(1-\frac{\hat{Q}}{L})v'(\hat{C})x]u'(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C})f(x)\,dx$$
$$-\int_0^L [\frac{\partial \hat{P}_f}{\partial \hat{C}}\hat{Q}+1]u'(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x - \hat{P}_f \hat{Q} - \hat{C})f(x)\,dx = 0. \tag{17}$$

From Equations (16) and (17), the optimal self insurance amount can be determined by the following condition:

$$v'(\hat{C}) = \frac{1}{(1+\lambda)\mu_f}. \tag{18}$$

where $\mu_f = \int_0^L x f(x)\,dx$.

After an integration by parts, Equation (16) can be rewritten as:

$$\frac{\partial \hat{H}}{\partial \hat{Q}} = \int_0^L [(1-\frac{\hat{Q}}{L})(1-v(\hat{C})]u''(W-(1-\frac{\hat{Q}}{L})(1-v(\hat{C}))x+\frac{\hat{Q}}{L}x-\hat{P}_f\hat{Q}-\hat{C})\hat{T}_{MS}(x,f)\,dx$$

$$+u'(W-(1-v(\hat{C}))L+\hat{Q}-\hat{P}_f\hat{Q}-\hat{C})\hat{T}_{MS}(L,f)=0, \tag{19}$$

where $\hat{T}_{MS}(x,f) = \int_0^x (\frac{1-v(\hat{C})}{L}t-\hat{P}_f)f(t)\,dt$. (20)

Theorem 5 $\hat{C}^*_g = C^*_f$.

Theorem 6 $\hat{Q}^*_g > \hat{Q}^*_f$ ***if and only if*** $\hat{T}_{MS}(x,g) > \hat{T}_{MS}(x,f), \forall x \in [0,L]$.

Like Theorems 3 and 4, Theorem 6 is nothing but Theorem 2 and mean-preserved spread in risk has no impact on self-insurance when both self insurance and market insurance are available.

Conclusions

This paper proves that, when either only market insurance or only self-insurance is available, the necessary and sufficient condition which can generate clear predictions is a special case of central risk dominance derived by Gollier (1995) when $\gamma = 1$. Moreover, the paper proves that, when both market insurance and self-insurance are available, a mean-preserved-spread in risk has no impact on self insurance and the necessary and sufficient condition which can generate clear predictions for market insurance is the same condition like when only market insurance is available. The paper also proves that the above finding is indifferent to whether the action of self insurance is observable by the insurer.

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