# 行政院國家科學委員會補助專題研究計畫成果報告

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中文摘要

本篇論文嘗試將 Gollier (1995)關於 Central Risk Dominance 的研究成果延伸到多決策變數的問題。論文以線性函數為例，進一步研究當 Central Risk 增加時，風險趨避的決策者在多個決策變數之間的互動。本篇論文研究進一步發現延伸 Gollier (1995)的研究成果到多決策變數的更一般化的模型。

關鍵詞：風險，隨機優越

**Abstract**

It is well known that increases in an asset's "risk" (as defined by Rothschild and Stiglitz, 1970) may not possess unambiguous comparative statics of demand for a risk averse decision maker. Gollier (1995) defined the concept of "central risk" and showed that greater central risk dominance is a necessary and sufficient condition for unambiguous comparative statics of demand. However, Gollier's theorem applies only in the case of one decision variable, whereas many important economic problems involve a multivariate setting. In this paper, we provide sufficient and/or necessary conditions for unambiguous comparative statics of demand for a class of problems with two or more decision variables.

Keywords: risk, stochastic dominance

**Introduction**

Gollier (1995) identified the necessary and sufficient condition for unambiguous comparative statics for demand under transformations of the asset's distribution. This least-constraining condition is called "Greater central riskiness" in the case of a linear payoff function. Clearly, Gollier's approach can be applied to numerous economic problems with one decision variable; however, many problems involve more than one such variable. In this paper, we extend Gollier's model for linear payoff cases with two or more than two decision variables and provide sufficient and/or necessary conditions for unambiguous comparative statics of demand.

**Model**

Let $X \in [a,b]$ be a random variable with probability distribution function $F(x)$. An individual's payoff $z(\underline{\alpha}, X)$ depends on the multivariate decision vector $\underline{\alpha} = [\alpha_1, \ldots, \alpha_n]$ and the random variable $X$. Assume that $u'(\cdot) > 0$ and $u''(\cdot) < 0$, and that the decision maker maximizes the expected utility, $E[u(z(\underline{\alpha}, X))]$. The problem can then be written as

$$\underset{\underline{\alpha}}{Max} \quad EU(\underline{\alpha}; u, F, z) = \int_a^b u(z(\underline{\alpha}, x))dF(x),$$

with first-order conditions

$$\frac{\partial EU(\underline{\alpha}; u, F, z)}{\partial \alpha_i} = \int_a^b z_{\alpha_i}(\underline{\alpha}, x)u'(z(\underline{\alpha}, x))dF(x) = 0, \ \forall i \quad (1)$$

It is easy to see that Gollier's condition

(1995) may not provide unambiguous comparative statics under condition (1).

Defining $T_F^i(x)=\int_a^x z_{\alpha_i}(\underline{\alpha}_F{}^*,t)dF(t)$ and $T_G^i(x)=\int_a^x z_{\alpha_i}(\underline{\alpha}_F{}^*,t)dG(t)$, where $\underline{\alpha}_F{}^*$ denotes the decision vector satisfying (1), it follows that Gollier's condition under the transformation $F\to G$ can be expressed as

$$\exists\gamma\in R^+ \ni T_G^i(x)\geq\gamma T_F^i(x),\forall x,\forall i.$$

Following Gollier (1995), it can then be shown that

$$\frac{\partial EU(\underline{\alpha}_F{}^*;u,G,z)}{\partial\alpha_i}=\int_a^b z_{\alpha_i}(\underline{\alpha}_F{}^*,x)u'(z(\underline{\alpha}_F{}^*,x))dG(x)>0,\ \forall i \tag{2}$$

Notice that, as shown in Gollier (1995), condition (2) implies an increase in the optimal solution when there is only one decision variable. However, in the case with more than one decision variables, condition (2) does not imply that $\underline{\alpha}_G{}^*>\underline{\alpha}_F{}^*$ (componentwise).

To address this problem clearly, we focus on a particularly useful class of models, linear payoff cases. As documented by Gollier (1995), linear payoff function can be applied to standard portfolio problems, including insurance and investment as special cases. In the linear payoff cases, Gollier's condition is specifically named as central risk dominance by Gollier (1995). We assume that $z(\underline{\alpha},X)$ can be expressed as

$$z(\underline{\alpha},X)=\theta(\underline{\alpha})X-\varphi(\underline{\alpha}). \tag{3}$$

We transform the original model into a two-step optimization problem with multiple decision variables in the first step, but only one decision variable in the second step.

For the first step, we have

$$\underset{\underline{\alpha}}{Min}\quad\varphi(\underline{\alpha})$$
$$s.t.\quad\theta(\underline{\alpha})=\omega, \tag{4}$$

which yields $\underline{\alpha}^*=\underline{\alpha}^*(\omega)$, so that

$$\varphi(\underline{\alpha}^*)=\varphi(\omega)\ \text{and}$$
$$z(\omega,X)=\omega X-\varphi(\omega). \tag{5}$$

Then, in the second step, the decision maker selects the optimal $\omega$ to maximize expected utility; i.e.,

$$\underset{\omega}{Max}\quad EU(\omega;u,F,z)=\int_a^b u(z(\omega,x))dF(x). \tag{6}$$

Define $T_F^\omega(x)=\int_a^x z_\omega(\underline{\alpha}_F{}^*,t)dF(t)$.

Two-step optimization has another niche, i.e., we can extend our results to analyze a more general change in the distribution.

Defining $T_F^\omega(x)=\int_a^x z_\omega(\underline{\alpha}_F{}^*,t)dF(t)$ and $T_G^\omega(x)=\int_a^x z_\omega(\underline{\alpha}_F{}^*,t)dG(t)$. Thus, in general, we can express central risk dominance under the transformation $F\to G$ as

$$\exists\gamma\in R^+ \ni T_G^\omega(x)\geq\gamma T_F^\omega(x),\forall x.$$

That is,

$$\exists\gamma\in R^+\ni\int_a^x[t-\varphi'(\omega)]dG(t)\geq\gamma\int_a^x[t-\varphi'(\omega)]dF(t),\forall x$$

This brings us to our principal result.

**Theorem**

(1) Let $Sgn\left(\frac{\partial\alpha_i{}^*}{\partial\omega}\right)=k_i\in\{-1,0,1\}$ be consta $\forall\omega$. $\exists\gamma\in R^+\ni T_G^\omega(x)\geq\gamma T_F^\omega(x),\forall x$, then $Sgn(\alpha_{i,G}{}^*-\alpha_{i,F}{}^*)=k_i,\forall i$.

(2) If and only if $\exists\gamma\in R^+\ni T_G^\omega(x)\geq\gamma T_F^\omega(x),\forall x$, then $\theta_G^*\geq\theta_F^*$ for all risk averse individuals.

## Conclusion

In the linear payoff cases, Gollier (1995) defined the concept of "central risk" and showed that greater central risk dominance is a necessary and sufficient condition for unambiguous comparative statics of demand. However, Gollier's theorem applies only in the case of one decision variable, whereas many important economic problems involve a multivariate setting. In this paper, we provide sufficient and/or necessary conditions for unambiguous comparative statics of demand for linear payoff problems with two or more decision variables. A new approach to analyze the linear payoff case with multiple decision variables is also suggested. Although the focus of the paper is to investigate the impact of an increase in risk, the same methodology can be applied to analyze the problem of an increase in risk aversion as well as an increase in background risk. A reasonable extension of the current model is to cope with cases with non-lineal payoff functions.

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