行政院國家科學委員會補助專題研究計畫成果報告

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摘要

當一個保險公司在考慮利率風險的情況下進行資產負債管理時，爲了控制利率風險，管理者通常將公司之資產存續期間安排等於負債比例與負債存續期間乘積。然而，上述的策略只針對保險公司面臨的利率風險，事實上忽略其他影響利率變動因素的風險。本篇論文將探討在利率變動是一個隨機過程時，保險公司的資產負債管理如何同時處理利率風險與其他參數的風險，論文利用目標規劃整合此兩項策略，論文模擬分析的結果發現本文提出的方法顯著的優於傳統的免疫策略。

Abstract

To hedge the interest-rate risk against a firm's surplus, insurance companies commonly set the firm's asset duration equal to the debt ratio times the firm's liability duration. However, this strategy focuses only on the fluctuation of interest rates; it does not address any of the uncertainty in the underlined factors, which guide the changes in interest rates. This paper first identifies parameter risks against a firm's surplus. We further propose to use goal programming to integrate the traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies against interest-rate risk and parameter risks, the immunization strategy suggested here includes classical immunization strategy as a special case.

Keywords: asset and liability management, immunization strategy, parameter risks

Introduction

Many papers (Bierwag, 1987; Grove, 1974; and Reitano, 1992) have recommended using classical immunization—setting the duration of assets equal to the asset/liability ratio times the duration of liabilities—for immunizing interest-rate risk against an insurance company's surplus. To recognize the stochastic behavior of interest rates as found in the literature,[1] Briys and Varenne (1997) and Tzeng, Wang and Soo (2000) have extended the traditional research of surplus management to the case where interest rates follow a stochastic process. The researchers have found that, under a stochastic process of interest rates, the traditional measurement of duration may miscalculate the firm's risk and may require further modification.

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[1] E.g., Vasicek, 1977; Dothan, 1978; Cox, Ingersoll, and Ross, 1979; Dothan and Feldman, 1986, Ho and Lee, 1986; Chan et al., 1992; and Heath, Jarrow, and Morton, 1992.

Although this line of research has provided many insightful strategies for asset-liability management of insurance companies, most papers focus on changes in interest rates in the case of given parameters. However, an insurance company may usually need to cope with the environment in which both interest rates and other factors guiding interest rates could be uncertain simultaneously.

For example, the current interest rates may fluctuate because of mean-reverting as recognized by the literature (e.g., Vasicek, 1977; Cox, Ingersoll, and Ross, 1985). On the other hand, long-term interest rates could also shift due to changes in many macroeconomic policies. From an insurance company's point of view, a change in the trend of interest rates could cause even more significant impacts on a firm's surplus than the stochastic changes of the current interest rates. In this case, insurance companies may have not much information to characterize their parameter risks.

Another example is estimation error in parameter estimates, which insurance companies may have more information to evaluate their parameter risks. The managers in insurance companies typically use unbiased point estimators for parameters in the process. However, the managers also recognize that there exists estimation error in parameter estimates. Thus, the practitioners may like to further control the risk caused by estimates' standard errors, even they have already employed the unbiased estimators.

In the above two cases, the insurance company may have few or some information to measure their risk exposure on parameter risks. But, without any doubt, the managers in the insurance company should have a need to further control unexpected shock from parameter risks. Thus, this paper intends to investigate the parameter risks of surplus management when interest rates follow a stochastic process. We employ the model proposed by Tzeng, Wang, and Soo (2000) because their model is shown to be a general model of many other traditional models. However, unlike Tzeng, Wang, and Soo, who examine the effects of a stochastic change on current interest rates, we focus on the changes in the underlined parameter factors that guide the process of the interest rates. We first identify parameter risks against a firm's surplus and provide the methods for immunizing those risks. Furthermore, we propose a goal-programming algorithm to integrate traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies against both interest-rate risk and parameter risks, this immunization strategy includes classical immunization strategy as a special case.

Model of Parameter Risks

Let $CI(t)$ and $CO(t)$ denote the cash inflows and cash outflows of an insurance company at period t. Let us assume that the return of the interest rate follows the stochastic process suggested by Cox, Ingersoll, and Ross (1985)[2] and can be expressed as

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dz, \tag{1}$$

where r_t is the spot rate at period t and a, b, and σ are constants.

In the above stochastic process, dz follows a standard Brownian motion. $a(b - r_t)$ is the drift rate of the interest rate and characterizes a mean-reverting process, where a and b represent the momentum of the drift rate and the mean of the long-term interest rate, respectively. The standard deviation of the interest rate is proportional to $\sqrt{r_t}$ and is denoted by $\sigma\sqrt{r_t}$.

Let r and $P(t)$ denote the current interest rate and the current value of a one-dollar zero-coupon bond at period t. From Cox, Ingersoll, and Ross (1985),

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[2] Many other stochastic models, such as Vasicek's (1979), can also be used. Although each model may have its

$$P(t) = \alpha(t)\exp(-\beta(t)r), \tag{2}$$

where $\alpha(t) = [\frac{2\sqrt{a^2+2\sigma^2}e^{\frac{t}{2}(a+\sqrt{a^2+2\sigma^2})}}{(\sqrt{a^2+2\sigma^2}+a)(e^{t\sqrt{a^2+2\sigma^2}}-1)+2\sqrt{a^2+2\sigma^2}}]^{2ab/\sigma^2}$, and

$$\beta(t) = \frac{2(e^{t\sqrt{a^2+2\sigma^2}}-1)}{(\sqrt{a^2+2\sigma^2}+a)(e^{t\sqrt{a^2+2\sigma^2}}-1)+2\sqrt{a^2+2\sigma^2}}.$$

Using Cox, Ingersoll, and Ross' method (1979),[3] we measure the present value of future cash flows of t periods by the amount of cash flows times the current price of a one-dollar zero-coupon bond, $P(t)$. Thus, the assets and liabilities of an insurance company, A and L, can be expressed as:

$$A = \sum_{t=0}^{n} CI(t)P(t), \text{ and}$$

$$L = \sum_{t=0}^{n} CO(t)P(t). \tag{3}$$

The surplus of insurance company S is then equal to

$$S = A - L. \tag{4}$$

Like many traditional papers, Tzeng, Wang, and Soo (2000) have proposed an immunization strategy by setting $\frac{\partial S}{\partial r} = 0$ (Interest Rate Immunization). Although the interest rate may change stochastically, the immunization strategy of $\frac{\partial S}{\partial r} = 0$ can protect the surplus of

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own strength, it would be easier to apply the model with a close form solution.

[3] Lai and Frees (1995) derived similar method of valuation to calculate the reserves for insurance contracts. This method of valuation was also used by Tzeng, Wang, and Soo (2000), who assumed that there is a spread between the discount rates of assets and liabilities. To focus on the integration of parameter risks and interest rate risk, for the sake of simplicity we assume that the discount rates of assets and liabilities are the same. However, the main results of the paper still hold after relaxing this assumption.

the firm, at least locally. However, the stochastic change in the interest rate is not the only source of risks against a firm's surplus. Let us recall Equation (1):

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}\,dz . \tag{1}$$

In the above stochastic process, a and b represent the momentum of the drift rate and the mean of the long-term interest rate, respectively. The level of long-term interest rates can be different from the one insurance company uses because of a change in government's financial policies or simply because of estimation error. However, the strategy of $\frac{\partial S}{\partial r} = 0$ implicitly assumes that the parameters in the interest model do not change.

To cope with the parameter risks in surplus management, asset-liability managers can mimic traditional immunization strategy and arrange the assets and liabilities of the firm as follows:

$$\frac{\partial S}{\partial a} = 0 \text{ (Momentum Immunization),} \tag{5}$$

$$\frac{\partial S}{\partial b} = 0 \text{ (Mean Immunization), and/or} \tag{6}$$

$$\frac{\partial S}{\partial \sigma} = 0 \text{ (Deviation Immunization).} \tag{7}$$

Mean immunization, momentum immunization, and deviation immunization, respectively, are used to hedge the risks of changes in the long-term interest rate level, the magnitude of the drift rate, and the variance in the interest rate. One advantage of using Cox, Ingersoll, and Ross' (1985) model is that the parameters in their model represent meaningful characteristics of the interest rate. Another advantage is that their model provides explicit solutions, such as Equation (2), for the price of a one-dollar zero-coupon bond. Thus, parameter risks can be measured easily by taking derivatives with respect to those parameters. However, it is very important to recognize that the existence of parameter risks in surplus management does not depend on the employment of any specific stochastic model for interest rates. Almost every model of stochastic interest rate is required to estimate certain parameters, which may have their own meanings in reality. Although we use Cox, Ingersoll, and Ross' (1985) model to demonstrate our methodology, the idea of this paper can be adjusted to fit into any other models.

Recalling Equations (2), (3), and (4), the surplus of an insurance company can be rewritten as

$$Surplus = S(r, a, b, \sigma), \tag{8}$$

where $S(.)$ denotes the surplus function.

Let Δ denote the difference. By Taylor's expansion series, the change in the surplus caused by the changes in interest rate and parameters can be expressed as:

$$\Delta S \approx \frac{\partial S}{\partial r}\Delta r + \frac{\partial S}{\partial a}\Delta a + \frac{\partial S}{\partial b}\Delta b + \frac{\partial S}{\partial \sigma}\Delta \sigma \tag{9}$$

Equation (9) provides some rationales for the strategies suggested by Equations (5), (6), and (7). From Equation (9), we know that

$E(\Delta S) \approx \frac{\partial S}{\partial r}E(\Delta r) + \frac{\partial S}{\partial a}E(\Delta a) + \frac{\partial S}{\partial b}E(\Delta b) + \frac{\partial S}{\partial \sigma}E(\Delta \sigma)$ and

$std(\Delta S) \approx \frac{\partial S}{\partial r}std(\Delta r) + \frac{\partial S}{\partial a}std(\Delta a) + \frac{\partial S}{\partial b}std(\Delta b) + \frac{\partial S}{\partial \sigma}std(\Delta \sigma)$, if the changes in interest rate and parameters are all independent. Further assume that the firm keeps $\frac{\partial S}{\partial r} = 0$ to avoid interest rate risk, then $E(\Delta S) \approx \frac{\partial S}{\partial a}E(\Delta a) + \frac{\partial S}{\partial b}E(\Delta b) + \frac{\partial S}{\partial \sigma}E(\Delta \sigma)$ and

$std(\Delta S) \approx \frac{\partial S}{\partial a}std(\Delta a) + \frac{\partial S}{\partial b}std(\Delta b) + \frac{\partial S}{\partial \sigma}std(\Delta \sigma)$. If the insurance company have no idea of the change in parameters, then the insurance company may set $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and $\frac{\partial S}{\partial \sigma} = 0$

to make $E(\Delta S) \approx 0$ and $std(\Delta S) \approx 0$ to eliminate their parameter risks.

If the source of parameter risk come from estimation errors. Given that the point estimators in the process are unbiased, we could consider that $E(\Delta a) = E(\Delta b) = E(\Delta \sigma) = 0$. Thus, If the firm does not take any risk on the changes in interest rate, then the firm can keep $\frac{\partial S}{\partial r} = 0$ and make $E(\Delta S) = 0$. Assume that the changes in interest rate and parameters are independent, then the standard deviation of the change of the surplus could be approximated by $\frac{\partial S}{\partial a} std(\Delta a) + \frac{\partial S}{\partial b} std(\Delta b) + \frac{\partial S}{\partial \sigma} std(\Delta \sigma)$ at $\frac{\partial S}{\partial r} = 0$. If the firm would like to further control any risk on the changes in parameters, then the best strategy is to set $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and $\frac{\partial S}{\partial \sigma} = 0$.

Thus, if the firm does not like to take any risk on the changes in interest rate and parameters, then the best strategy is to keep $\frac{\partial S}{\partial r} = 0$ as well as $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, and $\frac{\partial S}{\partial \sigma} = 0$. Separately, it may not be difficult for managers to cope with each risk, such as $\frac{\partial S}{\partial r} = 0$ or $\frac{\partial S}{\partial b} = 0$. However, immunization strategies may conflict with each other and/or may not even be completely compatible. To integrate the immunization strategies against interest-rate risk and parameter risks, we propose using the goal-programming algorithm as follows:

$$\min_{CI(t)} \quad d \tag{10}$$

$$s.t. \quad \left|\frac{\partial S}{\partial a}\right| \le d \cdot w_a ,$$

$$\left|\frac{\partial S}{\partial b}\right| \le d \cdot w_b ,$$

$$\left|\frac{\partial S}{\partial \sigma}\right| \le d \cdot w_{\sigma} ,$$

$$\left|\frac{\partial S}{\partial r}\right| \le d \cdot w_{r} .$$

where d is the risk position the firm takes and w_a, w_b, w_{σ}, and w_r are the weights of parameter risks and interest-rate risk, respectively.

Given the insurance company's liability schedule $CO(t)$, it is worth noting that $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, $\frac{\partial S}{\partial \sigma} = 0$, and $\frac{\partial S}{\partial r} = 0$ are all linear functions of asset allocation $CI(t)$, which is the decision variable of Equation (10). Thus, management can solve Equation (10) by linear programming.

The rationale of Equation (10) is that managers make the optimal allocation of a firm's assets and liabilities to cope simultaneously with parameter risks and interest-rate risk against a firm's surplus. If the optimal solution of Equation (10) is $d^{*} = 0$, then the strategies against parameter risks and interest-rate risk are completely compatible. If the optimal solution of Equation (10) is greater than zero, then managers can also easily know how much risk they take under various risk factors.

By means of their experience and judgment, asset-liability managers can further adjust the weights between parameter risks and interest-rate risk accordingly. The smaller the value of the weight given in a risk, the stricter the immunization strategy against the underlined risk the managers intend to take.[4] For example, managers can use the strategy setting $w_r = 0$ and $w_a = w_b = w_{\sigma} = \infty$ to implement the classical immunization against interest-rate risk. Furthermore, by setting $w_r = 0$ along with the appropriate weights for other parameter risks,

[4] One way to determine the weights is to set them proportional to the standard errors of estimators.

managers not only immunize a firm's interest-rate risk but also control the firm's parameter risks. Thus, the model suggested by Equation (10) can be considered as a general model of traditional classical immunization strategy, since it includes classical immunization strategy as a special case.

Conclusions

In this paper, we proposed to use goal programming to integrate the traditional immunization strategy against interest-rate risk and the strategies against parameter risks. Since the goal programming suggested in our paper is an integrated model of immunization strategies of interest-rate risk and parameter risks, this immunization strategy includes classical immunization strategy as a special case.