

行政院國家科學委員會專題研究計畫成果報告

計畫編號：NSC 92-2416-H-002-034

執行期限：92年8月1日至93年7月31日

主持人：曾郁仁教授 國立台灣大學財務金融學系暨研究所

計畫參與人員：專任助理 王憶萱

國立台灣大學財務金融學系暨研究所

研究生助理 張永郎

國立台灣大學財務金融學系暨研究所

中文摘要

本文研究在多風險下一個風險趨避且風險審慎的決策者會接受較高風險的條件，論文延伸 Gollier (1995) 的結果，Gollier (1995) 討論單一風險下一個風險趨避的決策者會接受較高風險的條件，本文研究擴充為兩個風險同時針對風險趨避且風險審慎的決策者，本論文證明以邊際報酬加權的隨機優越是獲得確定比較靜態的充要條件。

關鍵字：風險趨避，風險審慎，風險優越

Abstract

This paper exams the condition which makes a risk-adverse and risk-prudent individual take less risk under a change in multiple risks. We show that marginal-payoff weighted risk dominance is a necessary and sufficient condition of unambiguous comparative statics for all risk-adverse and risk-prudent individuals. The result is different from that of Gollier (1995), who showed that marginal-payoff weighted risk dominance is the necessary and sufficient condition of unambiguous comparative statics for all risk-averse individuals under a change in a single risk.

Key words: risk averse, risk prudent, risk dominance

Introduction

After Rothschild and Stiglitz (1970,

1971) pioneered the introduction of second-order stochastic dominance to measure an increase in risk of a random variable, many other researchers began to investigate whether an increase in risk makes a risk-averse individual—as defined by Pratt (1964)—demand less risky assets (Dreze and Modigliani, 1972; Diamond and Stiglitz, 1974; Meyer and Ormiston, 1983, 1985; Black and Bulkeley, 1989; Eeckhoudt and Hansen, 1980, 1983; Dionne and Eeckhoudt, 1987; and Briys, Eeckhoudt, and Dionne, 1989).

Gollier (1995) showed that a risk-averse individual will demand less risky assets if and only if the distribution of the random variable after changing the “marginal-payoff weighted risk” dominates the original distribution of the random variable. Although this line of research has generated many valuable findings, most of them are devoted to the case where there is only one source of risk. However, many economic problems involve more than one source of risk. Hadar and Seo (1990) made the first contribution on this issue. They found the necessary and sufficient conditions for all risk-averse investors to increase their investments on a risky asset in the case of independent risky assets. Meyer and Ormiston (1994) extended their results to the case of dependent risky assets. Dionne and Gollier (1996) further showed that Linear Stochastic Dominance, proposed by Gollier (1995), is the least-constrained necessary and sufficient condition in the case of dependent risky assets. However, all of these papers assumed that the marginal cumulative distribution of one risk remains unchanged when the conditional cumulative distribution

of the other risk is altered. Thus, they just consider an increase in the risk of a random variable rather than an increase in the risk of the joint underlined distribution.

This paper intends to fill this gap. We analyze the case when an individual's payoff depends on a decision variable and two random variables. We provide a necessary and sufficient condition of unambiguous comparative statics of an increase in risk for all risk-averse and risk-prudent individuals, as defined by Kimball (1990).

Marginal-Payoff Weighted Risk Dominance

Let us assume that an individual's payoff $Z(x, y, \alpha)$ depends on not only random variables x and y but also on a decision variable α . Let us assume that $Z_x > 0$, $Z_y > 0$, and $Z_{xy} \leq 0$. The decision maker chooses an optimal α to maximize his expected utility, $E[u(Z(x, y, \alpha))]$. Let us assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $u'''(\cdot) > 0$. The model can be written as

$$\text{Max}_{\alpha} E_F[u(Z(x, y, \alpha))] = \int_{\underline{y}}^{\bar{y}} \int_{\underline{x}}^{\bar{x}} u(Z(x, y, \alpha)) f(x, y) dx dy \quad (1)$$

The first-order condition of the model¹ is

$$H_F = \int_{\underline{y}}^{\bar{y}} \int_{\underline{x}}^{\bar{x}} Z_{\alpha}(x, y, \alpha_F^*) u'(Z(x, y, \alpha_F^*)) f(x, y) dx dy = 0, \quad (2)$$

where α_F^* is the optimal solution under distribution F .

After an integration by parts,

$$\begin{aligned} H_F &= u'(Z(\bar{x}, \bar{y}, \alpha_F^*)) \tau_F(\bar{x}, \bar{y}) \\ &- \int_{\underline{x}}^{\bar{x}} u''(Z(x, \bar{y}, \alpha_F^*)) Z_x(x, \bar{y}, \alpha_F^*) \tau_F(x, \bar{y}) dx \\ &- \int_{\underline{y}}^{\bar{y}} u''(Z(\bar{x}, y, \alpha_F^*)) Z_y(\bar{x}, y, \alpha_F^*) \tau_F(\bar{x}, y) dy \\ &+ \int_{\underline{y}}^{\bar{y}} \int_{\underline{x}}^{\bar{x}} [Z_{xy}(x, y, \alpha_F^*) u''(Z(x, y, \alpha_F^*)) + \\ &Z_x(x, y, \alpha_F^*) Z_y(x, y, \alpha_F^*) u'''(Z(x, y, \alpha_F^*))] \tau_F(x, y) dx dy, \end{aligned} \quad (3)$$

where

$$\tau_F(x, y) = \int_{\underline{y}}^y \int_{\underline{x}}^x Z_{\alpha}(\alpha, t, s) f(t, s) dt ds. \quad (4)$$

Definition 1

Under two random variables, a joint cumulative distribution G is marginal-payoff weighted risk dominated by a joint cumulative distribution F if and only if

$$\exists \gamma \ni \tau_G(x, y) \leq \gamma \tau_F(x, y), \quad \forall x \in [\underline{x}, \bar{x}],$$

and $\forall y \in [\underline{y}, \bar{y}]$, where

$$\tau_G(x, y) = \int_{\underline{y}}^y \int_{\underline{x}}^x Z_{\alpha}(\alpha, t, s) g(t, s) dt ds$$

$$\text{and } \tau_F(x, y) = \int_{\underline{y}}^y \int_{\underline{x}}^x Z_{\alpha}(\alpha, t, s) f(t, s) dt ds.$$

It can be shown that the linear stochastic dominance proposed by Dionne and Gollier (1996) implies marginal-payoff weighted risk dominance in Definition 2 if the marginal cumulative distribution of one risk remains unchanged when the conditional distribution of the other risk is altered.

Theorem 1

If $\exists \gamma \ni \tau_G(x, y) \leq \gamma \tau_F(x, y)$, $\forall x \in [\underline{x}, \bar{x}]$, and $\forall y \in [\underline{y}, \bar{y}]$, then $\alpha_G^* \leq \alpha_F^*$, $\forall u' > 0, u'' < 0$, and $u''' > 0$.

When there is only one source of risk, Gollier (1995) showed that a risk-averse individual will demand less risky assets if and only if the distribution of the random variable after the change "marginal-payoff weighted risk" dominates the original distribution of the random variable. However, in the case of two sources of risk, we show in Theorem 1 that marginal-payoff weighted risk dominance is a necessary and sufficient condition for all risk-averse and risk-prudent individuals (not all risk-averse individuals) to demand less risky assets.

Conclusion

In the case of two risk sources, this

¹ Let us assume that the second-order condition of the model is satisfied to ensure an interior solution.

paper shows that marginal-payoff weighted risk, first proposed by Gollier (1995), is a necessary and sufficient condition of unambiguous comparative statics for all risk-averse and risk-prudent individuals. We also find that marginal-payoff weighted risk dominance is implied by linear stochastic dominance, as proposed by Dionne and Gollier (1996). Moreover, the result is different from the findings of Gollier (1995), who showed that marginal-payoff weighted risk dominance is the necessary and sufficient condition of unambiguous comparative statics for all risk-averse individuals in the case of one risk source. An obvious extension of this paper will be the search for the necessary and sufficient conditions of the problem with more than two sources of risk.

References

- Black, J.M., and G. Bulkeley. "A Ratio Criterion for Signing the Effects of an Increase in Uncertainty." *International Economic Review* 30 (February 1989): 119-30.
- Briys, E., G. Dionne, and L. Eeckhoudt. "More on Insurance as a Giffen Good." *Journal of Risk and Uncertainty* 2 (December 1989): 415-20.
- Diamond, P.A., and J.E. Stiglitz. "Increase in Risk and in Risk Aversion." *Journal of Economic Theory* 8 (July 1974): 333-61.
- Dionne, G., and L. Eeckhoudt. "Proportional Risk Aversion, Taxation and Labor Supply Under Uncertainty." *Journal of Economics* 47 (November 1987): 353-66.
- Dionne, G., and C. Gollier. "Comparative Statics Under Multiple Sources of Risk with Applications to Insurance Demand." *Geneva Paper on Risk and Insurance Theory* 17 (1992): 21-33.
- . "A Model of Comparative Statics for Changes in Stochastic Returns with Dependent Risky Assets." *Journal of Risk and Uncertainty* 13 (1996): 147-62.
- Dionne, G., L. Eeckhoudt, and E. Briys. "Proportional Risk Aversion and Saving Decisions Under Uncertainty." In *Risk, Information, and Insurance*, edited by Henri Louberge. Boston: Kluwer, 1990.
- Dionne, G., L. Eeckhoudt, and C. Gollier. "Increases in Risk and Linear Payoffs." *International Economic Review* 34 (1993): 309-319.
- Dreze, J., and F. Modigliani. "Consumption Decisions Under Uncertainty." *Journal of Economic Theory* 5 (1972): 308-335.
- Eeckhoudt, L., and C. Gollier. "Demand for Risky Assets and the Monotone Probability Ratio Order Stochastic Returns." *Journal of Risk and Uncertainty* 11 (1995): 147-62.
- Eeckhoudt, L., and P. Hansen. "Micro-economic Applications of Marginal Changes in Risk." *European Economic Review* 22 (July 1983): 167-76.
- . "Minimum and Maximum Prices, Uncertainty and the Theory of the Competitive Firm." *American Economic Review* 70 (December 1980): 1064-68.
- Eeckhoudt, L., and M.S. Kimball. "Background Risk, Prudence and the Demand for Insurance." In *Contributions to Insurance Economics*, edited by G. Dionne. Boston: Kluwer, 1992.
- Eeckhoudt, L., C. Gollier, and H. Schlesinger. "Changes in Background Risk and Risk-Taking Behavior." *Econometrica* 64 (1996): 683-89.
- . "Increases in Risk and Deductible Insurance." *Journal of Economic Theory* 55 (1991): 435-40.
- Gollier, C. "The Comparative Statics and Changes in Risk Revisited." *Journal of Economic Theory* 66 (1995): 522-35.
- Gollier, C., and J.W. Pratt. "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica* 64 (1996): 1109-23.
- Hadar J., and T.K. Seo. "Changes in Risk and Insurance." *Geneva Papers on Risk and Insurance Theory* 17 (December 1992): 171-79.
- . "The Effects of Shifts in a Return Distribution on Optimal Portfolios." *International Economic Review* 31

- (1990): 721-36.
- Kanbur, S.M. "Increases in Risk with Kinked Payoff Functions." *Journal of Economic Theory* 7 (1982): 219-28.
- Katz, E. "A Note on a Comparative Statics Theorem for Choice Under Risk." *Journal of Economic Theory* 25 (April 1981): 318-19.
- Kimball, M. "Precautionary Saving in the Small and in the Large." *Econometrica* 58 (1990): 53-73.
- . "Standard Risk Aversion." *Econometrica* 61 (1993): 589-611.
- Kraus, M. "A Comparative Statics Theorem for Choice Under Risk." *Journal of Economic Theory* 25 (December 1979): 510-17.
- Machnes Y. "Production Decisions in Case of Monotone Likelihood Ratio Shifts of Cumulative Distribution Functions." *Insurance: Mathematics and Economics* 13 (1993): 299-302.
- Meyer, J. "Beneficial Changes in Random Variables under Multiple Sources of Risk and Their Comparative Statics." *Geneva Papers on Risk and Insurance Theory* 17 (1992): 7-19.
- Meyer, J., and M.B. Ormiston. "The Comparative Statics of Cumulative Distribution Function Changes for the Class of Risk-Averse Agents." *Journal of Economic Theory* 31 (October 1983): 153-69.
- . "Demand for Insurance in a Portfolio Setting." *Geneva Papers on Risk and Insurance Theory* 20 (1995): 203-11.
- . "Deterministic Transformations of Random Variables and the Comparative Statics of Risk." *Journal of Risk and Uncertainty* 2 (June 1989): 179-88.
- . "The Effect on Optimal Portfolio of Changing the Return to a Risky Asset: The Case of Dependent Risky Returns." *International Economics Review* 35 (August 1994): 603-12.
- . "Strong Increases in Risk and Their Comparative Statics." *International Economics Review* 26 (June 1985): 425-37.
- Mossin, J. "Aspects of Rational Insurance Purchasing." *Journal of Political Economy* 76 (1968): 553-68.
- Ormiston, M.B., and E.E. Schlee. "Comparative Statics Under Uncertainty for a Class of Economic Agents." *Journal of Economic Theory* 61 (1993): 412-22.
- Pratt, J.W. "Risk Aversion in the Small and in the Large." *Econometrica* 32 (1964): 122-36.
- Pratt, J.W., and R.J. Zeckhauser. "Proper Risk Aversion." *Econometrica* 55 (1987): 143-54.
- Ross, S.A. "Some Stronger Measures of Risk Aversion in the Small and in the Large with Applications." *Econometrica* 3 (1981): 621-38.
- Rothschild, M., and J. Stiglitz. "Increasing Risk I: A Definition." *Journal of Economic Theory* 2 (September 1970): 225-43.
- . "Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 (June 1971): 66-84.
- Sandmo, A. "The Effects of Uncertainty of Saving Decisions." *Review of Economic Studies* 37 (July 1970): 353-60.
- . "On the Theory of the Competitive Firm Under Price Uncertainty." *American Economic Review* 61 (March 1971): 65-73.