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Application of spectral analysis to determine hydraulic diffusivity of a sandy aquifer (Pingtung County, Taiwan)

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Abstract:

The study demonstrates spectral relationships in the time–frequency domain for one-dimensional groundwater flow in aquifers bounded by fluctuating boundaries. By nature, the solutions of spectral equations are non-linear complex functions. To determine hydraulic diffusivity in the governing equations, it is required that the data are collected from the spectra of water levels at the fluctuating boundaries and observation wells. Hydraulic diffusivity thus can be obtained by an iterative inverse approach. This paper presents an application in Pingtung County of Taiwan to determine the hydraulic diffusivity of a sandy aquifer under confined conditions. Spectral density function of water level obtained from tidal boundaries and observation wells are used to approximate hydraulic diffusivity, which yields an averaged value of $1{\cdot}26 \times 10^6$ m^2/h. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS hydraulic diffusivity; groundwater; spectral analysis; tide

INTRODUCTION

The conventional method to estimate hydraulic parameters, such as transmissivity and storage coefficient, is often from aquifer testing, which is very costly and it is difficult to interpret the accurate transmission of groundwater flow in an aquifer system when dealing with a large number of wells in a large area. Conventional tests and methods for finding aquifer parameters can be found in the literatures (Bear, 1979; Freeze, 1979; Driscoll, 1986; Batu, 1998). For aquifers adjacent to an ocean, river or lake the groundwater table fluctuates with the change of water level in the nearby surface water (Godin, 1972, p. 9; Shih *et al.*, 1999, 2000; Shih, 1999b, 2002; Shih and Lin, 2002). Such a disturbed groundwater table can be expressed mathematically providing relevant boundary conditions, which can be analysed by the spectral relationships through time–frequency domain of tidal effects. With this approach, the spectral characteristics of field data are used to solve for hydraulic diffusivity as the ratio of transmissivity and storage coefficient (Shih, 1999a,b). Therefore, it is an alternative approach to the study of aquifer parameters for a large area by using data from relevant boundary conditions and water level fluctuation in observation wells.

The water level data obtained from five river stages and seven groundwater wells in the Taipei Basin have been analysed by spectral analysis in time–frequency domain (Shih *et al.*, 1999). The diurnal, semi-diurnal and quarter-diurnal tidal components of the Tanshui River appeared to be closely related to tides of K_1, M_2 and M_4, respectively. The diurnal tide, K_1, is a luni-solar diurnal constituent. This constituent, with O_1 and P_1, expresses the effect of the declination of the Moon and Sun (Godin, 1972). Its speed equals to 15·0410686° per solar hour. The O_1 tide is a lunar diurnal constituent whereas P_1 is the solar diurnal component. The speed of O_1 and P_1 are 13·9430356° and 14·9589314° per solar hour, respectively. The semi-diurnal tide M_2 is a principal lunar semi-diurnal constituent. This constituent represents the rotation of Earth with respect

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to the Moon. Its speed equals 28·9841042° per solar hour. The quarter-diurnal tide M_4 is a shallow water overtide of principal lunar constituent. Its speed equals to 57·9682084° per solar hour. The water level data obtained from 16 groundwater wells and two seawater-gauging stations, coupled with atmospheric pressure measurements in an alluvial plain in the central-west region of Taiwan, are also studied by spectral analysis in the time–frequency domain. The semi-diurnal tidal component is observed to be the most noticeable signal, and the diurnal component was the next distinct signal recorded at water-level stations. The spectral analysis indicates that the water level adjusted from atmospheric pressure is almost in phase and retained almost the same amplitude in the area. This implies that the effect of atmospheric pressure variations is not significant on nearby seawater and groundwater-level fluctuations; the astronomical tidal components were the main factor causing fluctuation of seawater and groundwater levels in the Choshuihsi alluvial plain (Shih *et al.*, 2000). Gelhar (1974) derives a mathematical expression for a one-dimensional horizontal flow in terms of water level spectra in an unconfined aquifer, with a periodically varying head boundary condition on one end and a no-flow boundary on the other. He used the reciprocal of time as the frequency term in his study and applied the transmissivity and storage coefficient in his derivation of mathematical formula for an unconfined aquifer. Shih (1999a) used cyclical frequency to derive theoretical spectral representation and studied the application of spectral density functions of groundwater level and hydraulic diffusivity for unsteady state, one-dimensional, horizontal, confined and unconfined aquifers with time-dependent head boundaries at both ends. The hydraulic diffusivity, a ratio of transmissivity and storage coefficient for confined aquifers, or a ratio of transmissivity and specific yield for unconfined aquifers, can be estimated theoretically from the derived non-linear form of spectral representation. Shih (2000) also performs an uncertainty and sensitivity analysis to evaluate the applicability for determining hydraulic diffusivity of one-dimensional flow in an unconfined aquifer with time-dependent fluctuation of lateral head and vertical recharge boundaries, using predefined and analytical time-series data.

The studies of spectral density functions of water level for demonstrating important groundwater aspects has been conducted in the northern and the central-west region of Taiwan (Shih *et al.*, 1999, 2000; Shih and Lin, 2002). The results show that the effect of pressure variations is not significant on seawater and groundwater level. It is found that the astronomical tidal components seem to be the main factors causing fluctuation of water levels in the areas. Earth tide is almost imperceptible and is too minute a phenomena to detect (Godin, 1972, p. 29). Accordingly, the influence of Earth tide is ignored in this study. The spectral analysis is a successful method to discover significant periodic components of time-series in the frequency domain. From the spectral density functions we can obtain the energy distribution of water level in the frequency domain. The relevant fluctuations in an aquifer system can be well analysed. A special application in central-west region of Taiwan was utilized to inversely determine hydraulic diffusivity in the field using the derived spectral representations (Shih, 1999b). In that case, hydraulic diffusivity has been evaluated as $1{\cdot}8 \times 10^6$ m^2/h for a Quaternary alluvial plain. This paper conducts an application in Pingtung County of Taiwan to determine the hydraulic diffusivity of a sandy aquifer under confined conditions. Groundwater aspects in the area studied are demonstrated and compared.

SPECTRAL REPRESENTATION FOR ONE-DIMENSIONAL CONFINED GROUNDWATER FLOW

Spectral representations by Fourier transform consider two stationary random processes of $\{\phi(t)\}$ and $\{\gamma(t)\}$, where Fourier transforms representing each process are given by (Bendat and Piersol, 1991)

$$\Phi(f) = \int_{-\infty}^{\infty} \phi(t)\mathrm{e}^{-2\pi i f t}\mathrm{d}t \quad \text{and}$$
$$\Gamma(f) = \int_{-\infty}^{\infty} \gamma(t)\mathrm{e}^{-2\pi i f t}\mathrm{d}t \tag{1}$$

The inverse functions are

$$\phi(t) = \int_{-\infty}^{\infty} \Phi(f) e^{2\pi i f t} df \quad \text{and}$$
$$\gamma(t) = \int_{-\infty}^{\infty} \Gamma(f) e^{2\pi i f t} df \tag{2}$$

where $\Phi(f)$ and $\Gamma(f)$ are Fourier transforms of $\phi(t)$ and $\gamma(t)$, respectively, f is the cyclical frequency (cycles/time), t is time and i is $\sqrt{-1}$.

From a practical viewpoint, a finite record length T_r is adopted. The one-sided spectral density functions, for $f > 0$, are defined as (Bendat and Piersol, 1991)

$$S_{\phi\phi}(f) = \lim_{T_r \to \infty} \frac{2}{T_r} E\left[|\Phi(f, T_r)|^2\right],$$
$$S_{\gamma\gamma}(f) = \lim_{T_r \to \infty} \frac{2}{T_r} E\left[|\Gamma(f, T_r)|^2\right] \quad \text{and}$$
$$S_{\phi\gamma}(f) = \lim_{T_r \to \infty} \frac{2}{T_r} E[\Phi^*(f, T_r) \times \Gamma(f, T_r)] \tag{3}$$

where the expected-value operator E denotes an averaging value over a sampling period, Φ^* is complex conjugate of Φ and $S_{\phi\phi}$ and $S_{\gamma\gamma}$ are autospectral density functions for $\phi(t)$ and $\gamma(t)$, respectively. The cross-spectral density function is expressed as

$$S_{\phi\gamma}(f) = C_{\phi\gamma}(f) - iQ_{\phi\gamma}(f) \tag{4}$$

where $C_{\phi\gamma}(f)$ is the coincident spectral density function and $Q_{\phi\gamma}(f)$ is the quadrature spectral density function (Bendat and Piersol, 1991).

Considering the one-dimensional, homogeneous flow equation of a confined aquifer bounded at both ends by time-dependent boundaries (Figure 1), the groundwater flow can be expressed as (Freeze, 1979)

$$\frac{S_s}{K} \frac{\partial h(x, t)}{\partial t} = \frac{\partial^2 h(x, t)}{\partial x^2}, \tag{5}$$

where S_s is specific storage [L^{-1}], K is hydraulic conductivity [LT^{-1}], h is the hydraulic head [L] and x is the distance [L].

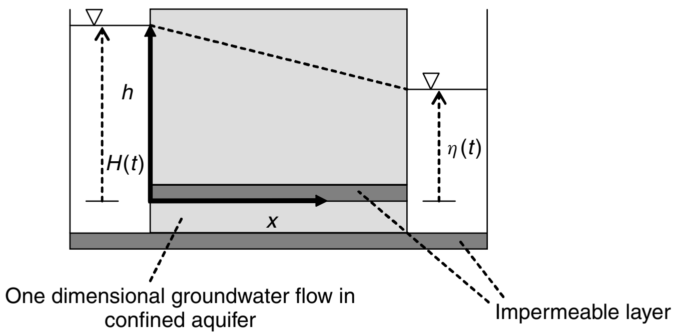

Figure 1. One-dimensional groundwater flow in a confined aquifer bounded by time-dependent boundaries at both ends (after Shih, 1999a,b)

For the case of a horizontal confined aquifer with constant thickness b, storage coefficient S and transmissivity T, Equation (5) is rewritten as

$$\frac{S}{T}\frac{\partial h(x,t)}{\partial t} = \frac{\partial^2 h(x,t)}{\partial x^2} \tag{6}$$

where S is dimensionless and T is in $[\mathrm{L}^2\mathrm{T}^{-1}]$.

For an aquifer bounded by a fluctuating water level at both ends, the relevant boundary conditions is posed as

$$h(0) = H(t) \text{ and } h(L) = \eta(t) \tag{7}$$

Let Z_h, Z_H and Z_η be Fourier components of h, H and η, respectively. From Equation (2) we have

$$\begin{aligned} h(x,t) &= \int_{-\infty}^{\infty} Z_h(x,f)\mathrm{e}^{2\pi i f t}\mathrm{d}f \\ H(t) &= \int_{-\infty}^{\infty} Z_H(f)\mathrm{e}^{2\pi i f t}\mathrm{d}f \\ \eta(t) &= \int_{-\infty}^{\infty} Z_\eta(f)\mathrm{e}^{2\pi i f t}\mathrm{d}f \end{aligned} \tag{8}$$

By using Equations (3), (4) and (8) the spectral relationship for an aquifer described by Equations (6) and (7) can be derived as (Shih, 1999a,b)

$$S_{hh} = S_{HH}F^*F + S_{\eta H}G^*F + S_{H\eta}F^*G + S_{\eta\eta}G^*G \tag{9}$$

or

$$S_{hh} = S_{HH}F^*F + C_{H\eta}(G^*F + F^*G) + iQ_{H\eta}(G^*F - F^*G) + S_{\eta\eta}G^*G \tag{10}$$

For the special case of $H(t) = \eta(t)$, Equation (9) is reduced to

$$S_{hh} = S_{HH}(F^*F + G^*G + G^*F + F^*G) \tag{11}$$

where $F(x) = \sinh(\lambda L(1-\ell))/\sinh(\lambda L)$, $G(x) = \sinh(\lambda L\ell)/\sinh(\lambda L)$, $\ell = x/L$, $\lambda^2 = 2\pi f i/\alpha$ and $\alpha = T/S$ is defined as the hydraulic diffusivity (McWhorter and Sunada, 1993).

In Equations (9)–(11), λL in F, F^*, G and G^* can be expressed as

$$\lambda L = \sqrt{\pi}(1+i)L\alpha^{-1/2}f^{1/2} = \sqrt{\pi}(1+i)\omega \tag{12}$$

We can solve for α in Equations (9) and (10) by using an iterative inverse method with the data obtained from the observed spectral density functions S_{hh}, S_{HH}, $S_{H\eta}$, $S_{\eta H}$ and $S_{\eta\eta}$. An initial guess for α is used in Equations (9) and (10) to calculate the groundwater level spectrum, which is subsequently compared with the observed spectrum. The process is continued until calculated spectrum match the actual data within a predetermined tolerance.

From the viewpoint of the application, Equations (9) and (10) are suitable for groundwater flow that encounters different time-dependent water level boundaries at both ends, and Equation (11) is used for

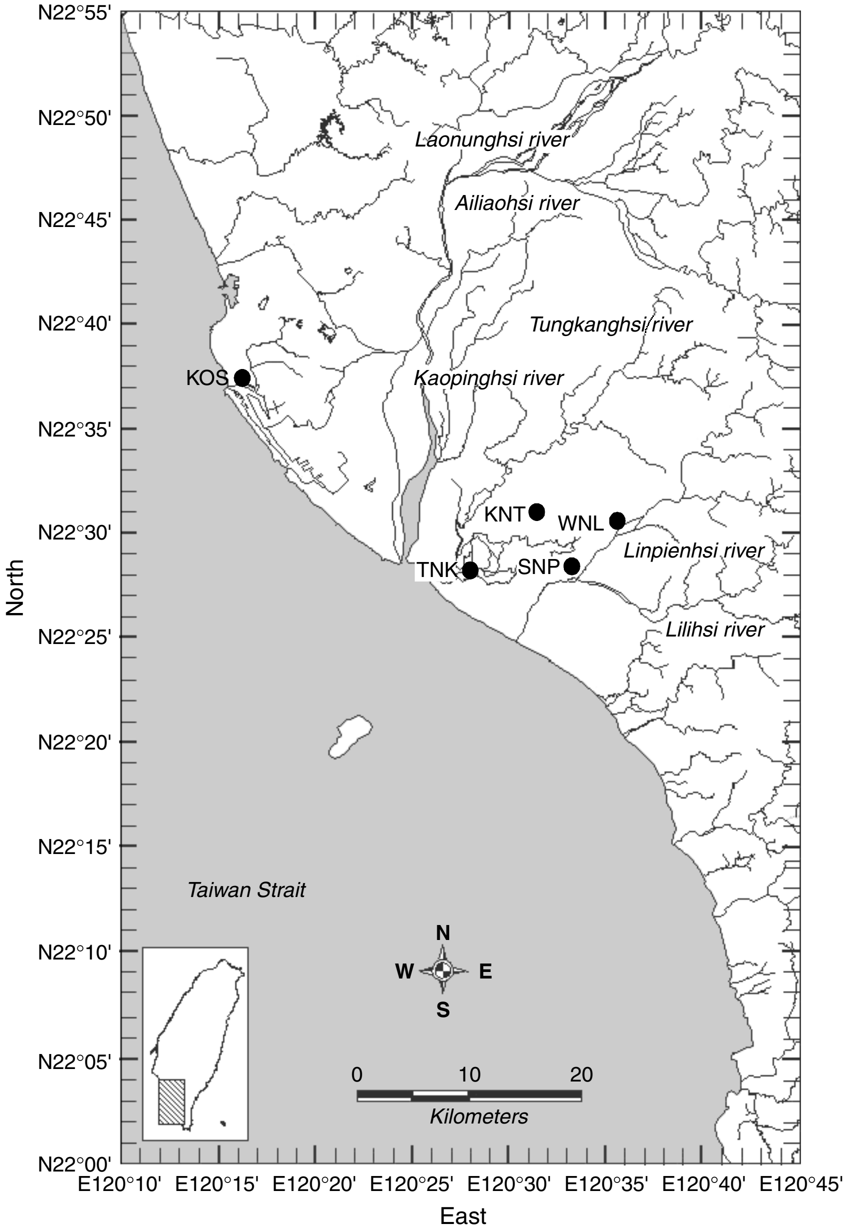

Figure 2. Map shows the location of water-level measurements for groundwater wells and seawater station

groundwater flow that encounters similarly disturbed boundary conditions at both ends. The later occurs in aquifers that are bounded at both ends by rivers subjected to the same tidal effect.

SPECTRAL ANALYSIS

Spectral analysis is a useful method to evaluate characteristics of the periodic fluctuation of time-series in the time–frequency domain. Original presentation of the spectral techniques can be cited in Bendat and Piersol

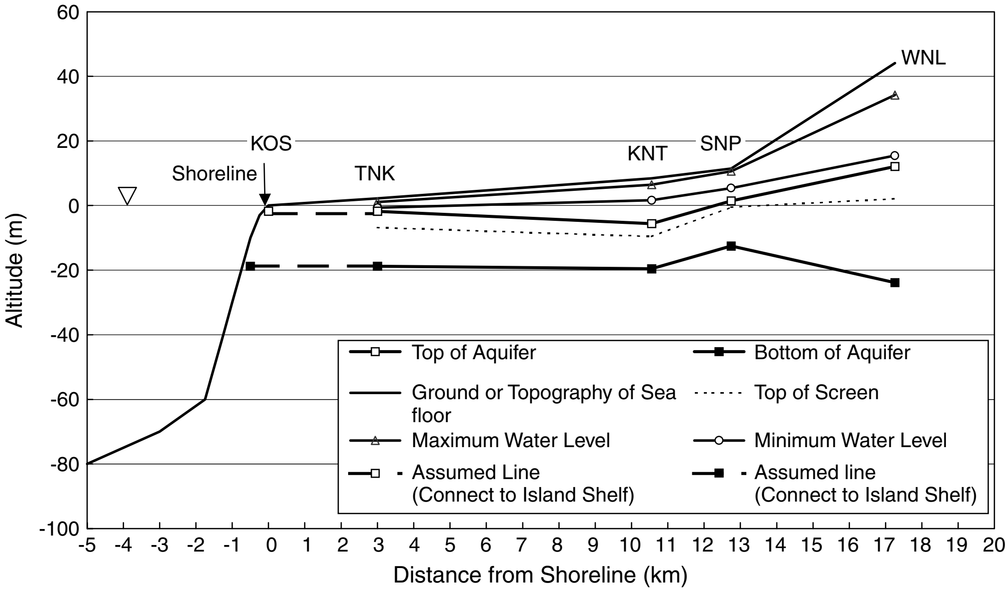

Figure 3. Cross-section of the aquifer studied

(1991). Shih has undertaken a wide range of applications of spectral analysis to identify fluctuation frequency and phase propagation in tidal water level of aquifers and river (Shih *et al.*, 1999, 2000; Shih, 2002; Shih and Lin, 2002) Autospectral density function is used to detect the stronger signal in time series, whereas cross-spectral density and coherence are measured to identify intensity of specific components between two time-series. The 95% confidence interval for the autospectral density function and the non-zero coherence level for cross-spectral density function are also evaluated to pick up significant components in the frequency domain. The basic development of spectral analysis and its expansion to the application in tidal hydrological aspects can be demonstrated in relevant literature (Bendat and Piersol, 1991; Shih *et al.*, 1999, 2000; Shih, 2002; Shih and Lin, 2002)

APPLICATION FOR INVERSE SOLUTION OF HYDRAULIC DIFFUSIVITY

The study area is located at Pingtung County in south-western Taiwan (Figure 2). The well log data indicate four groundwater wells tapping a confined aquifer. They are located at a distance of about 3 to 17 km from the shoreline (Table I and Figure 3). The bottom of the aquifer connecting the sea floor is situated at approximately 20 m below mean sea-level (Chinese Navy Chart 04504, 1995), and the wells TNK, KNT, SNP and WNL are located at 3·0, 10·6, 12·7 and 17·3 km from the shoreline, respectively (Figure 3). It is apparent that the aquifer tapped by the wells at TNK, KNT, SNP and WNL is similar to the aquifer system expressed in the case of Equation (9) or (10). The seawater-level measurements at KOS gauging station indicate that the tidal fluctuations of the water level are very distinct (Figure 4). It shows that the equipotential line of the tidal level along KOS–TNK is parallel to the coastline (Liu, 1999). Well KOS is then chosen to provide the computation reference of water-level spectra. For a semi-diurnal component, a Doodson band-pass filter (Godin, 1972, p. 232–233) is suitable for passing a semi-diurnal signal

$$\frac{1}{48}A_2(I + A_4)S_4S_6A_{12} \tag{13}$$

where $A_{2n} \leftrightarrow x^n + x^{-n}$, $S_{2n} \leftrightarrow x^n - x^{-n}$ and $I \leftrightarrow 1$.

Table I. Details of groundwater well stations and location of seawater gauging station KOS

Symbol	Station name	North	East	Altitude (m)				Type	Material	Distance to shoreline (km)
				Aquifer top	Aquifer bottom	Screen top	Screen bottom			
TNK	Tungkang	22°28′20·5″	120°26′51·4″	−1·783	−18·783	−6·783	−15·783	Confined	Medium sand	3·0
KNT	Kanting	22°30′59·6″	120°30′12·8″	−5·603	−19·603	−9·603	−21·603	Confined	Medium sand	10·6
SNP	Hsinpei	22°28′20·0″	120°32′35·0″	1·472	−12·528	−0·528	−12·528	Confined	Fine gravel with coarse sand	12·7
WNL	Wanlung	22°30′36·0″	120°34′48·0″	12·108	−23·892	2·108	−21·892	Confined	Fine gravel with coarse sand	17·3
KOS	Kaoshung	22°37′09″	120°16′50″							

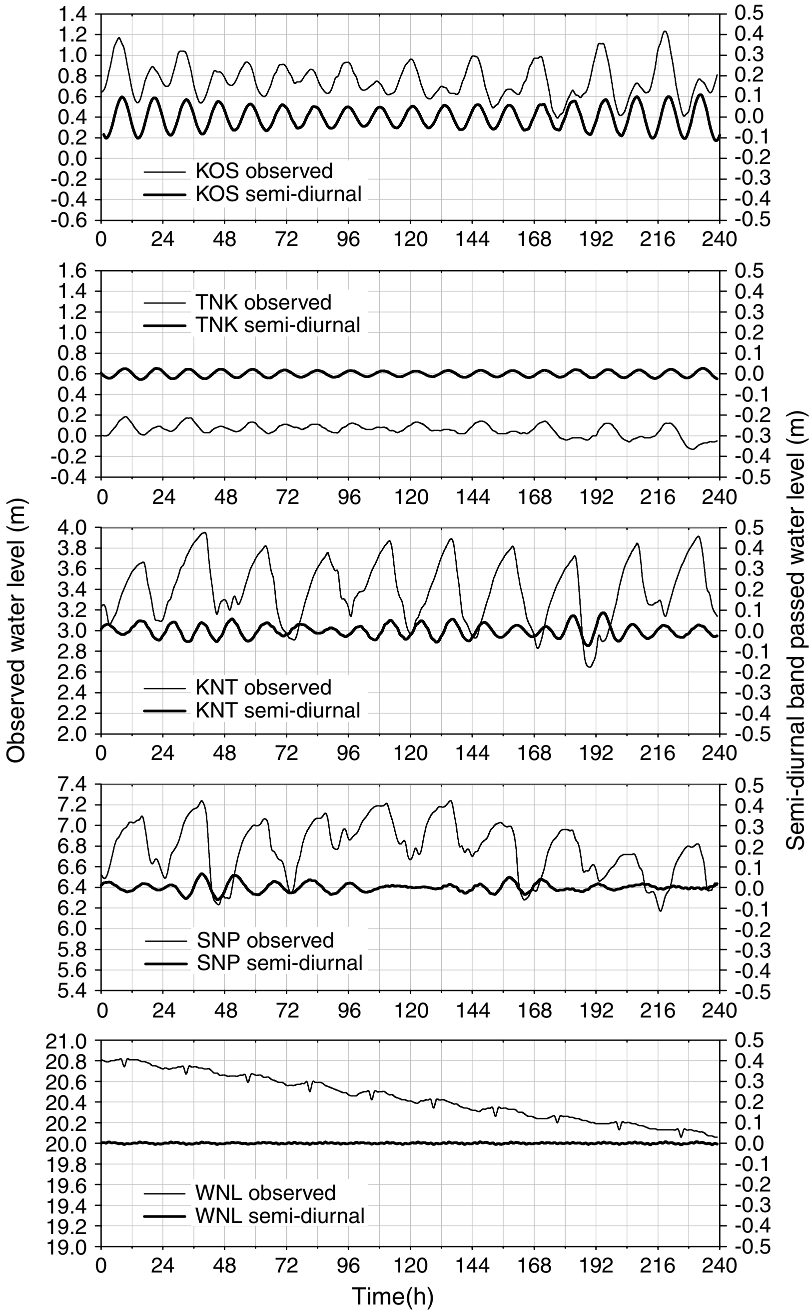

Figure 4. Observed and Doodson semi-diurnal band-passed water level for seawater level and groundwater fluctuation

The filtered time-series in Equation (13) is computed by a polynomial expansion and the power of each term of resultant polynomial formulation represents the index of time-series data and their coefficients as weighted factors. By using Doodson's semi-diurnal filter, it is found that the groundwater at TNK, KNT and SNP have a similar fluctuation to seawater-level changes at KOS for a semi-diurnal component, but with

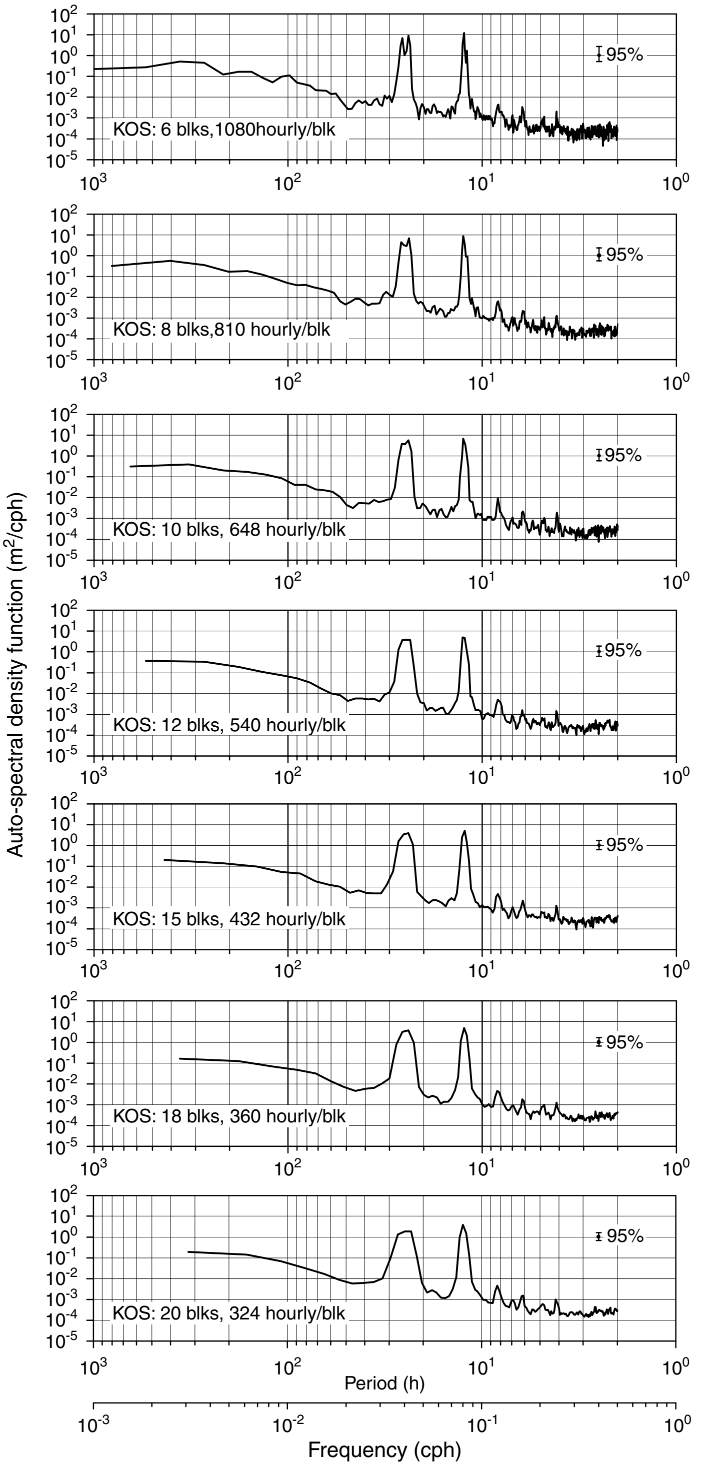

Figure 5. Demonstration for analysed spectral density function by different block number of subrecord in time series

smaller amplitude; the groundwater at WNL seems to have no significant fluctuation (Figure 4). In this case, infiltration is ignored and a conceptual flow model, such as Equations (6) and (7), is used.

The hourly data of the water level were collected for nearly 9 months, acquiring 6480 samples, from 0100 hours 4 February 1999 to 2400 hours 31 October 1999. In order to reduce random errors of the spectral density function, the data are divided into n_d subrecords to obtain a smooth estimate. By slicing the interval into smaller time segments before applying the Fourier transform, it reveals the actual non-seasonal fluctuation of water level in the observation wells. The autospectral density functions of a set of subrecords is analysed and shown in Figure 5, in which the majority of data blocks reveal well the diurnal and semi-diurnal components, except subrecords 6 and 8. The sample length for each hourly subrecord is then selected to be 432 for 15 subrecords of data, which will take approximately 18 days to collect. To suppress the seasonal variation in the time-series, the removal of a linear trend is adopted on each subrecord using the least-square method, and the Hanning window is used to suppress leakage problem (Bloomfield, 1976). The resolution of a discrete frequency is $0{\cdot}23148 \times 10^{-2}$ c.p.h. The autospectral density function with 95% confidence interval has lower and upper extremes of 0·6386 and 1·7867, respectively, and the non-zero coherence significant level (NZC) is calculated to be 0·5273. Although the confidence interval of the cross-spectral density function is dependent on coherence at each discrete frequency interval, it is reasonable that the significant peak in cross-spectral density function is evaluated by the non-zero coherence instead of the confidence interval (Shih *et al.*, 1999). Semi-diurnal and diurnal components are significant in the water-level spectrum at some relevant frequencies (Figure 6). The changes of water level at TNK, KNT, SNP, WNL and PTL are observed to be conformable, showing significant peaks at diurnal ($0{\cdot}41667 \times 10^{-1}$ c.p.h.; period 24 h) and semi-diurnal ($0{\cdot}83333 \times 10^{-1}$ c.p.h.; period 12 h) bands. The cross-spectral density function also shows that the coherence from pairs of KOS to TNK, KNT and SNP are significant at both semi-diurnal and diurnal components (Figure 7). Therefore, it shows that the resultant coherence for the KOS–TNK pair is significant at the 0·9 and 0·95 levels for diurnal and semi-diurnal bands, respectively. The coherence of the KOS well to other wells also reaches the non-zero coherence significant level. Shih and Lin (2002) concluded that the barometric variations do not affect seawater and groundwater levels. Barometric data (KOSp) at KOS is analysed to show the affect of the signal on water level. It is evident that coherences are low at the 12 h period, except KOSp–KOS, KOSp–TNK and KOSp–KNT (Figure 8). Phases are computed at nearly −60, −20 and −70 degrees for the pairs KOSp–KOS, KOSp–TNK and KOSp–KNT, respectively (Table II). It suggests that the semi-diurnal component of tidal fluctuation in the seawater is probably not affected by the barometric level. This study ignores the adjustment of barometric effect on water level. On the evidence, semi-diurnal components are selected as the affected signal in the aquifer. Spectral analysis of water level indicates that the fluctuations at TNK, KNT and SNP reveal a similar pattern to the water-level changes at KOS ocean station. Well WNL can be regarded as an undisturbed point, i.e. $\eta = 0$, for its attenuated water level. The study area is then suitable to be applied in the case described by Equations (9) or (10).

Well combinations KOS–TNK–KNT, KOS–TNK–SNP and KOS–TNK–WNL are chosen as combinations of $H(t)$, $h(t)$ and $\eta(t)$ and labelled respectively as models 1, 2 and 3 (Table III) to evaluate the hydraulic diffusivity. The subroutine ZANLY provided by IMSL (1997) is used to approximate α (or T/S) with a given convergence criteria of 10^{-7}(m^2/c.p.h.), based on non-linearity of Equations (9) and (10), which are

Table II. Phase between the pairs of KOSp to KOS, TNK, KNT, SNP, WNL

	KOSp–KOS	KOSp–TNK	KOSp–KNT	KOSp–SNP	KOSp–WNL
24 h phase (degree)	—	—	−8·5	−31·7	−32·9
Standard deviation	—	—	4·3	4·8	4·9
12 h phase (degree)	−60·8	−22·1	−69·5	—	—
Standard deviation	3·9	14·9	7·9	—	—

—, none detected.

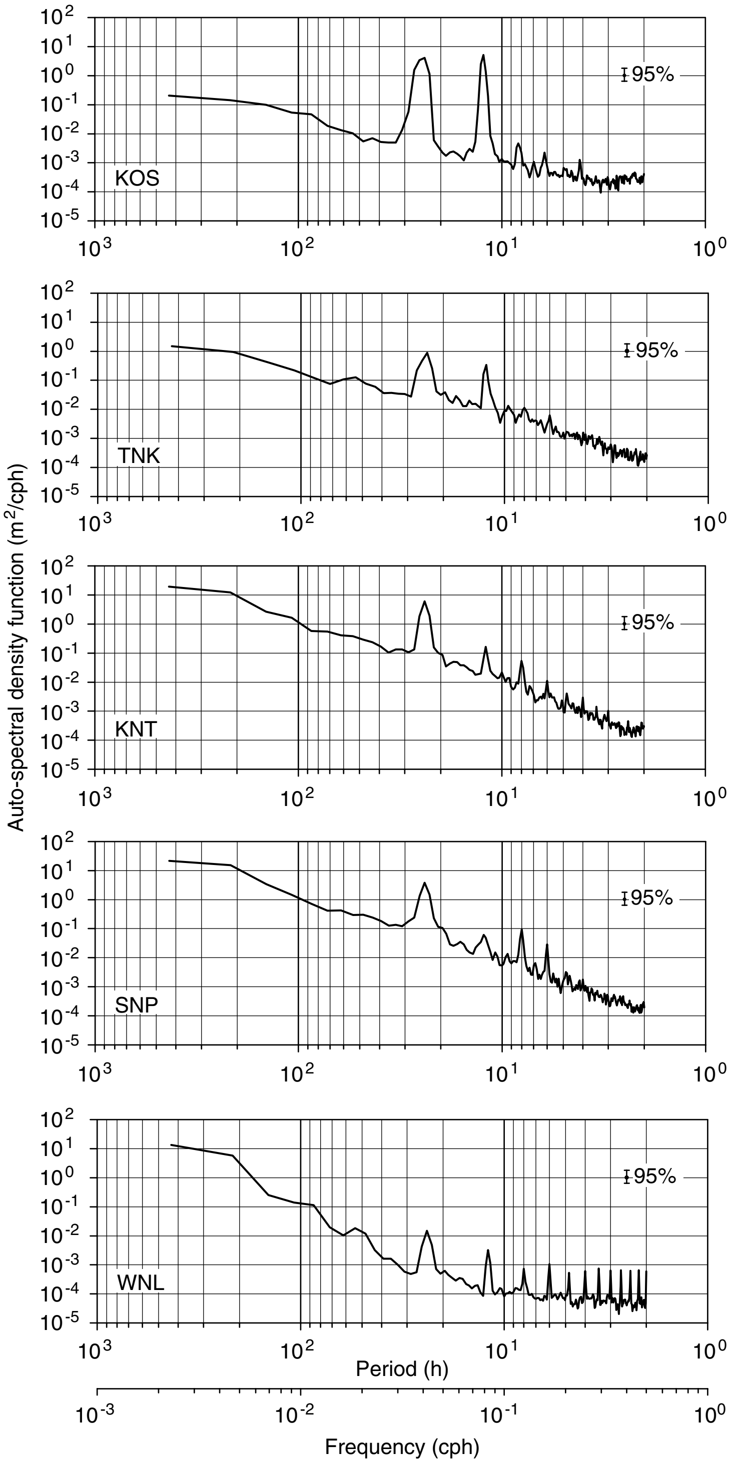

Figure 6. Autospectral density function of water level at KOS, TNK, KNT, SNP and WNL

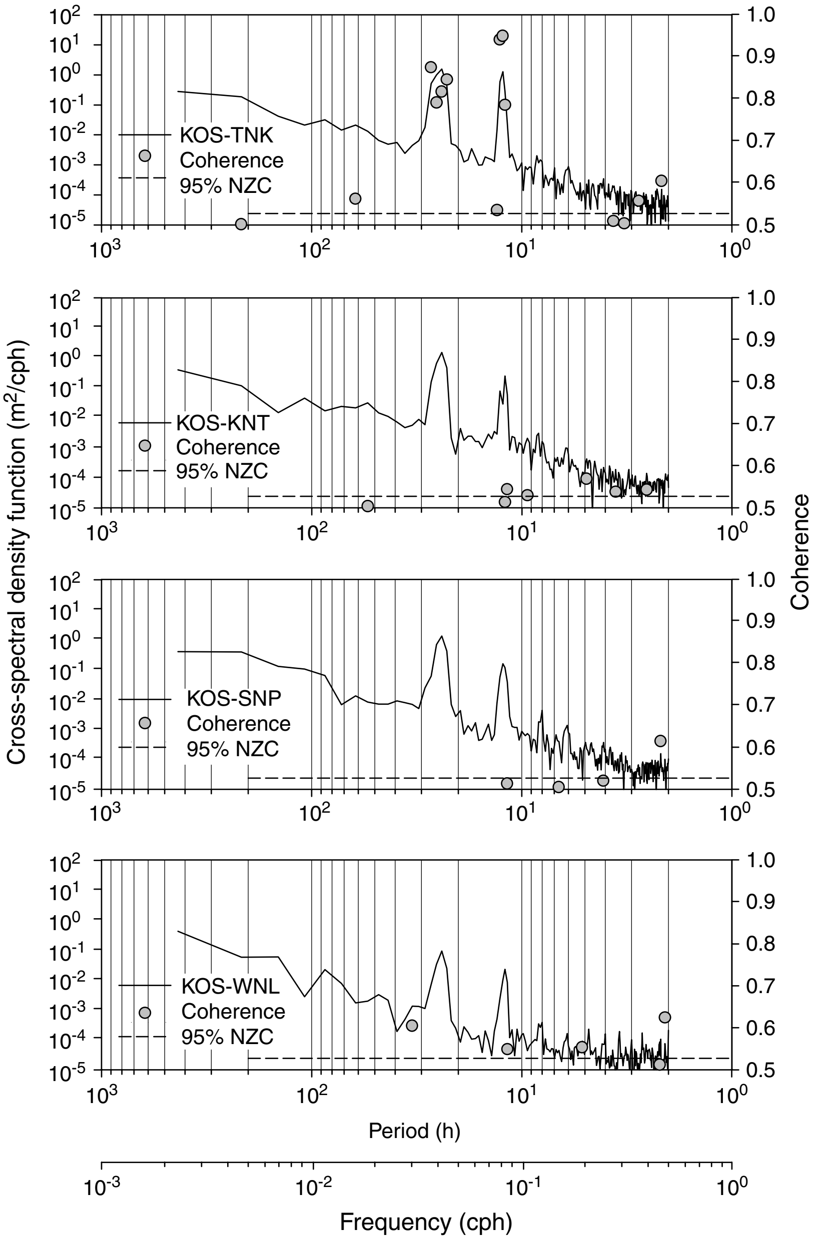

Figure 7. Cross-spectral density function and coherence for the pairing of KOS with TNK, KNT, SNP and WNL

rearranged as

$$S_{hh} - S_{HH}F^*F + S_{\eta H}G^*F + S_{H\eta}F^*G + S_{\eta\eta}G^*G = 0 \tag{14}$$

The spectral density function for the semi-diurnal component is then placed into Equation (14). Using different initial guess values of 10^6 to 10^{10}, α is then approached at $1{\cdot}25 \times 10^6$, $1{\cdot}27 \times 10^6$ and $1{\cdot}25 \times 10^6$ (m^2/h) for models 1, 2 and 3, respectively. Minimum and maximum iteration are 5 and 13 in the three models with the first (absolute) and the second (relative) tolerance of 10^{-7} (Table III). A zero is accepted if the difference in

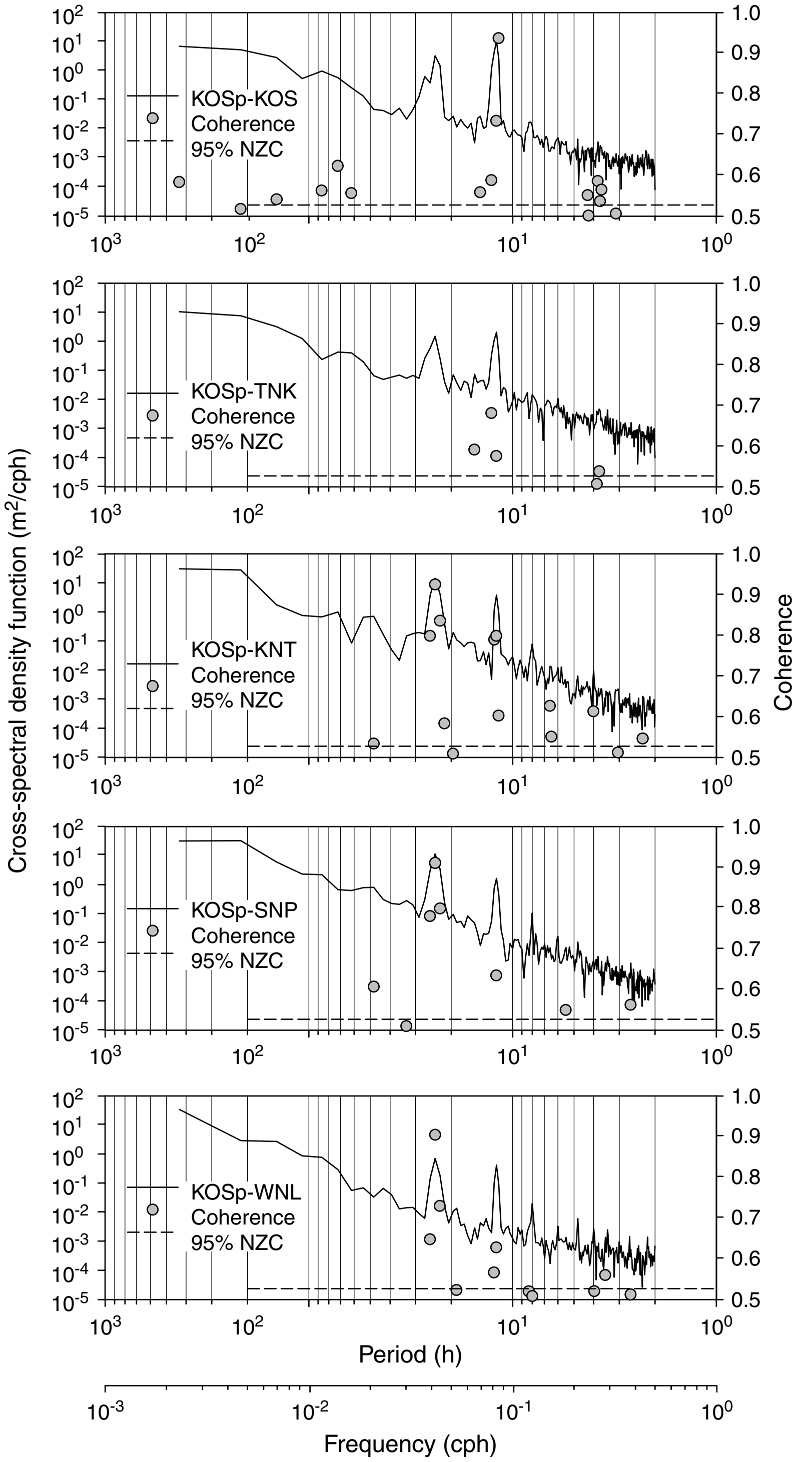

Figure 8. Cross-spectral density function and coherence for the pairing of KOSp with KOS, TNK, KNT, SNP and WNL

Table III. Inverse solutions of hydraulic diffusivity

Parameter	Model 1			Model 2			Model 3		
$H(t)$	KOS			KOS			KOS		
$h(t)$	TNK			TNK			TNK		
$\eta(t)$	KNT			SNP			WNL		
S_{hh}	$0{\cdot}340 \times 10^0$			$0{\cdot}340 \times 10^0$			$0{\cdot}341 \times 10^0$		
S_{HH}	$0{\cdot}512 \times 10^1$			$0{\cdot}512 \times 10^1$			$0{\cdot}512 \times 10^1$		
$S_{\eta\eta}$	$0{\cdot}613 \times 10^{-1}$			$0{\cdot}607 \times 10^{-1}$			$0{\cdot}890 \times 10^{-3}$		
$S_{H\eta}$	$(-0{\cdot}315 \times 10^{-2}, 0{\cdot}283 \times 10^{-1})$			$(-0{\cdot}146 \times 10^0, 0{\cdot}432 \times 10^{-1})$			$(0{\cdot}983 \times 10^{-2}, 0{\cdot}318 \times 10^{-3})$		
$S_{\eta H}$	$(-0{\cdot}315 \times 10^{-2}, 0{\cdot}283 \times 10^{-1})$			$(-0{\cdot}146 \times 10^0, -0{\cdot}432 \times 10^{-1})$			$(0{\cdot}983 \times 10^{-2}, -0{\cdot}318 \times 10^{-3})$		
	Guess	Estimated	Iteration	Guess	Estimated	Iteration	Guess	Estimated	Iteration
	1×10^6	$1{\cdot}25 \times 10^6$	6	1×10^6	$1{\cdot}24 \times 10^6$	5	1×10^6	$1{\cdot}25 \times 10^6$	5
α	1×10^8	$1{\cdot}25 \times 10^6$	8	1×10^8	$1{\cdot}33 \times 10^6$	10	1×10^9	$1{\cdot}25 \times 10^6$	10
	1×10^9	$1{\cdot}25 \times 10^6$	13	1×10^9	$1{\cdot}24 \times 10^6$	12	1×10^{10}	$1{\cdot}25 \times 10^6$	12
Average		$1{\cdot}25 \times 10^6$			$1{\cdot}27 \times 10^6$			$1{\cdot}25 \times 10^6$	
Average α					$1{\cdot}26 \times 10^6$				

Number in parentheses: Complex number

two successive approximations to this zero is within the stop criterion (IMSL, 1997). Respectively, the first and second stop criteria are both chosen as 10^{-7} for absolute and relative difference when using Equation (14). The averaged hydraulic diffusivity is then evaluated to be $1{\cdot}26 \times 10^6$ m^2/h.

DISCUSSION

From the spectral analysis in the time-frequency domain, the relationship of water level fluctuation can be measured in observation wells and at shoreline boundaries to determine hydraulic diffusivity. The study demonstrates that the method presented is applicable to the field in which a confined aquifer is subject to a disturbed boundary at one end and an undisturbed on the other. The result obtained by this study should represent the average property of the entire aquifer of interest and could be different, to a limited extent, from a pump test result accounting for the local aquifer property. Driscoll reported that S may vary from 10^{-3} to 10^{-5} for typical confined aquifers and indicated that hydraulic conductivity ranged from 10^{-3} to 10^2 m/h for sandy aquifers (Driscoll, 1986). Accordingly, the variation of *T/S* could vary from 10^2b to 10^5b m^2/h. However, T is always dependent on hydraulic conductivity and aquifer thickness. For the same type of aquifer, vertical extent of the aquifer may dominate the hydraulic diffusivity. The value of *T/S* is estimated approximately as 10^6 m^2/h in this study, which is located in the higher range of data compiled by Driscoll. Although it is close to the value determined for another area in Taiwan (Shih, 1999b), hydraulic diffusivity values evaluated in this research do not imply that aquifers have the same hydraulic conductivity. It only demonstrates a capability of water transmission in the aquifer. Unfortunately, there is no local pump test result to provide a comparison for this study. The advantage of this approach is to estimate an averaged value of *T/S* in an aquifer without adopting a large-scale pump test data. It indicates that the result of the suggested method in this research can be used to investigate the *T/S* under the averaged condition of aquifers. The estimate is helpful for understanding the groundwater resources in the area studied. The use of this approach requires a homogeneous aquifer with periodically fluctuated boundaries. Shih (2000) indicates that there is 10% error for the smaller aquifer length (100 m) when using 1400 data sets in an uncertainty scenario. It is also suggested that the larger aquifer length is less uncertain than a smaller one.

CONCLUSION

The research demonstrates the use of a stochastic method that is based on spectral relationships of a one-dimensional groundwater flow encountering time-dependent boundaries at both ends under confined conditions. The spectral equations, by nature, are non-linear complex functions. To determine the hydraulic diffusivity from these equations, the water-level spectra are needed for fluctuating boundaries, observation wells, or vertical infiltration. The hydraulic diffusivity then can be obtained by an iterative inverse approach.

An application of spectral analysis to determine the hydraulic diffusivity of a confined aquifer for a sandy aquifer in Pingtung County of Taiwan is presented. It uses the spectral density function of the water level obtained from tidal boundaries and observation wells in the aquifer to approximate hydraulic diffusivity through an inverse approach, which results in an averaged value of $1{\cdot}26 \times 10^6$ m^2/h.

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