

Dynamics of Trading Volume, Price, and Duration of Contractual Relationship

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Abstract

Agency theorists consider a firm as a nexus of contractual relationships. Contracting parties such as shareholders, debt holders, managers, input suppliers, advertising agencies and retailers may have information superior to outsiders concerning the firm's quality. In this paper, we show that the duration of contractual relationship within a firm has important implications on the dynamics of the price and trading volume of the firm's stock. If the duration of information asymmetry corresponds to that of contractual relationship, insiders with long-term relationship is shown to marginally prefer less aggressive strategies of trading than those who can only access superior

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information temporarily. Such behavior makes it more difficult for the market maker to infer the existence of informed traders. The market maker will henceforth provide smaller bid-ask spreads, which attract discretionary liquidity traders to gather their trades in that period.

With the long-term contractual relationship, uncertainty is resolved more slowly because the informed trader trades on his information advantage gradually. This strategy enlarges the liquidity traders' loss and discourages them from holding the firm's stock. We conclude that a firm's fundamental value will be influenced by its choice between changing partners regularly and committing to a partner. From the viewpoints of the firm and uninformed traders, changing partners regularly is always an optimal policy under the topics of market efficiency and information asymmetry.

1 Introduction

In Madhavan[1992], current trading mechanisms are categorized as two types, the order-driven type and the quote-driven one. In an order-driven mechanism, traders submit their orders without specifying any price-related actions; the market maker will aggregate market orders and set a price to clear the market. In a quote-driven one such as NASDAQ, however, each trader bases on the bid and ask prices set by the market maker to determine his orders.

The bid-ask spread has been a mainstream research topic in both the theoretic and empirical papers. The literature on this topic can be classified into two streams. Papers of the first stream discuss the relationship between the bid-ask spread and inventory cost, such as Garman[1976], Amihud and Mendelson[1980], Stoll[1978], Ho and Stoll[1981,83], Cohen, et. al.[1981], and O'Hara and Oldfield[1986]. They argue that the existence of the bid-ask spread is to avoid the market failure, reflect the market power of the monopolist specialist, and depict the excess return to compensate the exposure of the market maker in risk. They also emphasize that the bid-ask spread can bring only short-term influence on the trading

behavior in the financial market. In the long run, the market maker may adjust his cash as well as asset positions, and the true value of the asset will eventually turn out. However, these papers cannot give a persuasive and unambiguous explanation about the optimal inventory policy of the market maker.

The other stream of literature starts from the adverse-selection problem, such as Bagehot[1971], Copeland and Galai[1983], Glosten and Milgrom[1985], and Easley and O'Hara[1987]. They deem the existence of the bid-ask spread as the consequence of the information asymmetry between insiders and the market maker. Trading with insiders who have superior information causes the market maker to lose money since he will always trade on the wrong side, and therefore he has to set a bid-ask spread to ensure his breakeven.

This paper commences with a contract design within a firm, and bridges between the bid-ask spread and the duration of contractual relationship. Jensen and Meckling [1976] consider a firm as a nexus of contractual relationships, where contracting parties such as shareholders, debt holders, managers, input suppliers, advertising agencies and retailers may have information superior to outsiders concerning the firm's quality, see also Rajan [1992], Dewatripont [1994], and Villas-Boas [1994]. The duration of contractual relationship thus implies a period of time when these contracting parties hold their superior status of the firms' information.

In this paper, a firm chooses to change its counterpart regularly or to have a long-term cooperation with a specific partner. Here a long-term cooperation doesn't mean to simply choose a long-term collaborator, but to sign a long-term contract. As the firm resigns a contract with a specific partner again and again, we classify this as a short-term case. Within the duration of contractual relationship, the contracting party will have superior information with a positive probability lower than 1. If this party does not receive the private information, he will act as a liquidity trader; otherwise he will trade on his superior information before the expiration of contract.

Our model extends from the single-period framework in Easley and O'Hara [1987]. They discuss an insider's trading behavior with short-term information advantage and suggest that there exist two different equilibriums, the separating equilibrium and the pooling one. In the former equilibrium, insiders trade aggressively to exploit his private information. In the latter one, they disguise themselves by trading randomly as liquidity traders. Easley and O'Hara also claim that if this model is extended to be a multi-period one, insiders in each period will repeat their single-period optimal strategy.

While the insider possesses a long-term position of acquiring superior information, it is no longer an optimal strategy to simply maximize his profit in each period even though the acquired signal is valid for only one period. If a firm signs a long-term contract with his partner, this contracting party intentionally turns to trade randomly in the early stage of the contractual relationship, which makes the market maker more difficult to identify him, and therefore adjust downward the belief of his existence. As a result the bid-ask spread in the latter stage becomes narrower, which creates profitable room for the insider.

We have also introduced the discretionary liquidity traders¹ to discuss the dynamic interaction of the price and trading volume. We find that, if the firm signs a long-term contract, the discretionary liquidity traders will move forward to the latter stage of the current contractual relationship due to the relatively favorable prices. The moving-forward action of discretionary liquidity traders increases the market depth in the latter stage, which further enhances the incentive of insiders to camouflage the uninformed in the initial stage and

¹In present, many large-size institutions often split their trades among several markets or in round lots. These traders can choose when to trade but they shall fulfill their liquidity demand before some specific date. They are called "discretionary noise traders" referred to Admati and Pfleiderer[1988,1989], Foster and Viswanathan[1990], Seppi[1990], and Spiegel and Subramanham[1992]. Admati and Pfleiderer[1988,1989] argue that, in an order-driven market, the discretionary noise traders will aggregate their trades in a period of time under their assumption that these traders cannot split their trades, which increases the market depth of this period, and hence induces the informed traders.

results in a more likely breakdown of the separating equilibrium. On the other hand, the increment of liquidity traders also robs the trading opportunities from incumbent traders due to the quote-driven trading mechanism. Insiders may benefit from the narrow bid-ask spread, but this benefit realizes with lower probability².

We demonstrate that a firm's decision of contractual relationship will influence the market efficiency, the uninformed traders' welfare, and the firm's value. If the firm regularly changes his contractual partners, the insider will trade aggressively, reveal rapidly not only the content of information but also the fact that the contractual relationship has brought private information to the counterpart, and henceforth reduce future insiders' profit. If the firm signs a long-term contract, the market maker cannot by orders distinguish from insiders and liquidity traders, the market efficiency will be lower. Since the bid-ask spread in each stage will be affected by the duration of contractual relationship, investors' expected loss will also be affected due to their risk attitude or the timing of liquidity demand, and hence the capital collected will be changed when the firm makes financing.

Under the long-term contractual relationship, insiders' aggregate profit among all periods will be higher than the summation of short-term insiders' due to the informational externality. Since the stock turnover is a zero-sum game, the expected loss of uninformed traders will be higher under a long-term contractual relationship. Note that this conclusion conflicts the binding effect of the long-term contract pointed out by Hart and Moore [1988]. They demonstrate that when one party constructs a relationship with a specific party, the long-term contract prevents outside competitors from harming either side of them. However, we focus neither on the incomplete contract, nor on the contracting power against outsiders. From the viewpoints of the firm and the uninformed traders, changing partners regularly is always an optimal strategy under the topics of market inefficiency and information asymmetry.

²The result is different from the order-driven market. In Kyle[1985], the liquidity traders will cover the trading behavior of the insiders, and hence more liquidity traders will raise the profit of insiders.

This paper is organized as follows. In Section 2, the multi-period model is introduced. Section 3 lists some results of the single-period model already obtained in Easley and O'Hara [1987]. We derive the market equilibrium of our model in Section 4, and summarize main results in Section 5. Finally, we conclude this paper in Sec. 6. Detailed proofs are presented in the Appendix.

2 The Model

We consider an economy with 4 periods, period 0, 1, 2, and 3. There is a firm doing an investment plan whose payoff is expected in period 3. In the initial period, the firm chooses to cooperate with three different managers to execute its plan in each period, or with the same one within the first two periods and with another one in the last period³. As long as it signs contracts with its partners, the duration of contractual relationship becomes common knowledge. The contractual relationship will bring information superior to outsiders with a probability strictly lower than 1, denoted α .⁴ If information asymmetry occurs, it will last for three periods, i.e., managers in all periods will enjoy information advantage; otherwise, they know nothing and act like liquidity traders⁵. If the firm chooses to change its partners regularly, I_1, I_2 and I_3 denote these managers in periods 1, 2, and 3, respectively. If the firm hires the same manager for the first two periods, we call this person I_L .

Two assets are held: the stock and the cash. The value of the stock is $(\xi_1 + \xi_2 + \xi_3)$, where ξ_i is acquired by the informed trader at the beginning of period i and will become

³This paper only depicts hiring a manager with different duration of contractual relationship as an illustrative example. Actually, our deduction can also apply to other contracting parties.

⁴We must clarify that information advantage doesn't guarantee a manager to acquire private information; it only represents an access and the managers "may" be informed through the access.

⁵Our conclusion is still valid while extending to the multi-period economy. The existence of the last period represents that there will be management turnovers for at least one time, and there will be other accessible dates for the discretionary liquidity trader.

public at the end of this period. All traders except the informed trader neither recognize the occurrence of information nor see the content of information. Suppose that ξ_i has two equally probable outcomes, the bad one and the good one. Let L and H denote these two outcomes, and have respectively value 0 and 1 for simplicity. Hence the stock price in the initial period shall be 1/2.

There are n nondiscretionary liquidity traders in each period trading only in that period for fulfilling their liquidity demands, which consists of buying and selling with equal probabilities. Moreover, the liquidity demand will be small-quantity orders with probability ϵ and large-quantity ones with probability $1 - \epsilon$. For simplicity, the quantities of orders are assumed to be respectively one unit and two units.

The discretionary liquidity trader, called n_D , shall fulfill his liquidity demand in periods 2 and 3. If the liquidity demand is not satisfied before the expiration of period 3, there will be some cost, or say penalty, denoted C per unit of stock. We assume that the penalty is large enough for the nondiscretionary liquidity traders so that they will not give up their trading opportunities.

The market maker sets the bid-ask spread to clear the market and trades with only one person in each period. Since a liquidity trader will submit two different-sized orders, the market maker should set prices for a trader when he buys a small or large quantity (denoted B^1 and B^2), or sells a small or large quantity (denoted S^1 and S^2). All traders are risk-neutral.

For convenience we set the proportion of the informed trader to nondiscretionary liquidity traders is $\mu : 1 - \mu$. It is easy to verify that $\mu = \frac{1}{1+n}$.

3 A Review of the Single-Period Equilibrium

In this section we review the single-period model depicted by Easley and O'Hara [1987]. They demonstrate that based on the relative number of liquidity traders, there exist two

types of equilibriums, the separating one and the pooling one. In the former equilibrium, the informed trader will trade aggressively to exploit his superior information; while in the latter one, he turns to trade randomly so that he will not be identified by the market maker and henceforth face poor prices.

Proposition 1 *If $2 > \frac{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$, there exists a separating equilibrium, in which the informed trader will buy 2 units when the signal is H, and sell 2 units when the signal is L. The stock price of buying or selling a small quantity is 1/2 and that of a large quantity are described below:*

$$b(S^2) = \frac{\frac{1}{2}(1-\alpha\mu)(1-\epsilon)}{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}, a(B^2) = \frac{\alpha\mu + \frac{1}{2}(1-\alpha\mu)(1-\epsilon)}{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}.$$

The expected profit of the informed trader π_s is $\frac{(1-\alpha\mu)(1-\epsilon)}{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}$.

Making good use of his information advantage, the informed trader trades aggressively in the separating equilibrium. The market maker absorbs a buying or selling small-quantity order at price 1/2 because it is certainly submitted by a liquidity trader. The price of a large-quantity order is set according to the proportion it comes from the informed trader or from liquidity ones. The necessary condition for the separating equilibrium represents that only when the number of liquidity traders is large enough to cover the informed trader's action will he follows a pure strategy. The informed trader's expected profit while submitting a large-quantity order is larger than that while submitting a small-quantity one.

Proposition 2 *A pooling equilibrium exists when no separating one is able to sustain. In a pooling equilibrium, the informed trader is indifferent between submitting a small-quantity order and a large-quantity one. He will randomly submit a small-quantity order with probability β , and a large-quantity order with probability $1 - \beta$. The bid and ask prices are*

$$b(S^1) = \frac{\frac{1}{2}(1 - \alpha\mu)\epsilon}{\alpha\mu\beta + (1 - \alpha\mu)\epsilon}, a(B^1) = \frac{\alpha\mu\beta + \frac{1}{2}(1 - \alpha\mu)\epsilon}{\alpha\mu\beta + (1 - \alpha\mu)\epsilon},$$

$$b(S^2) = \frac{\frac{1}{2}(1 - \alpha\mu)(1 - \epsilon)}{\alpha\mu\beta + (1 - \alpha\mu)(1 - \epsilon)}, a(B^1) = \frac{\alpha\mu\beta + \frac{1}{2}(1 - \alpha\mu)(1 - \epsilon)}{\alpha\mu\beta + (1 - \alpha\mu)(1 - \epsilon)}.$$

The expected profit of the informed trader π_p is $\frac{(1-\alpha\mu)\epsilon}{\alpha\mu\beta+(1-\alpha\mu)\epsilon} = (1 - \alpha\mu)(2 - \epsilon)$.

If the number of liquidity traders is relatively small, there exists a pooling equilibrium, in which it is indifferent for the informed trader to submit a small-quantity order or a large-quantity one. The market maker will set fair bid and ask prices considering the ratio of the submitted order from the informed trader to liquidity ones.

4 Equilibrium Analysis under Different Contractual Relationships

In this section, we discuss the manager's optimal strategies under short-term contractual relationship and long-term cooperation; and evaluate if other market participants will be influenced by these different strategies.

Equilibrium Concept: PBE

First we define the PBE (perfect Bayesian equilibrium), which is a bundle of strategies and beliefs held by players. In a PBE, all participants in the market will update their information sets period by period. Thus, in each period the market maker will set an updated bid-ask spread to ensure his breakeven. I_1 and I_2 will maximize their one-period profits based on the signals they have received, and I_L maximizes his total profits, i.e., those of period 1 and period 2. Nondiscretionary liquidity traders will trade for fulfilling their liquidity demands, and the discretionary liquidity trader trades to maximize his utility in consideration of the penalty and the expected loss.

We further discuss the object functions and strategies of the players:

Market Maker

In the first period as an order is submitted, the market maker will take into consideration the proportion that this order comes from the informed trader and set a fair bid-ask spread. After the first-period trading, the market maker examines how likely this order comes up when the information event occurs, and thus updates his posterior belief of the existence of information event. In the second period the market maker does what he did in the first period.

Short-Term Informed Trader

If the information advantage lasts for only one period, the manager in each period will follow the optimal single-period strategy since his object function is the same as in Easley and O'Hara[1987]. Let α_i denote the posterior belief of the existence of the information event held by the market maker in period i . If $2 > \frac{\alpha_i\mu + (1-\alpha_i)\mu(1-\epsilon)}{(1-\alpha_i)\mu(1-\epsilon)}$, the informed trader will buy two units of stock when he receives H, and sell two units when L; otherwise, he will follow a mixed strategy and hence a pooling equilibrium comes out.

Long-Term Informed Trader

For the long-term manager the information advantage lasts for two periods, and therefore he will choose to maximize his two-period profit. The informed trader will also buy when he receives H, and sell when L. But in the first period, how many units he shall buy or sell depends on the profit in the current period plus the expected payoff in the second period. In most cases the large-quantity order brings a worse α_2 , and hence reduces his second-period profit; by submitting the small-quantity order, he collects less payoff but induces a favorable

bid-ask spread in the second period. Apparently in the first period his optimal strategy is no longer that one in Easley and O'Hara.

In the second period, given α_2 , he will act as an informed trader with single-period information advantage since there is no consequent period.

Discretionary Liquidity Trader

Since the bid-ask spread of a small-quantity order is narrower than that of a large-quantity one, the discretionary trader has incentive to split his demand into two periods. Which order he will submit in period 2 depends on the bid-ask spreads in both periods, the opportunity to trade in period 3, and the penalty he shall take while his liquidity demand is not fulfilled.

If the penalty C is large, he will speed up his trading in case he can catch more chances to trade successfully. If C is so large that its effect outweighs the expected loss originated from the bid-ask spread, he will submit a two-unit order in period 2 when he seizes the trading opportunity. Another pulling force to large-quantity trade is the bid-ask spread. If the bid-ask spread of a two-unit order in period 2 is small enough while compared to that in period 3, a large-quantity order is expected. Otherwise, if the penalty C is small or the spread of the two-unit order is large, he'd better to split his orders to two periods

4.1 The Economy Without the Presence of the Discretionary Liquidity Trader

We first consider the economy without the presence of the discretionary liquidity trader, and provide several static comparisons. In the next section we will introduce this player and see what will happen.

We present our major results by several lemmas and propositions:

Lemma 1 *If there exists a separating equilibrium in period 1, $\alpha(S^2) > \alpha(S^1), \alpha(B^2) > \alpha(B^1)$.*

Lemma 1 demonstrates that in a separating equilibrium, the probability that the information asymmetry exists will be higher when a large-quantity order is submitted. In our model, $\alpha(S^1) = \alpha(B^1), \alpha(S^2) = \alpha(B^2)$. We therefore in the following mention only the left-hand side for simplicity.

Lemma 2 *The single-period expected profit of the informed trader decreases in α .*

Intuitively, the expected profit of the informed trader will be lower if the market maker's belief of the existence of the information asymmetry.

Lemma 3 *If α increases, the pooling equilibrium will come out more likely.*

If the market maker believes that the information asymmetry exists with a higher probability, the adverse selection cost increases. Henceforth if the informed trader follows a pure strategy, his order will be priced more unfavorably. The optimal strategy for him is to disguise as a liquidity trader.

Proposition 3 *Given that the first-period equilibrium is separating, if in the first period a small-quantity order is submitted, the market maker will revise downward his belief and set a favorable bid-ask spread in the next period, which makes the separating equilibrium stand more firmly. In contrast, if the trading counterpart submits a large-quantity order in one period, the market maker will interpret that the information asymmetry exists more likely, and thus set worse prices in the next period. In such a case, the equilibrium in the next period turns out to a pooling one.*

Proposition 4 *If the firm signs a long-term contract, in the first period a pooling equilibrium will come out more likely than when it chooses a short-term one.*

This proposition points out that the threshold for a pooling to sustain under a long-term contract is strictly lower than that under a short-term one. Henceforth there exists a range that a separating equilibrium sustains under a short-term contract and a pooling equilibrium sustains under a long-term contract. In the following discussion we will focus on the innovations within this range.

Here we introduce a condition originated from Proposition 4:

Condition C:

$$\frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{(1 - \alpha\mu)(1 - \epsilon)} < 2 < \frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{(1 - \alpha\mu)(1 - \epsilon)} + \frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{1/2(1 - \alpha\mu)(1 - \epsilon)} \mu [\pi(\alpha_s(S^1)) - \pi(\alpha_s(S^2))]$$

This condition ensures that the first-period equilibriums under the short-term and long-term contractual relationship are, respectively, separating and pooling ones.

Lemma 4 *Suppose Condition C holds. $\alpha_s(S^1) < \alpha_p(S^1), \alpha_p(S^2) < \alpha_s(S^2)$.*

Lemma 4 demonstrates that if there exists a separating equilibrium in the first period, the belief of the information event held by the market maker when a large-quantity order is submitted will be higher than that as the first-period equilibrium is a pooling one; if the first-period order is a small-quantity one, the information event will be more unlikely when there is a pooling equilibrium in the first period.

Let $E_2(Y|X)$ denote the equilibrium when the equilibrium in the first period is X, and the executed order is Y, where $X \in \{S(\textit{Separating}), P(\textit{Pooling})\}$ and $Y \in \{B^1, B^2, S^1, S^2\}$.

With the help of these parameters we introduce the following proposition:

Proposition 5 *If both $E_2(S^1|S)$ and $E_2(S^2|S)$ are separating, then $E_2(S^1|P)$ and $E_2(S^2|P)$ shall also be separating. If both $E_2(S^1|S)$ and $E_2(S^2|S)$ are pooling, $E_2(S^1|P)$ and $E_2(S^2|P)$ shall be pooling, too. However, the agreement of $E_2(S^1|P)$ and $E_2(S^2|P)$ does not guarantee that $E_2(S^1|S)$ and $E_2(S^2|S)$ are of the same type. In other words, the degree of disagreement between $E_2(S^1|P)$ and $E_2(S^2|P)$ is higher than that between $E_2(S^1|S)$ and $E_2(S^2|S)$.*

Proposition 5 depicts that the trading behavior of the informed trader in period 2 will be more volatile when a separating equilibrium shows up in the first period. Since a separating equilibrium brings more information content to the market maker, the magnitude by which he adjusts his posterior belief is higher.

Proposition 6 *Suppose Condition C holds. The first-period expected profit of I_L is higher than that of I_1 .*

Proposition 6 is somewhat weird and against our intuition. The idea that long-term insider shall camouflage a liquidity trader starts from giving-up some information advantage to protect himself from being identified. Here we interpret it in another way. Other things being equal, in a separating equilibrium the ratio that a large-quantity order comes from the informed trader is higher than that in a pooling one, and hence the large-quantity price in a separating equilibrium shall be less favorable to traders. Note also that the existence of the second period provides a stronger support to a first-period pooling equilibrium because the insider's single-period profits of submitting a small-quantity order and a large-quantity one need not be the same.

Proposition 7 *Suppose Condition C holds. Given that the first-period order is submitted by the informed trader, the expected second-period profit of I_L is higher than that of I_2 .*

This proposition depicts that, if the first-period informed trader concerns only his single-period profit, his trading strategy will bring externality to his follower.

Proposition 8 *Suppose Condition C holds. Within a wide range the two-period profit of I_L is higher than the aggregate profits of I_1 and I_2 .*

Intuitively, a person maximizing the two-period profit will gain more than the one maximizing his profit period by period. This is no longer valid if other players will adjust their actions as they realize this. Although that the first-period profit of a long-term insider is

higher than that of a short-term insider, and given that the first-period order is submitted by the insider, the expected second-period profit of the long-term insider is higher, the long-term insider will be worse off if the first-period opportunity is occupied by a liquidity trader.

Lemma 5 *Suppose that a pooling equilibrium sustains in the first period. If $\beta < \frac{1}{2}, \alpha_p(S^1) < \alpha_p(S^2)$.*

Proposition 9 *Suppose that $2 < \frac{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$. Let β^* and β' denote respectively the probability with which in the first period I_1 and I_L will submit a small-quantity order. If $\beta' < \frac{1}{2}$, $\beta' > \beta^*$. Let $\alpha_p^L(Y)$ denote $\alpha_p(Y)$ if I_L exists and $\alpha_p^{1,2}(Y)$ denote $\alpha_p(Y)$ if I_1 and I_2 exist. If $\beta' < \frac{1}{2}$, $\alpha_p^L(S^1) > \alpha_p^{1,2}(S^1)$ and $\alpha_p^L(S^2) > \alpha_p^{1,2}(S^2)$.*

4.2 The Economy with the Discretionary Liquidity Trader

From now on we introduce the discretionary liquidity trader.

Proposition 10 *Suppose that the firm chooses the long-term contractual relationship. If there exist discretionary liquidity traders and their population is not too large, in the first period a pooling equilibrium will come out with a higher probability.*

The reason that a pooling equilibrium will exist more likely is that the presence of the discretionary liquidity trader brings more profit for the long-term insider in the second period, and henceforth the threshold for a pooling equilibrium to sustain is lower. The restriction on the population of the discretionary liquidity trader is due to the trading opportunity of the long-term insider in the second period.

Proposition 11 *Suppose Condition C holds. Given that the first-period order is submitted by the informed trader, in the second period more likely the discretionary liquidity traders will participate in the market when the firm chooses a long-term contractual relationship than when a short-term contract is signed.*

This proposition follows from Lemma 4. Given that the first-period order is submitted by the informed trader, the second-period price will be more favorable if the first-period equilibrium is a pooling one, and the discretionary liquidity trader will therefore be willing to participate in. The next two propositions are obvious observations.

Proposition 12 *If there exists a separating equilibrium in period 2, the discretionary liquidity trader will for certain compete the second-period trading opportunity.*

Proposition 13 *If only the discretionary liquidity traders with small-quantity demand are induced to the second period, the type of second-period equilibrium is the same as that without the existence of the discretionary liquidity traders. Conversely, the first-period equilibrium is affected by the presence of them when the firm hires a long-term manager.*

5 Discussion

With the introduction of the long-term contractual relationship, the single-period separating equilibrium in Easley and O'Hara[1987] will turn into a pooling one. The long-term insider's trading behavior will affect the market maker's belief, induce the discretionary liquidity trader to move forward, and henceforth change the equilibriums in both periods. If the bid-ask spread in the second period is favorable enough so that the discretionary traders are willing to submit large-quantity orders, the sustainability of the separating equilibrium is more strongly supported.

Due to the single-trader setup in each period, the discretionary liquidity trader will defer his order to the third period if he cannot make his deal in the second period. Roughly speaking, the traders' distribution will not vary apparently and the equilibrium in the last period will hold the same.

Although in the second period the moving-forward action of the discretionary liquidity trader will make the spread more favorable, the trading opportunity of the informed trader

will be detracted from this new participation. If such a detraction of opportunity is so severe that the expected profit of this long-term manager is worse off, he is not willing to induce the discretionary liquidity traders and hence a separating equilibrium will turn out in the first period.

We summarize our main results in the following four theorems.

Theorem 1 *The duration of contractual relationship changes the bid-ask spread.*

Under the long-term cooperation, the manager may abandon his information advantage in the early stage of contractual relationship even though this advantage is short-lived. Hence an adverse-selection problem occurs when a small-quantity order is submitted, which makes the small-quantity spread broader and the large-quantity spread becomes narrower concurrently. In the latter stage, the market maker will reduce the bid-ask spread more likely.

Theorem 2 *The duration of contractual relationship changes the firm's value.*

Considering how risk-averse they are, when their liquidity demand will occur and what prices in that moment will be, liquidity traders may adjust their expectation of their loss based upon the firm's choice, and henceforth their evaluation of the firm's stock varies. If the firm proposes to raise funds by IPO and its contracts do not coincide with investors' preferences, it has to turn its eyes on debt or other financing approaches to execute its positive-NPV plan, and eventually the firm's value will be changed.

Theorem 3 *Long-term cooperation reduces liquidity traders' welfare.*

Since the aggressive trading of the manager in the initial stage will disclose the existence of information asymmetry, the short-term contractual relationship will do injury to the manager in the latter stage, and hence their overall profit is less than that of the long-term manager. Due to the zero-sum nature, liquidity traders will on average lose more if the firm chooses a long-term contractual relationship.

Theorem 4 *Long-term cooperation harms market efficiency.*

As the long-term manager camouflages himself as a liquidity trader in the early stage, the market maker cannot tell if this contractual relationship has brought information asymmetry. Here the deficiency of market efficiency does not mean that the market is unaware of the result of investment plans but of the existence of information asymmetry.

6 Conclusion

This paper conducts the connection between the firm's contractual relationship and the capital market. We point out that the duration of contractual relationship will coincide with the period of information advantage held by the contracting party, and explain changes of equilibriums induced by the informed trader's strategic behavior. We suggest that the long-term contractual relationship of a firm will make insiders to camouflage at initial stages, induce discretionary liquidity traders to move forward, and hence reduce the market efficiency and the liquidity traders' welfare. Though many past papers have mentioned a lot of benefits brought by long-term commitment, this paper points out several possible flaws and provides persuasive explanations.

Proof of Lemma 1:

Since

$$\alpha(S^2) = \frac{\alpha\mu + \alpha(1-\mu)(1-\epsilon)}{\alpha\mu + (1-\mu)(1-\epsilon)}, \alpha(S^1) = \frac{\alpha(1-\mu)}{1-\alpha\mu},$$

subtract $\alpha(S^1)$ from $\alpha(S^2)$, we obtain that

$$\alpha(S^2) - \alpha(S^1) = \frac{(1-\alpha)(1-\alpha\mu)(1-\mu)(1-\epsilon)}{(1-\alpha\mu)[\alpha\mu + (1-\mu)(1-\epsilon)]} > 0.$$

Proof of Lemma 2:

Suppose that $\alpha_2 > \alpha_1$, and E_1 and E_2 are the corresponding equilibriums when $\alpha = \alpha_1$ and $\alpha = \alpha_2$. We now discuss this case by case.

(a) Both E_1 and E_2 are separating ones.

Since

$$\frac{\partial}{\partial \alpha} \pi_s(\alpha) = \frac{-\mu(1-\epsilon)(\alpha\mu\epsilon + 1 - \epsilon) - \mu\epsilon(1-\alpha\mu)(1-\epsilon)}{[\alpha\mu + (1-\alpha\mu)(1-\epsilon)]^2} < 0,$$

we obtain that $\pi(\alpha_2) < \pi(\alpha_1)$.

(b) Both E_1 and E_2 are pooling ones.

Since $\frac{\partial}{\partial \alpha} \pi_s(\alpha) = -\mu(2-\epsilon) < 0$, $\pi(\alpha_2) < \pi(\alpha_1)$.

(c) $(E_1, E_2) = (S, P)$.

Suppose α_3 is the critical value of which makes $\frac{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} = 2$, by the above discussion, $\pi(\alpha_2) \leq \pi(\alpha_3) \leq \pi(\alpha_1)$.

Note that (E_1, E_2) will never be (S,P) because $\alpha_2 > \alpha_1$, the lemma holds for all possible conditions.

Proof of Proposition 3:

This proposition is a combination of Lemmas 1 and 2.

Proof of Lemma 3:

Since a pooling equilibrium will exist when $2 \leq \frac{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$ and

$$\frac{\partial}{\partial \alpha} \left[\frac{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} \right] = \frac{\mu\epsilon(1-\alpha\mu)(1-\epsilon) + \mu(1-\epsilon)(1-\epsilon + \epsilon\alpha\mu)}{[(1-\alpha\mu)(1-\epsilon)]^2} > 0,$$

the lemma holds.

Proof of Proposition 4:

Suppose that a separating equilibrium exists in the first period. If the long-term manager submits a one-unit order, his total expected profit is $1 \times \frac{1}{2} + \mu\pi(\alpha(S^1))$; if instead a two-unit order is submitted, his total profit is $2 \times \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)} + \mu\pi(\alpha(S^2))$. The necessary condition for a separating equilibrium to survive is

$$2 \times \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)} + \mu\pi(\alpha(S^2)) > 1 \times \frac{1}{2} + \mu\pi(\alpha(S^1))$$

Rewrite it, we obtain

$$2 > \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} + \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}\mu[\pi(\alpha(S^1)) - \pi(\alpha(S^2))]. \quad (1)$$

By Lemma 1, $\alpha(S^2) > \alpha(S^1)$, and Lemma 2 ensures that $\pi(\alpha(S^2)) < \pi(\alpha(S^1))$. Since the second term of the right-hand side in Eq. (1) is strictly positive, if for some α the condition holds, it shall satisfy $2 > \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$. Thus the range that a separating equilibrium will sustain under the long-term contractual relationship is narrower than that under the short-term one, i.e., the pooling equilibrium will come up more likely.

Proof of Lemma 4:

It is easy to verify that

$$\alpha_p(S^1) = \frac{\alpha\mu\beta + \alpha(1-\mu)\epsilon}{\alpha\mu\beta + (1-\alpha\mu)\epsilon}, \alpha_s(S^1) = \frac{\alpha(1-\mu)}{1-\alpha\mu}.$$

By subtracting them, we obtain that

$$\alpha_p(S^1) - \alpha_s(S^1) = \frac{\alpha\mu\beta(\alpha-1)}{(1-\alpha\mu)[\alpha\mu\beta + (1-\alpha\mu)\epsilon]} < 0,$$

and henceforth $\alpha_p(S^1) > \alpha_s(S^1)$. Similarly,

$$\alpha_p(S^2) = \frac{\alpha\mu(1-\beta) + \alpha(1-\mu)(1-\epsilon)}{\alpha\mu(1-\beta) + (1-\alpha\mu)(1-\epsilon)}, \alpha_s(S^2) = \frac{\alpha\mu + \alpha(1-\alpha\mu)(1-\epsilon)}{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}.$$

and

$$\alpha_s(S^2) - \alpha_p(S^2) = \frac{\alpha\mu\beta(1-\alpha)(1-\epsilon)}{[\alpha\mu + (1-\alpha\mu)(1-\epsilon)][\alpha\mu(1-\beta) + (1-\alpha\mu)(1-\epsilon)]} > 0.$$

Thus $\alpha_p(S^2) < \alpha_s(S^2)$. The other two claims can be shown analogously.

Proof of Proposition 5:

By Lemmas 3 and 4, the proposition holds.

Proof of Proposition 6:

The expected first-period profit of I_L is

$$2\mu \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu(1-\beta) + (1-\alpha\mu)(1-\epsilon)},$$

and that of I_1 is

$$2\mu \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu + (1-\alpha\mu)(1-\epsilon)}.$$

Due to the larger denominator, the first-period profit of I_L is higher.

Proof of Proposition 7:

According to Lemma 2, the expected second-period profit of the informed trader is decreasing in α , and by Lemma 4, $\alpha_p(S^1), \alpha_p(S^2) < \alpha_s(S^2)$. The expected profit of I_2 is

$$\pi(\alpha_s(S^2)) = \beta\pi(\alpha_s(S^2)) + (1-\beta)\pi(\alpha_s(S^2)) < \beta\pi(\alpha_p(S^1)) + (1-\beta)\pi(\alpha_p(S^2)),$$

where the right-hand side of the inequality is the expected profit of I_L .

Proof of Proposition 8:

The expected two-period profit of the informed traders is

$$\mu^2(\pi_1 + E[\pi_2|1_I]) + \mu(1 - \mu)\pi_1 + (1 - \mu)\mu E[\pi_2|1_N], \quad (2)$$

where $E[\pi_2|1_I], E[\pi_2|1_N]$ represent the expected second-period profit given that the first-period order is submitted by, respectively, an informed trader and a liquidity trader. By Propositions 5 and 6, the first two terms of I_L are higher than that of I_1 and I_2 . $E[\pi_2|1_N]$ can be explicitly expressed as $\epsilon\pi(\alpha(S^1)) + (1 - \epsilon)\pi(\alpha(S^2))$, and we have obtained that $\pi(\alpha_s(S^1)) > \pi(\alpha_p(S^1))$. Unless $(1 - \mu)\mu(1 - \epsilon)\pi(\alpha(S^2))$ outweighs all the other terms in Eq.(2), the expected profit of I_L will be higher.

Proof of Lemma 5:

It is easy to verify that

$$\alpha_p(S^2) - \alpha_p(S^1) = \frac{\alpha\mu(1 - 2\beta)(1 - \alpha)(1 - \epsilon)}{[\alpha\mu\beta + (1 - \alpha\mu)\epsilon][\alpha\mu(1 - \beta) + (1 - \alpha\mu)(1 - \epsilon)]},$$

and the lemma follows directly.

Proof of Proposition 9:

First note that if $2 < \frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{(1 - \alpha\mu)(1 - \epsilon)}$, the first-period equilibrium under the short-term contractual relationship will be a pooling one. According to Proposition 4, I_L will also randomize his order in the first period, and his incentive compatibility condition is as follows:

$$2 \times \frac{1/2(1 - \alpha\mu)(1 - \epsilon)}{\alpha\mu(1 - \beta) + (1 - \alpha\mu)(1 - \epsilon)} + \mu\pi(\alpha_p(S^2)) > 1 \times \frac{1/2(1 - \alpha\mu)\epsilon}{\alpha\mu\beta + (1 - \alpha\mu)\epsilon} + \mu\pi(\alpha_p(S^1)).$$

By the continuity of the above equation, there exists a solution β' . Since $\beta' < \frac{1}{2}$, $\pi(\alpha_p(S^2)) < \pi(\alpha_p(S^1))$ by Lemma 5. One can easily verify that $\beta' > \beta^*$. The last claim follows directly.

Proof of Proposition 10:

Since $\alpha_s(S^1) < \alpha_s(S^2)$, by Lemma 2 the price under $\alpha_s(S^1)$ is strictly preferred. Discretionary liquidity traders will therefore show up in the second period more likely when the order executed in the first period is a small-quantity one. In other words, the second-period profit of informed trader is higher than that without the discretionary liquidity traders. Moreover, the expected profit difference $\pi(\alpha_s(S^1)) - \pi(\alpha_s(S^2))$ is higher. Thus Eq.(1) holds with a higher probability. For I_L the incentive to randomize his order is strengthened by the discretionary liquidity traders if they do not share the trading opportunity significantly. If the population of the discretionary liquidity trader is large, leading them to the second-period market will reduce the informed trader's trading opportunity.

Proof of Proposition 11:

By Lemma 4, $\alpha_p(S^1), \alpha_p(S^2) < \alpha_s(S^2)$. Therefore the bid-ask spreads in the second period under $\alpha_p(S^1)$ and $\alpha_p(S^2)$ are strictly favorable to traders than that under $\alpha_s(S^2)$. Given that the first-period order is submitted by the informed trader, $\alpha_2 = \alpha_p(S^1)$ or $\alpha_2 = \alpha_p(S^2)$ when the first-period equilibrium is pooling and $\alpha_2 = \alpha_s(S^2)$ when it is separating. The proposition follows immediately.

Proof of Proposition 12:

To verify that a discretionary liquidity trader with small-quantity demand will participate is easy, and therefore we consider that with two-unit demand. Suppose that the discretionary liquidity trader succeeds to trade with the market maker in period 2. If he submits a one-unit order, there is no expected loss during this round of trade and he still holds the opportunity to trade in the next period. Such a strategy is strictly preferred to waiting for the last chance and hence he will certainly compete in period 2.

Proof of Proposition 13:

It is easy to verify that the necessary condition of the existence of the second-period separating equilibrium is still

$$2 \times \frac{1/2(1 - \alpha_2\mu)(1 - \epsilon)}{\alpha_2\mu + (1 - \alpha_2\mu)(1 - \epsilon)} > 1 \times \frac{1}{2},$$

irrelevant to the presence of the discretionary liquidity traders. However, if the second-period equilibrium is pooling, both the small-quantity and large-quantity spreads are different from those without the presence of them, and therefore the expected second-period profit of the informed trader will vary, on which the criterion of the first-period equilibrium depends.

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