行政院國家科學委員會專題研究計畫　成果報告

風險轉換及保險需求

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計畫主持人：曾郁仁

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中文摘要

本文研究風險轉換與保險需求之間的關係，首先本文說明如果保險是共保而且保費不會隨損失分配改變，風險轉換與保險需求之間的關係是Gollier（1995）的應用，更重要的是本文證明當保費會隨損失分配改變而改變時，Gollier（1995）的結果不能直接套用。因此，本文試圖延伸Gollier（1995）的研究，嘗試尋找當保費會隨損失分配改變而改變時風險轉換與保險需求的充分和必要條件。

關鍵字：風險轉換、保險需求、保費、共保

Abstract

Gollier (1995) first identified a necessary and sufficient condition for unambiguous comparative statics for demand under transformations of a risky asset's probability distribution. In this project, we extend Gollier's approach to study the demand for insurance, when the price of insurance is not preserved. We begin by demonstrating how Gollier's result may be applied to the case of proportional insurance with premiums preserved by the transformation of the loss random variable. Moreover, we show that Gollier's result can not be directly employed when the price of insurance is not preserved. We then try to find the necessary and sufficient condition of comparative statics for transformations of the loss distribution that may or may not preserve premiums.

Key words: Risk Transformations, Insurance Premiums, Proportional Insurance

Introduction

Rothschild and Stiglitz (1970, 1971) pioneered to study how an increase in risk affects a risk-averse decision maker's demand for a risky asset, various researchers have provided ingenious finding in this topic. Some researchers (e.g., Dreze and Modigliani, 1972; Diamond and Stiglitz, 1974; Dionne and Eeckhoudt, 1987; and Briys, Dionne, and Eeckhoudt, 1989) have found conditions on a decision maker's utility function that can generate unambiguous comparative statics with a mean-preserving transformation (MPT) of the asset's probability distribution. Others (e.g., Eeckhoudt and Hansen, 1980, 1983; Meyer and Ormiston, 1983, 1985; Black and Bulkley, 1989; and Dionne and Gollier, 1992) have found constraints on the increase in risk that provide clear prediction. An important step was taken by Gollier (1995), who first identified the necessary and sufficient condition for unambiguous comparative statics for demand under transformations of an asset's probability distribution.

Although the literature has well studied how an increase in risk affects a risk-averse decision maker's demand, relative few papers investigate this issue under state-dependent preference. In fact, many important economic topics, such as war or irreplaceable commodity, fall into this category.

In this article, we extend Gollier's approach to study how an increase in risk affects a risk-averse decision maker's demand under state-dependent preference. Furthermore, most papers in the literature assumed that the increase in risk only shifts the underlined distribution of the random variable, but the risk premium remains unchanged through the shift of the distribution. However, in reality, the risk premium could change when the underlined

distribution of the random variable changes. For example, insurance may charge more insurance premium when observing an increase in risk on the loss distribution. Thus, in this article, we assume that the increase in risk not only shifts the underlined distribution of the random variable but also changes the risk premium individual needs to pay. We then try to find the necessary and sufficient condition of comparative statics for transformations of the distribution while the risk premiums may not be preserved.

Model

Let us assume that the utility of the individual depends on a critical event with a chance of happening π. If the event does not happen (happens), the utility of the individual is $u_N(z_N)$ $(u_L(z_L))$, where z_N and z_L are the payoff in each state respectively. Let α denotes a decision variable, while x denote a random variable and follows a distribution $f(x)$, $x \in [a,b]$. Further assume that the risk premium under $f(x)$ is $p_f(\alpha)$. z_N and z_L are then defined as $z_N = y_N(\alpha) - p_f(\alpha)$ and $z_L = y_L(\alpha, x) - p_f(x)$, where $y_N(\alpha)$ and $y_L(\alpha, x)$ are the revenue individual receive when the event does not happen (happens). The decision maker maximizes his expected utility and the problem can be written as:

$$\underset{\alpha}{\text{Max}}\, H = (1-\pi)u_N(y(\alpha) - p_f(\alpha)) + \pi\int_a^b u_L(y(\alpha, x) - p_f(\alpha))f(x)dx. \tag{1}$$

The first order condition of Equation (1) can be expressed as

$$\tau_f = (1-\pi)(y'(\alpha_f^*) - p_f'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_f(\alpha_f^*)) + \pi\int_a^b (\frac{\partial y(\alpha_f^*, x)}{\partial \alpha} - p_f'(\alpha_f^*))u_L'(y(\alpha_f^*, x) - p_f(\alpha_f^*))f(x)dx = 0. \tag{2}$$

Consider that $F(.) \to G(.)$ and $p_f \to p_g$ accordingly. Assume that $p_g \geq p_f$. Therefore,

$$\tau_g = (1-\pi)(y'(\alpha_f^*) - p_g'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_g(\alpha_f^*)) + \pi\int_a^b (\frac{\partial y(\alpha_f^*, x)}{\partial \alpha} - p_g'(\alpha_f^*))u_L'(y(\alpha_f^*, x) - p_g(\alpha_f^*))g(x)dx = 0. \tag{3}$$

The necessary and sufficient condition for unambiguous comparative statics for the decision variable is then: $\alpha_g^* \leq \alpha_f^*$, if and only if $\exists \gamma \in \Re$ such that

$$\tau(p_g, G, \alpha_f^*) - \gamma\tau(p_f, F, \alpha_f^*) \leq 0. \tag{4}$$

Equation (4) holds for all individuals with $u_N(.)$ and $u_L(.)$ if and only if there exists a γ, such that

$$(y'(\alpha_f^*) - p_g'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_g(\alpha_f^*)) - \gamma(y'(\alpha_f^*) - p_f'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_f(\alpha_f^*)) \leq 0, \tag{5}$$

and

$$\int_a^b (\frac{\partial y(\alpha_f^*, x)}{\partial \alpha} - p_g'(\alpha_f^*))u_L'(y(\alpha_f^*, x) - p_g(\alpha_f^*))g(x)dx - \gamma\int_a^b (\frac{\partial y(\alpha_f^*, x)}{\partial \alpha} - p_f'(\alpha_f^*))u_L'(y(\alpha_f^*, x) - p_f(\alpha_f^*))f(x)dx \leq 0. \tag{6}$$

For simplicity, assume that $y'(\alpha_f^*) - p_f'(\alpha_f^*) < 0$. Equation (5) can be rewritten as

$$\frac{(y'(\alpha_f^*) - p_g'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_g(\alpha_f^*))}{(y'(\alpha_f^*) - p_f'(\alpha_f^*))u_N'(y(\alpha_f^*) - p_f(\alpha_f^*))} \geq \gamma. \tag{7}$$

Since $p_g \geq p_f$ and Equation (7) holds for all individuals with $u_N(.)$ with $u_N''(.) < 0$, Equation (7) can be rewritten as

$$\frac{y'(\alpha_f^*) - p_g'(\alpha_f^*)}{y'(\alpha_f^*) - p_f'(\alpha_f^*)} \geq \gamma.$$

Conclusion

Following Gollier (1995) we analyze

how an increase in risk affects a risk-averse decision maker's demand under state-dependent preference. Specifically, we study whether an increase in risk make a risk-averse individual demand more insurance when the increase in risk not only shifts the underlined distribution of the random variable but also changes the risk premium individual needs to pay. We find the necessary and sufficient condition of comparative statics for transformations of the distribution while the risk premiums may not be preserved. The contribution of our paper further extends Gollier (1995) to a more realistic world and is generally more useful.

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