行政院國家科學委員會補助專題計畫

保險公司資產負債管理

執行機關:國立台灣大學財務金融學系

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執行期限:88/08/01~89/07/31

計畫編號:NSC 89-2416-H-002-032

Surplus Management of Insurance Companies

Abstract

To immunize the surplus of an insurance company against the fluctuation of the interest rate, asset-liability managers commonly adopt the so-called classical immunization strategy, maintaining the duration of assets equal to the liability/asset ratio times the duration of liabilities. However, it is well known that this strategy is valid only when the shift in the interest rate is parallel and infinitesimal. This paper examines the immunization strategy with a non-parallel and non-infinitesimal yield curve shift. Instead of assuming that the interest rates of assets are all equal in each period, we consider the strategy under a linear term structure. We find that the immunization strategy under a linear term structure requires further adjustment in the convexity of the surplus. Moreover, we show that maximizing the convexity of the surplus subject to the classical immunization constraints is the optimal strategy for an insurance company when the shift of the interest rate is not small. We also demonstrate that this optimal strategy can be implemented by linear programming.

Introduction

Staking and Babbel (1995) provide empirical evidence to support the relationship between surplus duration and market value for insurance companies. Barney (1997) demonstrates that the relationship found by Staking and Babbel (1995) may be at least partially caused by the choice of the measure of surplus duration. Traditionally, to immunize the surplus of an insurance company against the fluctuation of the interest rate, asset-liability managers need to carefully arrange both the cash inflows and outflows of the firm. A classical immunization strategy serves to maintain the duration of assets equal to the asset/liability ratio times the duration of liabilities (Bierwag, 1987; Grove, 1974; Reitano, 1991). Although this strategy is commonly used by asset-liability managers, several issues have been raised to question this theory. First, this classical immunization strategy is known to be valid only when the shift in the interest rate is small. Second, Reitano (1992) demonstrates that the classical immunization theory works only in the case of the parallel yield curve shift. Furthermore, Barney (1997) shows that the interest spread between assets and liabilities is crucial to immunize the interest rate risk for an insurance company. In an intriguing article, Briys and Varenne (1997) provide an alternative measure of effective surplus duration under stochastic interest rates. However, most insurance companies may consider a discrete time model easier to adopt in their ordinary operation. This paper intends to integrate the models of Reitano (1992) and Barney (1997) with a non-infinitesimal yield curve shift. Instead of assuming that the interest rates of assets are the same as those of liabilities in each period, we examine the immunization strategy under a linear term structure. We find that the immunization strategy under liner term structure requires further adjustment in the convexity of the surplus. Moreover, we show that maximizing the convexity of the surplus, subject to the classical immunization constraints, is the optimal strategy for an insurance company when the shift in the interest rate is not small. We also demonstrate that this optimal strategy can be implemented by linear programming.

Non-Parallel Shift of the Yield Curve

Let us consider an insurance company with assets A and liabilities L. Assume that the assets and liabilities are valued as the present value of the future cash inflows and cash outflows of n periods. Thus,

$$A = \sum_{i=0}^{n} \frac{A_i}{(1+r_{Ai})^i}, \text{ and}$$

$$L = \sum_{i=0}^{n} \frac{L_i}{(1+r_{Li})^i}, \tag{1}$$

where A_i is the cash inflow at period *i*,

L_i is the cash outflow at period *i*,

r_{Ai} is the interest rate for the cash inflow at period *i*, and

r_{Li} is the interest rate for the cash outflow at period *i*.

From an accounting point of view, the surplus of firm S is equal to the difference between its assets and liabilities. Thus,

$$S = \sum_{i=0}^{n} \frac{A_i}{(1+r_{Ai})^i} - \sum_{i=0}^{n} \frac{L_i}{(1+r_{Li})^i}. \tag{2}$$

The surplus of the firm is immunized locally by the change in interest rate if $\frac{dS}{dr} = 0$, where r is the current interest rate.

Differentiating the surplus with respect to the interest rate gives

$$\frac{dS}{dr} = \sum_{i=0}^{n} \frac{-iA_i}{(1+r_{Ai})^{i+1}} \frac{dr_{Ai}}{dr} - \sum_{i=0}^{n} \frac{-iL_i}{(1+r_{Li})^{i+1}} \frac{dr_{Li}}{dr} = 0. \tag{3}$$

To consider a non-parallel yield curve shift, we assume that

$$r_{Ai} = (\alpha_A + \beta_A i)r + C_A + K_A i = (\alpha_A r + C_A) + (\beta_A r + K_A)i \text{ and}$$

$$r_{Li} = (\alpha_L + \beta_L i)r + C_L + K_L i = (\alpha_L r + C_L) + (\beta_L r + K_L)i, \tag{4}$$

where α_A, β_A, α_L, β_L, C_A, C_L, K_A, and K_L are all constants.

There are two important issues in Equation (4). First, the decision-maker needs to choose appropriate definitions for r_{Ai}, r_{Li}, and r. For example, r_{Ai} and r_{Li}, respectively, can be the rates of return for lending and borrowing at period *i,* while r is the current central rate of return between lending and borrowing. Furthermore, r can be any factor that can influence the term structure of the interest rate. For example, if the decision-maker is interested in how to hedge the risk of the inflation rate embedded in the interest rate risk. It is easy to extend Equation (4) to cope with this problem. The decision-maker can let r denote the current inflation rate and let r_{Ai} and r_{Li} denote the rate of return for lending and borrowing in period *i,* respectively. Second, it is critical for the decision-maker to get a set of parameters for α_A, β_A, α_L, and β_L. One way to estimate α_A, β_A, α_L, and β_L is to regress r_{Ai} and r_{Li} with respect to r and ri. This method looks straightforward on the basis of Equation (4). Basically, Equation (4) implies a linear term structure of interest rates for both assets and liabilities. That is, Equation (4) can simulate a rising, declining, or flat yield curve when $\beta_A r + K_A$ and $\beta_L r + K_L$ are positive, negative, or zero, respectively. Furthermore, this equation is a general model of parallel yield curve shift. That is, if $\alpha_A = \alpha_L$ and $\beta_A = \beta_L = 0$, Equation (4) simply implies a parallel shift of the term structure. Moreover, the Equation (4) anticipates the problem of interest rate spread between assets and liabilities.

From Equation (4), Equation (3) can be rewritten as

$$\sum_{i=0}^{n} \frac{iA_i}{(1+r_{Ai})^{i+1}}(\alpha_A + \beta_A i) = \sum_{i=0}^{n} \frac{iL_i}{(1+r_{Li})^{i+1}}(\alpha_L + \beta_L i) \tag{5}$$

By arranging the terms, Equation (5) can be rewritten as

$$[(\alpha_A - \beta_A)A][\frac{1}{A}\sum_{i=0}^{n}\frac{iA_i}{(1+r_{Ai})^{i+1}}]+[\beta_A A][\frac{1}{A}\sum_{i=0}^{n}\frac{i(i+1)A_i}{(1+r_{Ai})^{i+1}}]$$
$$=[(\alpha_L - \beta_L)L][\frac{1}{L}\sum_{i=0}^{n}\frac{iL_i}{(1+r_{Li})^{i+1}}]+[\beta_L L][\frac{1}{L}\sum_{i=0}^{n}\frac{i(i+1)L_i}{(1+r_{Li})^{i+1}}]. \quad (6)$$

Let us define the modified duration and convexity of assets and liabilities as follows:

$$D_A^m = \frac{1}{A}\sum_{i=0}^{n}\frac{iA_i}{(1+r_{Ai})^{i+1}},$$

$$D_L^m = \frac{1}{L}\sum_{i=0}^{n}\frac{iL_i}{(1+r_{Li})^{i+1}},$$

$$V_A^m = \frac{1}{A}\sum_{i=0}^{n}\frac{i(i+1)A_i}{(1+r_{Ai})^{i+1}}, \text{ and}$$

$$V_L^m = \frac{1}{L}\sum_{i=0}^{n}\frac{i(i+1)L_i}{(1+r_{Li})^{i+1}},$$

where D_A^m , D_L^m, V_A^m, and V_L^m denote the modified duration and the modified convexity of assets and liabilities, respectively.

Thus, substituting D_A^m , D_L^m, V_A^m, and V_L^m into Equation (6), we get

$$D_A^m = (\frac{\alpha_L - \beta_L}{\alpha_A - \beta_A})(\frac{L}{A})D_L^m + (\frac{1}{\alpha_A - \beta_A})[\beta_L \frac{L}{A}V_L^m - \beta_A V_A^m]. \quad (7)$$

Equation (7) shows that the asset-liability managers may need to match not only the surplus's duration but also the surplus's convexity when coping with interest rate risk under a linear term structure. Obviously, the terms $V_A^m = (\frac{\beta_L}{\beta_A})(\frac{L}{A})V_L^m$ and $D_A^m = (\frac{\alpha_L - \beta_L}{\alpha_A - \beta_A})(\frac{L}{A})D_L^m$ are sufficient conditions of Equation (7). Moreover, the immunization strategy on the basis of Equation (7) is more general than the classical immunization strategy and that suggested by Barney (1997), since they are only the special cases of Equation (7).

To demonstrate this point, let $\alpha_A = \alpha_L = \alpha$ and $\beta_A = \beta_L = 0$ (i.e., $r_A == \alpha r + C_A$ and

$r_L == \alpha r + C_L$); then

$$\frac{A}{1+r_A}D_A = \frac{L}{1+r_L}D_L, \tag{8}$$

where $D_A = \frac{1}{A}\sum_{i=0}^{n}\frac{iA_i}{(1+r_A)^i}$, and $D_L = \frac{1}{L}\sum_{i=0}^{n}\frac{iL_i}{(1+r_L)^i}$.

We note that Equation (8) is the same as the results derived by Barney (1997). Obviously, if $\alpha_A = \alpha_L = \alpha$, $\beta_A = \beta_L = 0$, and $C_A = C_L$ (i.e., $r_A = r_L$), then $D_A = \frac{L}{A}D_L$, which is the classical immunization strategy commonly used by asset-liability managers.

Non-Infinitesimal Shift in Interest Rates

It is well known that $\frac{dS}{dr}$ is a good measure for the interest rate risk only if the movement of the interest rate is small. From Taylor's expansion series, we know

$$\Delta S = \sum_{k=1}^{m}\frac{d^k S}{dr^k}\frac{(\Delta r)^k}{k!}. \tag{9}$$

If the shift in the interest rate is small, the change in the surplus can be approximated by Equation (9) at $m = 1$. That is,

$$\Delta S \approx \frac{dS}{dr}\Delta r. \tag{10}$$

It is obvious that the surplus of the firm is immunized by the change in the interest rate if $\frac{dS}{dr} = 0$, which is also the rationale of Equation (3). However, if the shift in the interest rate is not small, then Equation (9) with $m = 2$ will provide a more accurate approximation. Thus,

$$\Delta S \approx \frac{dS}{dr}\Delta r + \frac{1}{2}\frac{d^2 S}{dr^2}(\Delta r)^2. \tag{11}$$

It is noteworthy that the second term of Equation (11), $\frac{1}{2}\frac{d^2 S}{dr^2}(\Delta r)^2$, will always be positive if

$\frac{d^2S}{dr^2} > 0$. In other word, if $\frac{d^2S}{dr^2} > 0$, then the surplus of the firm increases whether or not interest rate increases or decreases. Therefore, if the shift in the interest rate is not small, the best strategy for immunization should be

$$\max_{A_i, L_i} \quad \frac{d^2S}{dr^2}$$

$$s.t. \quad S = A - L, \text{ and}$$

$$\frac{dS}{dr} = 0. \tag{12}$$

Equation (12) can be rewritten as

$$\max_{A_i, L_i} \quad \sum_{i=0}^{n} \frac{i(i+1)(\alpha_A + \beta_A i)^2}{(1+r_{Ai})^{i+2}} A_i - \sum_{i=0}^{n} \frac{i(i+1)(\alpha_L + \beta_L i)^2}{(1+r_{Li})^{i+2}} L_i$$

$$s.t. \quad \sum_{i=0}^{n} \frac{1}{(1+r_{Ai})^i} A_i - \sum_{i=0}^{n} \frac{1}{(1+r_{Li})^i} L_i = S, \text{ and}$$

$$\sum_{i=0}^{n} \frac{i(\alpha_A + \beta_A i)}{(1+r_{Ai})^{i+1}} A_i - \sum_{i=0}^{n} \frac{i(\alpha_L + \beta_L i)}{(1+r_{Li})^{i+1}} L_i = 0. \tag{13}$$

Although the manager of an insurance company can control the firm's future liabilities by means of marketing and reinsurance strategies, such strategies are relatively difficult to achieve. A better way to implement this asset-liability management strategy is to make appropriate investment decisions given the firm's future liability schedule. To demonstrate this point, Equation (13) can be rewritten as

$$\max_{A_i} \quad \sum_{i=0}^{n} \frac{i(i+1)(\alpha_A + \beta_A i)^2}{(1+r_{Ai})^{i+2}} A_i$$

$$s.t. \quad \sum_{i=0}^{n} \frac{1}{(1+r_{Ai})^i} A_i = S + \sum_{i=0}^{n} \frac{1}{(1+r_{Li})^i} L_i, \text{ and}$$

$$\sum_{i=0}^{n} \frac{i(\alpha_A + \beta_A i)}{(1+r_{Ai})^{i+1}} A_i = \sum_{i=0}^{n} \frac{i(\alpha_L + \beta_L i)}{(1+r_{Li})^{i+1}} L_i. \tag{14}$$

In Equation (14), A_i is the investment decision for asset-liability management in each period for insurance company. It is very important to recognize that, when surplus S and both the liability schedule and the term structure are given as parameters, the terms $\frac{i(i+1)(\alpha_A+\beta_A i)^2}{(1+r_{Ai})^{i+2}}$, $\frac{1}{(1+r_{Ai})^i}$, $\frac{i(\alpha_A+\beta_A i)}{(1+r_{Ai})^{i+1}}$, $S+\sum_{i=0}^{n}\frac{1}{(1+r_{Li})^i}L_i$, and $\sum_{i=0}^{n}\frac{i(\alpha_L+\beta_L i)}{(1+r_{Li})^{i+1}}L_i$ are all constants with respect to A_i. Therefore, Equation (14) can be solved by linear programming.

Simulation

To demonstrate the application of our model, we construct a hypothetical insurance company. Solvency and non-negative constraints are further considered to make the case more realistic. The results of the simulation show that the immunization strategy of our model can be implemented by using linear programming. We also find that the effect of the convexity cannot be ignored under a linear term structure.

For simulation purposes, we create a balance sheet for the hypothetical insurance company at period 0 (Exhibit 1).

Exhibit 1 Balance Sheet of a Sample Insurance Company

Assets	Liabilities	Surplus
\$238,798M	\$211,303M	\$27,495M

Without losing any generosity, we assume the liabilities are to be paid out in five years as shown in Exhibit 2.

Exhibit 2 Liability Schedule of the Sample Insurance Company

Periods	Liabilities
1	\$45,500M
2	\$49,000M
3	\$52,700M
4	\$56,500M
5	\$60,900M

Let us assume that the current interest rate is 5 percent. Let us also assume that $\alpha_A = 1.00$, $\alpha_L = 1.10$, $\beta_A = 0.05$, $\beta_L = 0.06$, $C_A = 0.00$, and $C_L = 0.01$. By Equation (4), we know

$r_{Ai} = 0.05 + 0.0025i$ and

$r_{Li} = 0.065 + 0.003i$.

As mentioned above, the best immunization strategy suggested by Equation (14) should be:

$$\underset{A_i}{Max} \quad \sum_{i=0}^{n} \frac{i(i+1)(1+0.05i)^2}{(1.05+0.0025i)^{i+2}} A_i$$

$$s.t. \quad \sum_{i=0}^{n} \frac{1}{(1.05+0.0025i)^i} A_i = 238{,}798\text{, and}$$

$$\sum_{i=0}^{n} \frac{i(1+0.05i)}{(1.05+0.0025i)^{i+1}} A_i = 773{,}330.5 \qquad (15)$$

It is reasonable to assume that no A_i is less than zero. That is,

$$A_i \geq 0, i = 0,1,2,3,4,5. \qquad (16)$$

An insurance company may sometimes need to maintain solvency in each period because of the regulation. This constraint can be expressed as

$$\sum_{i=0}^{j}(A_i - L_i)(1+r_{i,j})^{j-i} \geq 0, j = 0,1,2,3,4,5, \tag{17}$$

where $r_{i,j}$ is the interest rate from the ith period to the jth period.

Let us also assume that the expectation hypothesis holds. Thus, $r_{i,j}$ can be estimated rationally from the current term structure. If all L_i are given, then Equation (17) is, indeed, a linear function of A_i and can be expressed as

$1.05A_0 + A_1 \geq 45{,}500$,

$1.11A_0 + 1.06A_1 + A_2 \geq 97{,}116$,

$1.18A_0 + 1.12A_1 + 1.06A_2 + A_3 \geq 155{,}888$,

$1.26A_0 + 1.20A_1 + 1.13A_2 + 1.07A_3 + A_4 \geq 222{,}916$, and

$1.35A_0 + 1.29A_1 + 1.22A_2 + 1.14A_3 + 1.07A_4 + A_5 \geq 29{,}990$.

After considering the non-negative constraints of Equation (6) and the solvency constraints of Equation (17), the problem of the asset/liability manager can be expressed as:

$$\underset{A_i}{Max} \quad \sum_{i=0}^{n}\frac{i(i+1)(1+0.05i)^2}{(1.05+0.0025i)^{i+2}}A_i$$

$$s.t. \quad \sum_{i=0}^{n}\frac{1}{(1.05+0.0025i)^i}A_i = 238{,}798,$$

$$\sum_{i=0}^{n}\frac{i(1+0.05i)}{(1.05+0.0025i)^{i+1}}A_i = 773{,}330.5,$$

$1.05A_0 + A_1 \geq 45{,}500$,

$1.11A_0 + 1.06A_1 + A_2 \geq 97{,}116$,

$1.18A_0 + 1.12A_1 + 1.06A_2 + A_3 \geq 155{,}888$,

$1.26A_0 + 1.20A_1 + 1.13A_2 + 1.07A_3 + A_4 \geq 222{,}916$,

$1.35A_0 + 1.29A_1 + 1.22A_2 + 1.14A_3 + 1.07A_4 + A_5 \geq 299{,}990$, and

$$A_i \geq 0, i = 0,1,2,3,4,5. \tag{18}$$

Obviously, Equation (18) can be solved by linear programming. The results from the equation are as follows:

Exhibit 3: Optimal Asset Allocation

Periods	Assets
0	\$58,844M
1	\$0M
2	\$31,624M
3	\$52,700M
4	\$56,498M
5	\$84,262M

In addition, to demonstrate the immunization effect of the strategy suggested in this paper, we generate two counter-examples. Example A shows the cost of failing to recognize the existence of a term structure for the interest rates; and example B shows the cost of failing to recognize a non-infinitesimal shift in interest rates.

Case of Failing to Recognize the Existence of a Term Structure for Interest Rates

In counter-example A, the asset-liability manager fails to recognize the existence of a term structure for the interest rates. Therefore he/she assumes that $\alpha_A = 0.00$, $\alpha_L = 0.00$, $\beta_A = 0.00$, $\beta_L = 0.00$, $C_A = 0.05$, and $C_L = 0.065$, where they are actually $\alpha_A = 1.00$, $\alpha_L = 1.10$, $\beta_A = 0.05$, $\beta_L = 0.06$, $C_A = 0.00$, and $C_L = 0.01$. Thus, the asset-liability manager allocates the firm's assets as follows:

Exhibit 4: Asset Allocation of Counter-Example A

Periods	Assets
0	$78,308M
1	$0M
2	$11,123M
3	$54,164M
4	$58,846M
5	$70,459M

Let us assume that the interest rate shifts from 5 percent to 3 percent, 4 percent, 6 percent, or 7 percent. The surpluses of the optimal case and counter-example A are as follows:

Exhibit 5: The Cost of Mismatch in Example A

(1)	(2)	(3)	(4)=(3)-(2)([4]/$27,495M)
Interest Rate	Optimal Case	Example A	Differences
5%	$27,495M	$21,444M	$-6,051M (-22.01%)
6%	$27,495M	$22,281M	$-5,215M (-18.97%)
4%	$27,495M	$20,561M	$-6,934M (-25.22%)
7%	$27,497M	$23,074M	$-4,424M (-16.09%)
3%	$27,493M	$19,623M	$-7,896M (-28.62%)

Exhibit 5 shows that the cost of failing to recognize the existence of the term structure is extremely high, ranging from –16 percent to nearly –29 percent. Furthermore, in all the situations, the surplus of the firm in the optimal case is almost immunized against the interest rate risk, whereas the surplus of the firm in counter-example A faces volatile fluctuations.

Case of Failing to Recognize a Non-infinitesimal Shift of Interest Rates

In counter-example B, the asset-liability manager recognizes that there is a term structure for the interest rate but fails to maximize the convexity of the surplus. Therefore, the asset allocation in counter-example B satisfies all the constraints in Equation (18) except for maximizing the objective function.

Exhibit 6: Asset Allocation of Counter-Example B

Periods	Assets
0	$ 0M
1	$ 45,500M
2	$ 49,000M
3	$ 52,701M
4	$ 127,058M
5	$ 8,580M

Let us assume that the interest rate shifts from 5 percent to 3 percent, 4 percent, 6 percent, or 7 percent. The surpluses of the optimal case and counter-example B are as follows:

Exhibit 7: The Effects of Immunization

(1) Interest Rate	(2) Optimal Case	(3) Example B	(4)=(2)-(3)([4]/$27,495M) Differences
5%	$27,495M	$27,494M	$ 1M (0.0035%)
6%	$27,495M	$27,461M	$ 35M (0.1258%)

4%	$27,495M	$27,457M	$ 37M (0.1362%)
7%	$27,497M	$27,367M	$130M (0.4732%)
3%	$27,493M	$27,340M	$153M (0.5574%)

From Exhibit 7, the effect of immunization is better in the optimal case than in the counter case. When the change in the interest rate is 1 percent, the surplus of the firm is immunized in the optimal case, whereas the surplus of the firm has little fluctuation in counter-example B (the cases when the interest rate is 6 percent and 4 percent). When the change in the interest rate is 2 percent, the surplus of the firm in the optimal case is still almost immunized, whereas the change in the surplus of the firm in counter-example B is shown to be large (the cases when interest rate is 7 percent and is 3 percent). Most important, in all situations the surpluses of the firm in the optimal case are always higher than those in counter-example B.

Conclusions

In this paper, we find that the immunization strategy under a linear term structure requires further adjustments in the convexity of the surplus. The immunization strategy in our model is more general than the classical immunization strategy and that suggested by Barney (1997). Moreover, we show that maximizing the convexity of the surplus, subject to the classical immunization constraints, is the optimal strategy for an insurance company when the shift of the interest rate is not small. We also demonstrate that this optimal strategy can be implemented by linear programming. By simulation, we further demonstrate the application of our model. The results of our simulation show that the immunization strategy of our model can be implemented easily by using linear programming. We also find that the costs of failing to recognize the term structure are extremely high, and the effect of the convexity cannot be ignored. As a future extension of this paper, we suggest considering a more general term structure or to that assume interest rates follow a stochastic process.

References

Barber, Loel R., and Mark L.Copper. "Is Bond Convexity a Free Lunch?" *The Journal of Portfolio Management*, Fall 1997, pp. 113-119.

Barney, L. Dwayne. "The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry: Comment. " *The Journal of Risk and Insurance*, Vol. 64 (1997), pp. 733-38.

Bierwag, Gerald O. *Duration Analysis: Managing Interest Rate Risk.* Cambridge, MA: Ballinger Publishing Company, 1987.

Bierwag, Gerald O., Charles J. Corrado and George G. Kaufman. "Duration for portfolios of binds priced on different term structures." *Journal of Banking and Finance*, Vol. 16 (1992), pp. 705-714.

Bierwag, Gerald O., Iraj Fooladi and Gordon S. Roberts. "Designing an Immunized Portfolio: Is M-squared the Key?" *Journal of Banking and Finance*, Vol. 7 (1993), pp. 1174-1170.

Briys, Eric, and Francois de Varenne, "On the Risk of Insurance Liabilities: Debunking Some Common Pitfalls," *The Journal of Risk and Insurance*, Vol. 64 (1997), 673-694.

Christensen, peter Ove, and Bjarne G. Sorensen. "Duration, Convexity, and Time Value." *The Journal of Portfolio Management*, Winner 1994, pp. 51-60.

Fong, H. Gifford, and Oldrich A. Vasicek. "A Risk Minimizing Strategy for Portfolio Immunization." *The Journal of Finance*, Vol. 5 (1984), pp. 1541-1546.

Gagnon, Louis, and Lewis D. Johnson., "Dynamic Immunization under Stochastic Interest Rates." *The Journal of Portfolio Management*, Spring 1994, pp. 48-54.

Grove, M.A. "On Duration and Optimal Maturity Structure of the Balance Sheet. " *Bell Journal of Economics and Management Sciences*, Vol. 5 (1974), pp. 696-709.

Hegde, Shantaram P., and Kenneth P. Nunn, Jr. "Non-infinitesimal Rate Changes and Macaulay

Duration." *The Journal of Portfolio Management*, Winner 1988, pp. 69-73.

Reitano, Robert R. "Non-Parallel Yield Curve Shifts and Immunization." *The Journal of Portfolio Management*, Spring 1992, pp. 36-43.

---. "Non-Parallel Yield Curve Shifts and Spread Leverage." *The Journal of Portfolio Management*, Spring 1991, pp. 82-87.

Staking, Kim B., and David F. Babble. "The Relation Between Capital Structure, Interest Rate Sensitivity, and Market Value in the Property-Liability Insurance Industry." *The Journal of Risk and Insurance* 62 (1995): 690-718.