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Optimal Taxi Market Control Operated with a Flexible Initial Fare Policy

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Abstract – *It is important issue that makes the commercial vehicle operations (CVO) system be operated efficiently and social resource be allocated optimally of. Particularly for taxi business, keep the vacancy rate of taxi riding under an optimal level is a main object of the taxi authority board. The flexible pricing mechanism is a useful tool to achieving the presented object. To match market demand and to hold on an optimal vacancy rate, this study establishes a stochastic optimal control law with a flexible initial Fare for a taxi riding. For verification and evaluation, six scenarios have been conducted for simulation. The result shows that the proposed model can work satisfyingly under every circumstance.*

Keywords: stochastic optimal control, CVO, taxi market, flexible pricing.

1 Introduction

"Commercial vehicle operations" (CVO) is one of important subsystems of ITS, see Figure 1. In general, commercial vehicles include taxi, bus, truck, and emergency cars, etc.

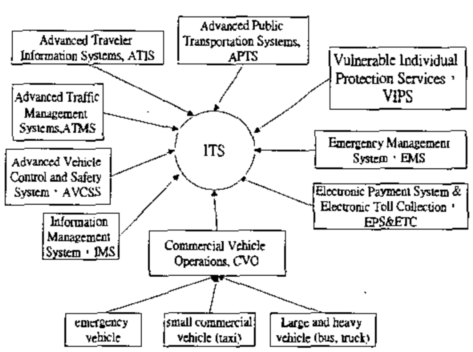

Figure 1. The subsystems of ITS
(Source: www.its-taiwan.org.tw/)

With ITS technologies, commercial vehicle operators can improve efficiency, productivity and safety; and also reduce operating cost. In a case, utilizing the global positioning system (GPS) and wireless communication for vehicle dispatching has become a useful management tool for taxi operators.

For a public policy researcher, how to control the vacancy rate under an ideal level, further achieving an optimal allocation of social resources ("Pareto-efficiency") is another key issue. *Flexible pricing* may be a feasible solution for this issue. Through the market operation, a flexible pricing mechanism is able to adjust the price to match market demand. Further more, such a mechanism can keep the vacancy rate under the ideal level.

Taxi is the most important paratransit in Taiwan urban area [1]. With the properties of privacy, conveniency, speediness, accessibility and decency, taxi has become a part of modern life of Taiwanese. The most common operation of a taxi in Taipei is cruising randomly around the city to search passengers. Since the taxi amount rapidly, it often causes traffic jam.

Because entering into the taxi market is rather easy, the vacancy rate of taxi riding is pretty high in Taipei. Up to the end of 2001, the amount of taxies has reached 107,527 vehicles in Taiwan. According to the survey of Chou[2], the vacancy rate of taxi riding is around 52% in Taipei city. Due to such a highly vacancy rate, what is the optimal vacancy rate and how to keep it under the optimal level through the pricing mechanism have become an interest topic of transportation academic.

Fairly with consideration of the rights of customers and operators simultaneously, the authority should deregulates the amount and pricing for taxi. However, the weakness of current pricing policy cannot automatically reflect the variety of market demands. According to the Motor Carrier Regulations of Fare for Passengers and Cargos, the fare of taxi is reviewed every 2 years. Due to the inflexibility of current pricing policy, some researches [3,4] presented the concept of a *flexible initial fare mechanism*. Through such a mechanism, the taxi market can approach a stable equilibrium and keep the vacancy rate of taxi riding under the ideal level.

In Taiwan, the taxi fare structure can be divided into two categories, namely initial fare and stepping price. "Initial fare" is a minimum charge for each riding, which is independent from riding mileages. "Stepping price" is composed with two components: distance rate and detention rate, which are counted in terms of riding mileages and detention time respectively. Recalling to current taxi fare structure in Taipei area, the initial fare is set at NT$70(US$2.00), for a ride less than 1.5 km, beyond 1.5km, the distance rate is NT$5.00per 300 meters (US$0.14 /0.3km) and the detention rate is NT$5.00 per 2min (US$0.14/2min).

The fare is the most important effective factor of vacancy rate [5]. Passengers have more sensitive to "initial fare" then "stepping fare". As the initial fare is changed, a passengers can alter his/her decision of take a taxi or not immediately. However, when the stepping fare changed, a passengers must estimate his/her traveling distance and detention time to counting how much payment will be, before that the traveler decides to take a taxi or not. Accordingly, the initial fare is more suitable to be a control variable than the stepping fare.

Based on the demand theory with a flexible initial fare policy, this paper aims to derive an optimal control model for the taxi operational market. For verification and evaluation purposes, six scenarios have been conducted for simulation. The result shows that the optimal control model can work satisfyingly under every circumstance.

2 Literature Review

For taxicab business, Douglas [6] discussed the effect of freight level upon different market segment. In Douglas' study, the demand is a function of price and service quality, where the term of service quality is evaluated by waiting time.

Beesley and Glaister [7] set the demand function as a function of price and waiting time. In their study, they focused on the variation of demand and waiting time when freight changed.

Chang and Sun [3] indicated that, current fixed initial taxi fare exists unfair between short and long distance travelers as well as between peak and off-peak travelers. In their conclusion, the concept of flexible initial fare was proposed. To reflect the cost of carriers and the demand of passengers and further to achieve the "Pareto-efficiency", the study suggested the authority board setting a flexible range of initial fare. Then, one carrier can decide his/her initial fare on account of demand at that moment within the regular range and show the fare via a displayer equipped on top of the taxicab.

Chang [4] studied the taxi service market and the pricing of taxi services for urban areas. His paper suggested that pricing of taxi service should be oriented toward maximizing the social benefits. The study reveals that the demand elasticity of taxi services exceed 1.0 and the average cost function of providing taxi service is steadily decreasing. It is also estimated that by using second-best theory to price taxi service, the vacancy rate of taxis can be reduced from 54% to 32%. To facilitate such an improvement, the study suggested that the government should revise the regulations allowing the initial fare of a taxi riding vary between NT$30 (US$0.85) and NT$50 (US$1.43).

Chang and Tu [8] developed time-based taxi fare structure. In this fare structure, cost components of taxis are divided into three categories, namely fixed cost, variable cost, and mixed cost, based on their correlation to taxi travel time and distance.

Chang and Huang [5] developed a taxi demand and cost function. The optimal fare and vacancy rate are simultaneously determined for the maximum social welfare objective with a break-even constraint. The study indicated that, vacancy rate is one of the service quality attributes for taxi and affects the quantity of demand and has great influence on operating cost and pricing. With the assumption that the demand function is the form of Cobb-Douglas, it is found that the optimal vacancy rate is the function of price elasticity, waiting time elasticity, and the sensitivity of vacant mileage with respect to waiting time. It is also found that price elasticity is the most effective factor affecting vacancy rate and operating cost. The optimal vacancy rate is concluded at around 33%.

3 Derivation of the Optimal Control Model

This study aims to control the vacancy rate under an optimal level. Because the fare is the most important effective factor of vacancy rate, the initial fare of taxi riding is chosen as the control variable in this study. Through the flexible pricing mechanism, the real vacancy rate will finally automatically turn to an optimal level.

3.1 Assumptions

For simplifying the problem, several assumptions are pointed as follows:

- The total supply mileage of taxi in the market is changeless; even the initial fare is opened for change arbitrarily.
- The operation cost has been considered in the optimal vacancy rate. The initial fare will directly affect the market phenomenon.
- The waiting time is fixed as fare changed, so that the fare is the only chosen variable of demand function.

3.2 Dynamic demand model formulation

As the previous indication, the fare structure of taxi in Taipei has two portions, namely initial fare and stepping fare. The average total fare in k^{th} stage is denoted as follows:

$$P_k = P_k^f + \frac{d_k - E}{N_d} P_k^d + \frac{t_k}{N_t} P_k^t \quad (1)$$

In which,

d_k is the averaged occupancy distances (km) per ride in k^{th} timing stage;

t_k is the averaged detention time(hr) per ride in k^{th} timing stage;

P_k^f is the initial fare in k^{th} timing stage;

P_k^d is the stepping distance rate in k^{th} timing stage;

P_k^t is the detention rate in k^{th} timing stage;

E is the basis of distance for the initial fare;

N_d is the length of distance for each unit of distance charge;

Nt is the length of distance for a unit of detention charge.

In the researches of Douglas [6], Beesley and Glaister [7], and Chang and Huang [5], the price elasticity of demand is fixed and the demand function is developed as the Cobb-Douglas form. Hence, such price and demand function are applied in this study. Supposing that the quantity of demand in $(k+1)^{th}$ stage is affected by k^{th} pricing, the demand function in $(k+1)^{th}$ stage can be written as following:

$$Q_{k+1} = \alpha_k P_k^{\beta_k} \quad (2)$$

Where, Q_{k+1} is the average occupancy distance (km) in $(k+1)^{th}$ stage;

P_k is the average fare in k^{th} stage;

α_k is a parameter in k^{th} stage;

β_k is the price elasticity of demand which is another parameter in k^{th} stage;

Since the initial price is the chosen control variable, let $\Phi_k = \frac{d_k - E}{N_d} P_k^d + \frac{t_k}{N_t} P_k^t$, eq. (1) can be simplified then:

$$P_k = P_k^f + \Phi_k \quad (3)$$

Substituting (3) into (2), the demand function becomes:

$$Q_{k+1} = \alpha_k (P_k^f + \Phi_k)^{\beta_k} \quad (4)$$

Take the logarithm and differential with respect to the control variable P_k^f, Eq. (4) implies that:

$$\frac{\partial \ln Q_{k+1}}{\partial P_k^f} = \frac{\beta_k}{P_k^f + \Phi_k} \quad (5)$$

3.3 Stochastic flexible initial fare control model

When P_{k-1} approaches to P_k, Q_k will get closed to Q_{k+1}. Rewrite the discrete form of (5) and substitute (4) into it; equation (6) can be obtained.

$$\frac{Q_{k+1} - Q_k}{Q_{k+1}} = \frac{\beta_k (P_k^f - P_{k-1}^f)}{P_k^f + \Phi_k}$$

$$\Rightarrow Q_{k+1} - Q_k = \alpha_k \beta_k (P_k^f + \Phi_k)^{\beta_k - 1} (P_k^f - P_{k-1}^f) \quad (6)$$

Equation (6) obviously is a nonlinear equation. Take the 1st Taylor expansion from Equation (6) at center of $[P_k^f \; P_{k-1}^f \; \alpha_k \; \beta_k \; \Phi_k]=[0 \; 0 \; \alpha_0 \; \beta_0 \; \Phi_0]$, equation (6) becomes as follows.

$$\begin{aligned} Q_{k+1} - Q_k = (P_k^f - P_0^f)\{[\alpha_k\beta_k(\beta_k - 1)(P_k^f + \Phi_k)^{\beta_k - 2}(P_{k+1}^f - P_k^f) \\ + \alpha_k\beta_k(P_k^f + \Phi_k)^{\beta_k - 1}\}_{[0 \; 0 \; \alpha_0 \; \beta_0 \; \Phi_0]}\} \\ - (P_{k-1}^f - P_0^f)\{\alpha_k\beta_k(P_k^f + \Phi_k)^{\beta_k - 1}\}_{[0 \; 0 \; \alpha_0 \; \beta_0 \; \Phi_0]}\} \end{aligned}$$

$$\Rightarrow Q_{k+1} - Q_k = \alpha_0\beta_0\Phi_0^{\beta_0 - 1}(P_k^f - P_{k-1}^f) \quad (7)$$

If the random error is considered, eq. (7) can be rewritten as:

$$Q_{k+1} - Q_k = \alpha_0\beta_0\Phi_0^{\beta_0 - 1}(P_k^f + P_{k-1}^f) + \varepsilon_{k+1} + \varepsilon_k$$

$$\Rightarrow (1 - Z^{-1})Q_{k+1} = \alpha_0\beta_0\Phi_0^{\beta_0 - 1}(1 - Z^{-1})P_k^f + (1 + Z^{-1})\varepsilon_{k+1}$$

$$\Rightarrow AQ_{k+1} = BP_k^f + C\varepsilon_{k+1} \quad (8)$$

Where, $A = 1 - Z^{-1}$;

$B = \alpha_0\beta_0\Phi_0^{\beta_0 - 1}(1 - Z^{-1})$;

$C = 1 + Z^{-1}$;

ε_k and ε_{k+1} is a white noise in k^{th} and $(k+1)^{th}$ stage respectively.

Eq.(8) is the state equation in this study.

Next we will formulate the optimal occupancy mileage. Define the k^{th} vacancy rate as the ratio of demand mileage and supply mileages in k^{th} stage. The optimal occupancy mileage can be derived as formula (9).

$$R_{k+1}=\frac{L-Q_{k+1}}{L}=1-\frac{Q_{k+1}}{L}=R^*_{k+1}$$

$$\Rightarrow Q^*_{k+1}=L(1-R^*_{k+1}) \quad (9)$$

Where, R_{k+1} is the reveal vacancy rate in $(k+1)^{th}$ stage;
R^*_{k+1} is the optimal vacancy rate in $(k+1)^{th}$ stage;
L is the supply mileage, which is assumed constant;
Q^*_{k+1} is the optimal occupancy distance (km) in $(k+1)^{th}$ stage.

Hence, the performance function can be expressed as Eq. (10).

$$J=E[(Q_{k+1}-Q^*_{k+1})^2] \quad (10)$$

3.4 The control law

The control object is to minimize the performance function. If we make a decomposition as the following identity:

$$A^{-1}C=F+Z^{-1}A^{-1}G \quad (11)$$

Where, F is the quotient of $\frac{C}{A}$, and G is the remainder.

We can rewrite (8) as equation (12).

$$\begin{aligned}Q_{k+1}&=A^{-1}BP^f_k+A^{-1}C\varepsilon_{k+1}\\&=A^{-1}BP^f_k+(F+Z^{-1}A^{-1}G)\varepsilon_{k+1}\\&=A^{-1}BP^f_k+F\varepsilon_{k+1}+A^{-1}G\varepsilon_k\end{aligned} \quad (12)$$

Besides, we may obtain the ε_k from equation (8).

$$\varepsilon_k=C^{-1}(AQ_k-Z^{-1}BP^f_k) \quad (13)$$

Substituting (13) into (12), it yield:

$$\begin{aligned}Q_{k+1}&=A^{-1}BP^f_k+F\varepsilon_{k+1}+A^{-1}GC^{-1}(AQ_k-Z^{-1}BP^f_k)\\&=F\varepsilon_{k+1}+A^{-1}GC^{-1}AQ_k+A^{-1}(1-Z^{-1}A^{-1}GC^{-1})BP^f_k\end{aligned} \quad (14)$$

Through the identity (11), $1-Z^{-1}GC^{-1}=AFC^{-1}$, Eq. (14), becomes:

$$Q_{k+1}=F\varepsilon_{k+1}+A^{-1}GC^{-1}AQ_k+FC^{-1}BP^f_k \quad (15)$$

Substituting (15) into (10), we have:

$$\begin{aligned}J&=E[(Q_{k+1}-Q^*_{k+1})]^2\\&=E[F\varepsilon_{k+1}+A^{-1}GC^{-1}AQ_k+FC^{-1}BP^f_k-Q^*_{k+1}]^2\end{aligned} \quad (16)$$

Supposing ε_{k+1} is independent to Q_k, Q_{k-1} and P^f_k, P^f_{k-1}, Eq. (16) implies

$$J=E[(F\varepsilon_{k+1})^2]+E[(A^{-1}GC^{-1}AQ_k+FC^{-1}BP^f_k-Q^*_{k+1})^2]$$

The minimization condition of above equation is $A^{-1}GC^{-1}AQ_k+FC^{-1}BP^f_k=Q^*_{k+1}$. Hence, the optimal control law is obtained.

$$(P^f_k)^*=-\frac{G}{FB}Q_k+\frac{C}{FB}Q^*_{k+1}$$

Moreover,

$$\frac{C}{A}=\frac{1+Z^{-1}}{1-Z^{-1}}=1+Z^{-1}\frac{2}{1-Z^{-1}}$$

This leads to F=1 and G=2. Substituting them into (17), then

$$\begin{aligned}(P^f_k)^*&=-\frac{G}{FB}Q_k+\frac{C}{FB}Q^*_{k+1}\\&=-\frac{2}{\alpha_0\beta_0\Phi_0^{\beta_0-1}(1-Z^{-1})}Q_{k+1}+\frac{1+Z^{-1}}{\alpha_0\beta_0\Phi_0^{\beta_0-1}(1+Z^{-1})}Q^*_{k+1}\\&=\frac{(1+Z^{-1})Q^*_{k+1}-2Q_k}{\alpha_0\beta_0\Phi_0^{\beta_0-1}(1-Z^{-1})}\end{aligned}$$

$$\Rightarrow (P^f_k)^*=\frac{Q^*_{k+1}+Q^*_k-2Q_k}{\alpha_0\beta_0\Phi_0^{\beta_0-1}}+(P^f_{k-1})^* \quad (18)$$

4 Model Verification and Evaluation

This study set a rational demand function on the basis of related papers [5,9]. The model is verified and evaluated by a simulation approach. Six scenarios are simulated herein:

(1) The parameter α_k, the price elasticity of demand β_k, average occupancy distance per ride d_k and average detention time per ride (t_k) are fixed, at the values of [2839.61, -1.4, 4.61, 4.36].
(2) Almost the same as scenario (1), but the price elasticity of demand β_k varies between (±0.05) with random.
(3) Similar to scenario (2), but the parameter α_k is altered in the range of (±5%) randomly.

(4) Similar to scenario (3), but the average occupancy distance per ride d_k is randomized within $\pm 10\%$.
(5) Similar to scenario (4), but the average delay time per ride t_k varies between $\pm 10\%$ with random.
(6) Similar to scenario (5), but the ideal vacancy rate (R^{*}_{k+1}) is randomized within the range of $\pm 10\%$.

Each scenario is simulated for 200 stages.

According to Chang and Huang [5], the taxi market in Taipei city can be briefly illustrated as Table 1.

Table 1. Taxi market in Taipei city [5]

Items	Years					
	1984	1987	1991	1995	1997	2000
Supply mileage (km/day/vehicle)	102.56	103.13	84.63	106.99	91.95	84.3
vacancy mileage (km/day/vehicle)	100.9	86.26	99.34	52.03	90.26	95.13
Vacancy rate (%)	49.3	45	54	32.06	48.69	52.48
Revenue (NT$/day)	1391	1498	1629	2218	2251	2017
Waiting time(min)	1	—	—	4.45	—	2.18
The number ofregistered taxi (veh)	48629	57191	62102	62426	69193	69106

Chang and Huang [5], had formulated the demand function as bellows:

$$Q = (7.2\times 10^{9})P^{-1.4}w^{-0.2} \qquad (19)$$

Where, Q is occupancy distance per day (km);
P is fare (NT$);
w is waiting time (min).

Supposing that the initial fare is adjusted every 20 minutes. According to the assumption, the waiting time is fixed. We can substitute the average value (2.18 min) from Table 1 into (19). Recalling to the related survey [9], the operation time of taxi is in general between 9.5 hours and 9.8 hours in Taipei metropolitan area. Substitute the average value of operation time (9.6 hours) into (19), we have:

$$q_{k+1} = 2839.61P_k^{-1.4} \qquad (20)$$

Where, q_{k+1} is the occupancy mileage per 20 min (km).

According to current fare structure, the initial fare is NT$70 of first 1.5km, the stepping distance rate is NT$5/300m, and the detention rate is NT$5/2 min. Hence, the total fare of a ride in a taxi tour can be expressed as equation (21).

$$P = 70 + \frac{(d-1.5)\times 5}{0.3} + \frac{t\times 5}{2} \qquad (21)$$

The study of Chang and Huang [5] found that on the circumstances of the price elasticity of demand of 1.4 and the cost per vehicle kilometer of NT$19.63, the optimal vacancy rate is 33.3%. Substitute the value and the data of 2000 year in Table 1 into equation (9), the optimal occupancy mileage is presented of 4.13 km per 20 min.

Figure 2 illustrates the simulation results in scenario 1. As shown in Figure 2, the current initial fare steadily converges to the optimum value NT$43.6 at first 1.5km (US$1.25/km) of a ride.

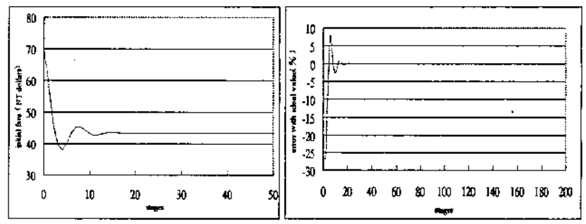

Figure 2. The simulation result of scenario1

Figure 3 depicts the convergent process for different given fares. It shows that this optimal control model can converge to the same optimum level within around 15 stages. This result implies that suppose operators adjust the initial fare every day then the market stable can be reached after 15 days later. This concludes that the initial fare can be deregulated to open status. Such a policy will make the taxi market convergent, not widely divergent.

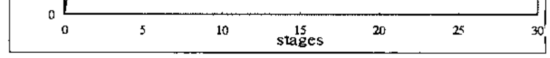

Figure 3. The convergent process for different initial solution

Figure 4 to Figure 8 depict that whatever the viration of market circumstance occurs, the optimum initial fare can be obtained efficiently by this model. The simulation results also indicate that the probability of error between real and ideal occupancy mileage less then 10% is in the range of 0.75-0.8. This means that the model can work satisfyingly.

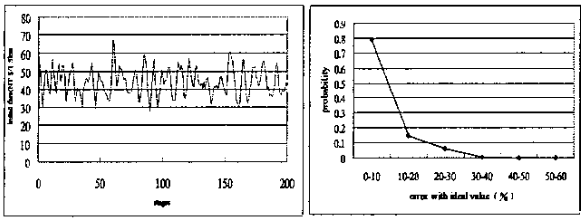

Figure 4. The simulation result of scenario 2

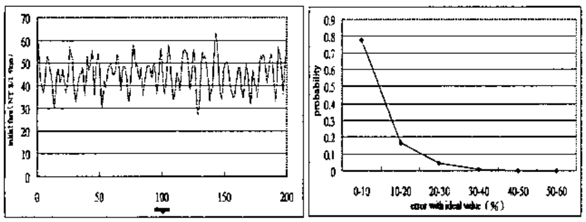

Figure 5. The simulation result of scenario 3

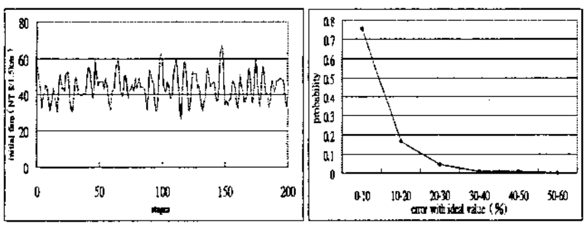

Figure 6. The simulation result of scenario 4

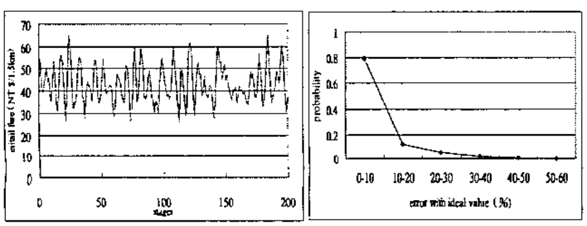

Figure 7. The simulation result of scenario 5

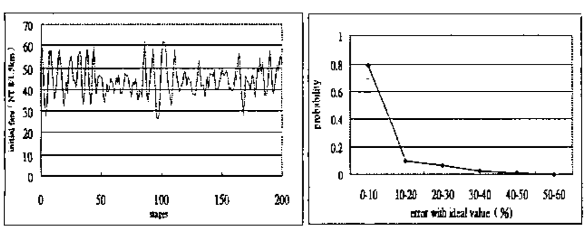

Figure 8. The simulation result of scenario 6

5 Concluding Remarks

This study aims to control the vacancy rate of taxi operation under the optimal level through the flexible initial fare mechanism. The some conclusions are summarized.

- Based on the demand theory, the optimal control law presented as following:

$$(P_k^f)^* = \frac{Q_{k+1}^* + Q_k^* - 2Q_k}{\alpha_0\beta_0\Phi_0^{\beta_0-1}} + (P_{k-1}^f)^*$$

- The comparative static analysis as shown in Figure 2 indecates that on the circumstance of the price elasticity of demand of 1.4 and the optimal vacancy rate of 33.3%, the minimum fare converges to the optimum value of NT$43.6 at first 1.5km(US$1.25/1.5km).

- For static circumstance, this optimal control model can converges to the same optimum level for arbitrary initial fare. This concludes that the initial fare can be deregulated to open status such a policy will make the taxi market convergent, not widely divergent.

- The control model and policy can work satisfyingly.

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