DETERMINATION OF AQUIFER PARAMETERS USING RADIAL BASIS FUNCTION NETWORK APPROACH

Gwo-Fong Lin* and Guo-Rong Chen

ABSTRACT

In this paper, a radial basis function network (RBFN) approach to the determination of aquifer parameters is developed. The approach is based on the combination of an RBFN and the Theis or the Hantuch-Jacob solution. The proposed RBFN approach has advantages over the existing back-propagation network (BPN) approach. It avoids inappropriate setting of a trained range. It also determines the aquifer parameters more accurately and needs less training time. Testing the BPN and RBFN approaches by synthetic data also demonstrates these advantages. As to the comparison between the RBFN approach and the type-curve graphical method, two applications to actual time-drawdown data show that the RBFN approach determines the aquifer parameters more precisely for both nonleaky-confined and leaky-confined aquifers. The RBFN approach is recommended as an alternative to the type-curve graphical method and the existing BPN approach.

Key Words: aquifer parameter, aquifer test, artificial neural network, radial basis function network, back-propagation network.

I. INTRODUCTION

The determination of aquifer parameters, such as the transmissivity and the storage coefficient, from aquifer test data has been studied for many years (for example, Jacob, 1940; Hantush, 1956; Walton, 1970; Wikramaratna, 1985; Aziz and Wong, 1992; Zhan *et al.*, 2001; Chen and Chang, 2002; Balkhair, 2002; Chen and Chang, 2003) because it holds a central position in groundwater modeling. The aquifer parameters obtained by the type-curve graphical method (Jacob, 1940) are of questionable reliability (Aziz and Wong, 1992; Balkhair, 2002). In recent years, some convenient and reliable approaches based on artificial neural networks (ANNs) have been developed. For example, Aziz and Wong (1992) and Balkhair (2002) used back-propagation networks (BPNs) to determine aquifer parameters. BPNs are a class of ANNs. Rumelhart et al. (1986) proposed a supervised learning scheme known as a back-propagation algorithm to train BPNs. However, BPNs have some disadvantages (Wasserman, 1993). First, BPNs tend to yield local optimal solutions. Secondly, BPNs may produce different results after the training process even when the same training data are used. Finally, BPNs are trained slowly.

Apart from the aforementioned disadvantages, the BPN approach has another drawback. It is not capable of producing aquifer parameter values accurately when the desired values are out of the trained range. Hence, the performance of the BPN approach is largely determined by the selection of a range of aquifer parameter values in the training phase. However, there is no established methodology for selecting an appropriate trained range, especially when there is no prior information on the aquifer parameters available. Such a limitation has prompted a search for an improved ANN approach to estimating aquifer parameters. In this paper, a radial basis function network (RBFN) approach is proposed.

RBFNs, which are another class of ANNs, have been widely used for nonlinear system identification because of simple topological structure. Radial basis functions were first introduced to solve real multivariate interpolation problems (Powell, 1987). Broomhead and Lowe (1988) exploited the use of radial basis function in the design of neural networks.

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Poggio and Girosi (1990) developed regularization networks from approximation theory with radial basis function networks. Girosi and Poggio (1990) have proven that RBFN is a kind of universal approximator. Given a network with enough hidden layer neurons, RBFN can approximate any continuous function with arbitrary accuracy.

RBFN has been employed in time series prediction (Broomhead and Lowe, 1988) and nonlinear systems identification (Moody and Darken, 1989). Park and Sandberg (1991) studied the universal approximation problem using the RBFN. More recently, RBFNs have also found increasing applications in various aspects of hydrology, such as spatial interpolation (Lin and Chen, 2004a) and rainfall-runoff modeling (Lin and Chen, 2004b).

Based on the combination of an RBFN and the Theis or the Hantuch-Jacob solution, an alternative approach to the determination of aquifer parameters from aquifer test data is proposed. The RBFN approach has three advantages over the BPN approach. Firstly, it avoids the aforementioned problem regarding the selection of an appropriate trained range. Secondly, it determines the aquifer parameter values more accurately. Finally, the RBFN is trained more rapidly.

This paper is organized as follows. First, an RBFN approach is established which is capable of determining the aquifer parameters from aquifer test data. Then, applications are performed and their results are presented to demonstrate the advantages of the RBFN approach. Finally, conclusions are drawn.

II. RBFN APPROACH

1. Radial Basis Function Network

An RBFN can be presented as a three-layer feedforward structure (Fig. 1). As shown in Fig. 1, an RBFN is composed of a number of interconnected processing elements. These elements, called neurons, are joined together with weighted connections. The parameters associated with each of these connections are called weights. The input vector $\boldsymbol{X}$, which consists of N_{in} components, is received by N_{in} input neurons. Then the input vector is transmitted to each hidden neuron that includes a center and an activation function. A center is a vector and its dimensionality is the same as that of the input vector. The output of the jth hidden neuron is obtained by

$$h_j(\boldsymbol{X})=\phi_j(\|\boldsymbol{X}-\boldsymbol{C}_j\|),\ j=1,\ 2,\ \cdots,\ N_h \qquad (1)$$

where $\|\cdot\|$ denotes the Euclidean norm, $\boldsymbol{C}_j$ is the center of the jth hidden neuron, $\phi_j(\ )$ is the activation function of the jth hidden neuron, and N_h is the number of hidden neurons.

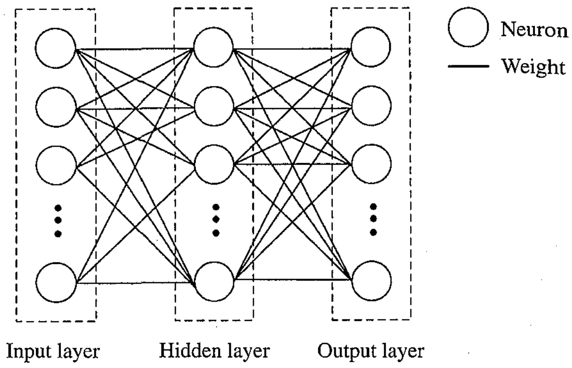

Fig. 1 Architectural graph of an RBFN

The activation function is a nonlinear function that is radially symmetric in the input space. The output of the jth hidden neuron depends only on the radial distance between the center of the jth hidden neuron and the input vector. The activation function used herein is the Gaussian function:

$$\phi_j(\boldsymbol{X})=e^{\left(-\frac{\|X-C_j\|^2}{2\sigma^2}\right)} \qquad (2)$$

where σ is the width of the hidden neurons. A Gaussian function is chosen because it is closely related to the expression of the analytical solutions, Theis and Hantuch-Jacob solutions. This choice makes the RBFN approach a more cost-effective solution than the BPN approach.

In this paper, σ is obtained by (Haykin, 1999)

$$\sigma=\frac{d_{\max}}{\sqrt{2N_h}} \qquad (3)$$

where $d_{\max}$ is the maximum distance between the centers of hidden neurons. Once the outputs of the hidden neurons are obtained, the kth component of the output vector, $\hat{y}_k$, can be calculated as

$$\hat{y}_k=w_0+\sum_{j=1}^{N_h} w_{jk}h_j,\ k=1,\ 2,\ \cdots,\ N_{out} \qquad (4)$$

where w_0 is the bias, w_{jk} is the weight between the jth hidden neuron and the kth output neurons, and N_{out} is the number of out neurons. Once the centers and the width σ are chosen, the bias and the weights connecting the hidden and output neurons can be obtained using the least mean square method. Eq. (4) can be represented in matrix notation:

$$\hat{\boldsymbol{Y}}=\boldsymbol{W\Phi} \qquad (5)$$

where

$$\boldsymbol{W} = \begin{bmatrix} w_0 & w_{11} & w_{21} & \cdots & w_{N_h1} \\ w_0 & w_{12} & w_{22} & \cdots & w_{N_h2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ w_0 & w_{1N} & w_{2N} & \cdots & w_{N_hN_{out}} \end{bmatrix} \quad (6)$$

and

$$\boldsymbol{\Phi} = [1 \;\; \phi_1 \;\; \phi_2 \;\; \cdots \;\; \phi_{N_h}]^T \quad (7)$$

The square error, SE, between the actual output of the network $\hat{y}_k$ and the desired output y_k can be written as

$$\mathrm{SE} = \frac{1}{2}\sum_{k=1}^{N_{out}} [\hat{y}_k - y_k]^2 = \frac{1}{2}(\hat{\boldsymbol{Y}} - \boldsymbol{Y})^T(\hat{\boldsymbol{Y}} - \boldsymbol{Y}) \quad (8)$$

Substituting Eq. (5) into Eq. (8) yields

$$\mathrm{SE} = \frac{1}{2}(\boldsymbol{Y}^T\boldsymbol{Y} - 2\boldsymbol{Y}^T\boldsymbol{W\Phi} + \boldsymbol{W}^T\boldsymbol{\Phi}^T\boldsymbol{W\Phi}) \quad (9)$$

The W can be obtained by minimizing SE. Differentiating Eq. (9) with respect to W gives

$$\frac{\partial \mathrm{SE}}{\partial W} = -\boldsymbol{\Phi}^T\boldsymbol{Y} + \boldsymbol{\Phi}^T\boldsymbol{\Phi W} \quad (10)$$

Setting $\partial\mathrm{SE}/\partial W=0$ gives

$$\boldsymbol{W} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{Y} \quad (11)$$

2. Description of the RBFN Approach

The RBFN approach is developed based on the combination of an RBFN and an analytical solution that can calculate the drawdown from known aquifer parameters. For a leaky confined aquifer, Hantush and Jacob (1955) presented an analytical solution to calculate the drawdown s at a distance r at time t from the transmissivity T, the storage coefficient S, leakage factor B, and the discharge Q. The Hantuch-Jacob solution can be written as

$$s = \frac{Q}{4\pi T}W(u, \frac{r}{B}) \quad (12)$$

where

$$W(u, \frac{r}{B}) = \int_u^\infty \frac{1}{y}\exp(-y - \frac{r^2}{4B^2y})dy \quad (13)$$

and

$$u = \frac{r^2S}{4Tt} \quad (14)$$

The analytical solution for a nonleaky confined aquifer is a special case of the Hantush-Jacob solution in the limit when the leakage factor B approaches infinity, and it is written as (Theis, 1935)

$$s = \frac{Q}{4\pi T}W(u) \quad (15)$$

where

$$W(u) = \int_u^\infty \frac{1}{y}\exp(-y)dy \quad (16)$$

The type-curve graphical method is developed based on the Theis solution for the nonleaky condition or the Hantush-Jacob solution for the leaky condition. In the type-curve graphical method for the determination of aquifer parameters, one has to fit observed time-drawdown data to the one of families of type curves of $W(u, r/B)$ versus $1/u$ and find a match point. In this paper, the above procedures are done by an RBFN.

In order to determine the aquifer parameters from a set of N observed time-drawdown data, the RBFN constructed herein includes N–1 neurons in the input layer. These N–1 neurons are designed to process an input vector consisting of N–1 components. The components of the input vector are obtained from N observed time-drawdown data as

$$x_i = \log(s_{i+1}) - \log(s_1) = \log(\frac{s_{i+1}}{s_1}), \quad i=1, 2, \cdots, N-1 \quad (17)$$

where s_i is the corresponding drawdown observed at time t_i.

For the leaky-confined condition, the proposed RBFN is designed to produce the $1/u$ and r/B coordinates of the match point as

$$\hat{y}_1 = \log[\frac{1}{u_m}] \quad (18)$$

$$\hat{y}_2 = (r/B)_m \quad (19)$$

For the nonleaky-confined condition, r/B is set to zero and only one neuron is needed in the output layer. The components of the input vector and the output of the RBFN are shown in Fig. 2.

Training of the RBFN consists of generating the training patterns and determining the weights and biases by the least mean square method. A training pattern includes an input vector and a target output vector. To generate training patterns, a trained range of the output values must be specified. According to the type curves presented by Walton (1970), the RBFN output, $\log(1/u)$ and r/B, values are selected from –0.5 to 4.0 and 0.0 to 2.5 respectively. Once a specific combination of $\log(1/u)$ and r/B values is chosen, the corresponding $W(u, r/B)$ value is calculated and the corresponding input vector components are generated as

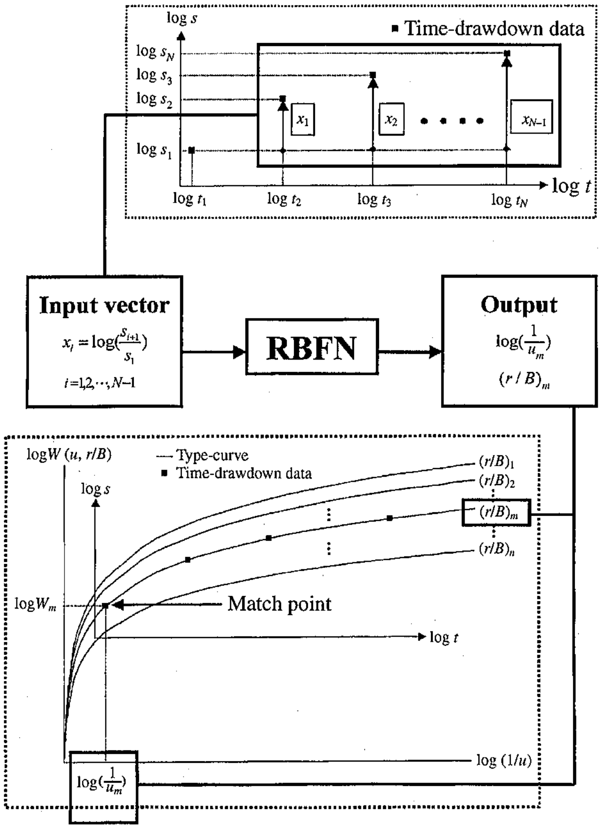

Fig. 2 The components of input vector and the output of the RBFN

$$x_i = \log[\frac{W(u_m t_1/t_{i+1})}{W(u_m)}],\ i=1,\ 2,\ \cdots,\ N-1 \tag{20}$$

Figure 3 illustrates the input vector and target output of the training pattern. For the nonleaky-confined condition, r/B is set to zero and only a log(1/u) value is chosen.

Once the RBFN is trained, it is capable of producing an output vector when an input vector transformed from observed data is processed. Then the coordinates of the match point W_m, s_m, $1/u_m$ and t_m are determined from the output vector and time-drawdown data as follows

$$\frac{1}{u_m} = 10^{\hat{y}_1} \tag{21}$$

$$W_m = W[u_m, (r/B)_m] = W(\frac{1}{10^{\hat{y}}}, \hat{y}_2) \tag{22}$$

$$s_m = s_1 \tag{23}$$

$$t_m = t_1 \tag{24}$$

Finally, transmissivity T and storage coefficient S are determined by

$$T = \frac{Q \cdot W_m(u, \frac{r}{B})}{4\pi s_m} \tag{25}$$

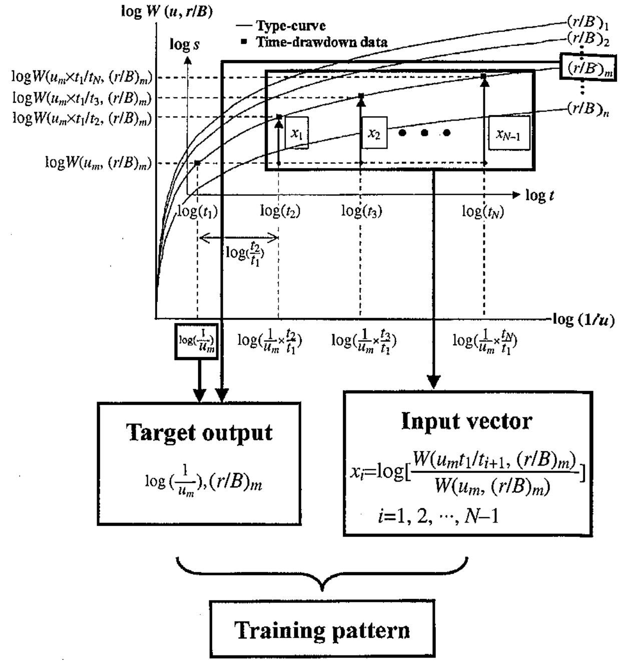

Fig. 3 The input vector and target output of the training pattern

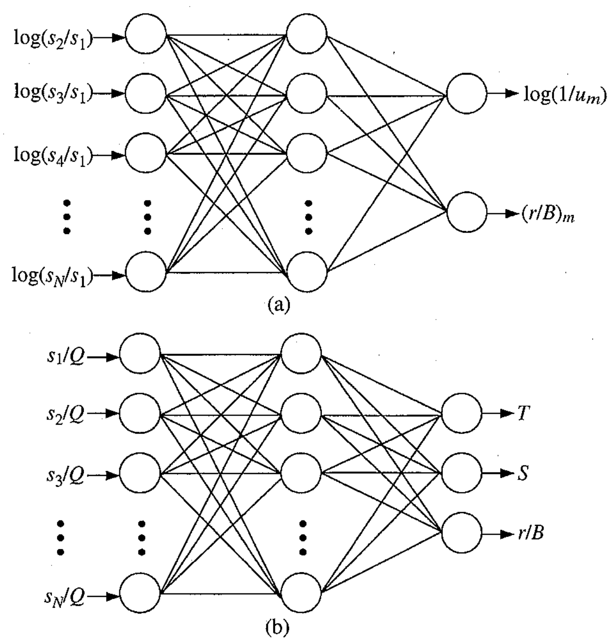

Fig. 4 Structures of (a) the RBFN and (b) the BPN

and

$$S = \frac{4Tu_m t_m}{r^2} \tag{26}$$

Apart from using an RBFN instead of a BPN, another major difference between the RBFN and the BPN approaches is the design of the input and output components. Fig. 4 shows the structures of the RBFN and the BPN. Our design allows the RBFN approach

Table 1 The BPN influential parameters used during training

Parameter	Value
Learning rate	0.5
Momentum constant	0.6
Convergence criterion	0.001
Maximum training cycle	10,000
Number of training patterns	1,024

to successfully avoid the problem regarding the selection of an appropriate trained range. Moreover, the RBFN is trained more rapidly than the BPN. These advantages will be further demonstrated in the next section.

III. APPLICATION AND DISCUSSION

In this section, the RBFN approach is applied to four examples. In Example 1, 1000 sets of synthetic data are used to test the accuracy of the BPN and the RBFN approaches for a nonleaky-confined aquifer. Then a set of actual time-drawdown data, which are taken from Walton (1970), is used to test the applicability and reliability of the RBFN approach in Example 2. The time-drawdown data were measured during pumping at a constant discharge of 1,199 m^3/day and the distance between observed well and pumping wells is 502 m.

The RBFN approach is applied to leaky-confined aquifers in Examples 3 and 4. In a like manner, 1000 sets of synthetic data and a set of actual time-drawdown data are used in Examples 3 and 4, respectively. The time-drawdown data, which are taken from Dawson and Istok (1991), were measured during pumping at a constant discharge of 382 m^3/day and the distance between observed well and pumping wells is 49 m.

1. Example 1: Testing the BPN and the RBFN Approaches for a Nonleaky-Confined Aquifer Using Synthetic Data

In the BPN approach, a three-layered BPN is designed to produce values of T and S when the values of s/Q are received. The components of the input and output vectors are shown in Fig. 4. To generate training patterns, a trained range of the BPN output values for both T and S must be specified and then combinations of these values are used during the generation process. Once a specific combination of T and S values is chosen, the corresponding input data are calculated using the Theis solution, Eq. (15). In this example, 22 and 2 neurons are constructed in the input and output layers, respectively. Then the BPN is trained with 1,024 training patterns which are generated using T values ranging from 10 to 100 m^2/day with a step size of 2.9 m^2/day and S values ranging from 10^{-6} to 10^{-5} with a step size of 2.9×10^{-7}. The BPN influential parameters used during training are given in Table 1.

Table 2 The required training times for the BPN and the RBFN approaches (Example 1)

Approach	Required training time (min)
BPN	13
RBFN	1

The RBFN constructed in this example includes 21 and 2 neurons in the input and output layers, respectively. To train the RBFN, a total of 102 center patterns and 1,024 training patterns are generated using $\log(1/u)$ values ranging from -0.5 to 4.0 with step sizes of 0.044 and 0.0044, respectively. The BPN and the RBFN are trained with the same number of training patterns.

After the BPN and the RBFN are trained with the training patterns, the trained BPN and RBFN are capable of yielding the corresponding estimated output values when the input values of the training patterns are processed. Table 2 summarizes the required training times for the BPN and the RBFN. The required training times for the BPN and the RBFN are 13 and 1 min on a 1.8 Ghz personal computer, respectively.

To assess the generalization performances of the BPN and the RBFN approaches, 1000 tested patterns not used during the training process are employed. The tested patterns are randomly generated from combinations of idealized T and S values ranging from 10^1 to 10^4 m^2/day and 10^{-2} to 10^{-6}, respectively. Figs. 5 and 6 show the scatter plots for the idealized and estimated aquifer parameters, T and S, obtained by the BPN and the RBFN approaches.

2. Example 2: Testing the RBFN Approach for a Nonleaky-Confined Aquifer Using Field Data

When the BPN approach is applied to this example, it is very difficult to select an appropriate trained range because there is no prior information on the aquifer parameter available. Hence, only the RBFN approach is applied herein. Testing the trained RBFN with the time-drawdwn data results in T and S values of 119 m^2/day and 2.223×10^{-5}, respectively. The values of T and S obtained by the type-curve graphical method are 125 m^2/day and 2.0×10^{-5} (Walton, 1970). Given a set of T and S values, the

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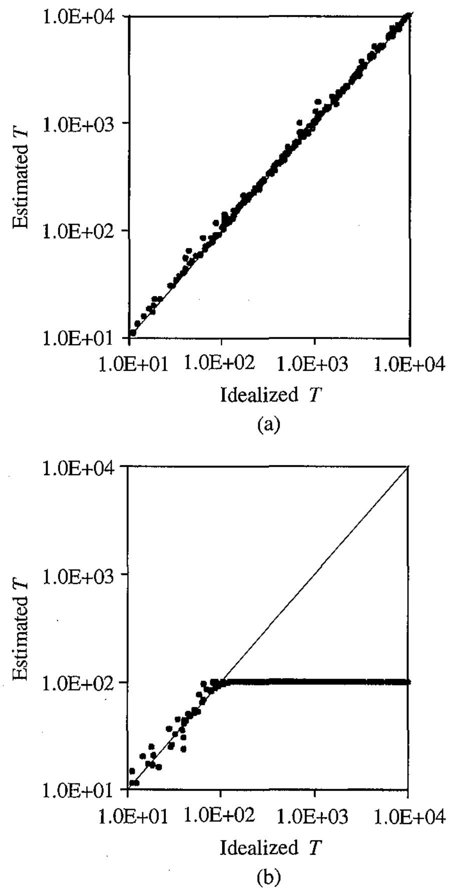

Fig. 5 Idealized *T* values versus estimated *T* values obtained by (a) the RBFN and (b) the BPN approaches (Example 1)

estimated drawdown can be calculated by the Theis solution, Eq. (15). Then, the relative root mean square error (RRMSE) of the estimated drawdown is computed according to

$$\text{RRMSE} = \sqrt{\frac{1}{N_s}\sum_{i=1}^{N_s}\left(\frac{\hat{s}_i - s_i}{s_i}\right)^2} \qquad (27)$$

where $\hat{s}_i$ is the estimated drawdown, s_i is the observed drawdown and N_s is the number of drawdowns. Table 3 summarizes the estimated aquifer parameters and the RRMSE values for the RBFN approach and the type-curve graphical method. The RRMSE values for the RBFN approach and the type-curve graphical method are 2.8% and 9.8%, respectively.

3. Example 3: Testing the BPN and the RBFN Approaches for a Leaky-Confined Aquifer Using Synthetic Data

The BPN constructed herein includes 28 and 3 neurons in the input and output layers, respectively. Fig. 4 shows the components of the input and output vectors. Then the BPN is trained with 1,024 training patterns that are generated using *T* values ranging from 10 to 100 m^2/day with a step size of 6 m^2/day, *S* values ranging from 10^{-6} to 10^{-5} with a step size of 6.0×10^{-7}, and *r*/*B* values ranging from 0.0 to 1.0 with a step size of 0.33. In the RBFN approach, 27 and 2 neurons are constructed in the input and output layers, respectively. A total of 256 center patterns are generated from log(1/*u*) values ranging from -0.5 to 4.0 with a step size of 0.3 and *r*/*B* values ranging from 0.0 to 2.5 with a step size of 0.167. The RBFN is

Table 3 The estimated aquifer parameters and RRMSE values for the RBFN approach and the type-curve graphical method (Example 2)

Method	Aquifer parameter		RRMSE (%)
	T (m^2/day)	*S* (10^{-5})	
RBFN approach	119	2.223	2.8
Type-curve graphical method	125	2.000	9.8

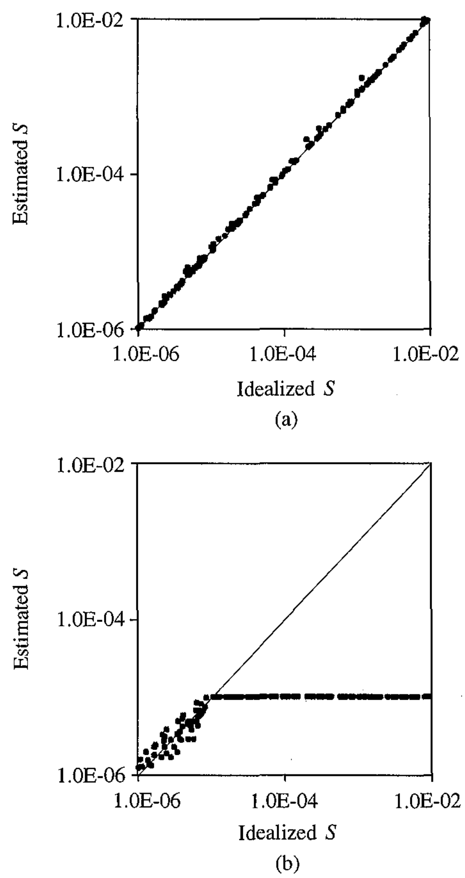

Fig. 6 Idealized S values versus estimated *S* values obtained by (a) the RBFN and (b) the BPN approaches (Example 1)

Table 4 The required training times for the BPN and the RBFN approaches (Example 3)

Approach	Required training time (min)
BPN	31
RBFN	2

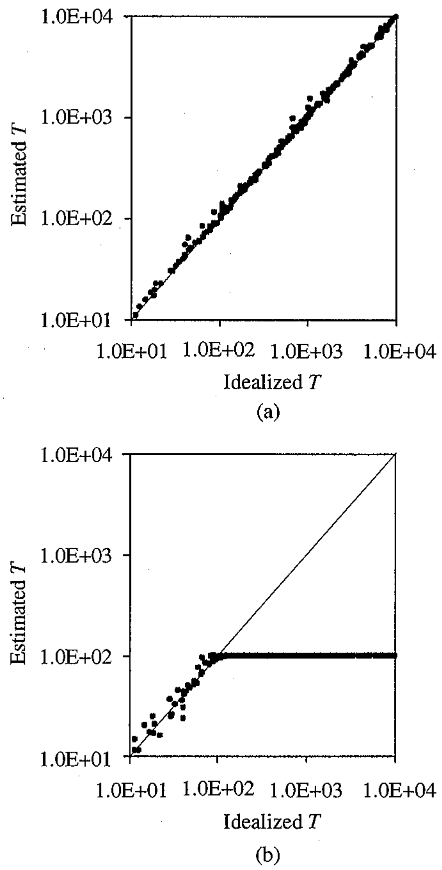

Fig. 7 Idealized T values versus estimated T values obtained by (a) the RBFN and (b) the BPN approaches (Example 3)

trained with 1,024 trained patterns, which are generated from $\log(1/u)$ values ranging from -0.5 to 4.0 with a step size of 0.071 and r/B values ranging from 0.0 to 2.5 with a step size of 0.167. Table 4 summarizes the required training times for the BPN and RBFN approaches. The required training times for the BPN and RBFN approach are 31 and 2 min on a 1.8 Ghz personal computer, respectively.

Then the trained BPN and the RBFN are tested with 1000 tested patterns that are not used during training. The testing data are randomly generated from T values ranging from 10^1 to 10^4 m^2/day, S values ranging from 10^{-2} to 10^{-6}, and r/B values ranging from 0 to 2.5. Figs. 7, 8 and 9 show the scatter plots for the idealized and estimated aquifer parameters obtained by the BPN and the RBFN approaches.

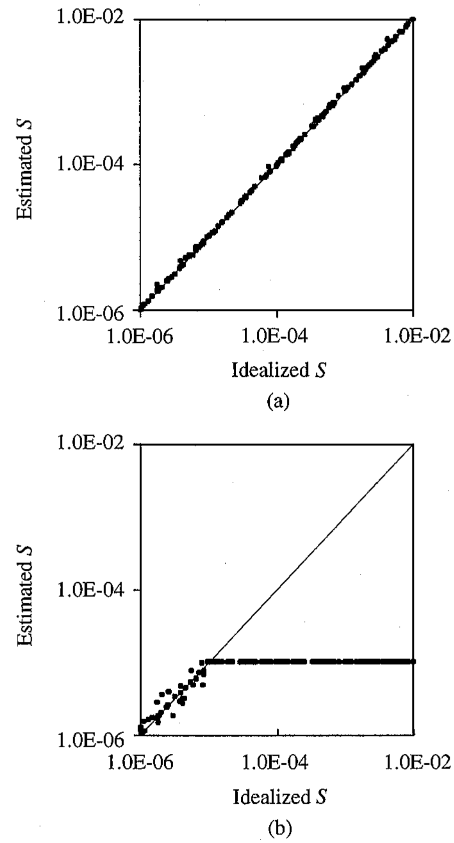

Fig. 8 Idealized S values versus estimated S values obtained by (a) the RBFN and (b) the BPN approaches (Example 3)

4. Example 4: Testing the RBFN Approach for a Leaky-Confined Aquifer Using Field Data

The T, S and r/B values determined by the RBFN approach from the actual time-drawdown data are 112 m^2/day, 1.552×10^{-4} and 0.30, respectively. The values of T, S and r/B obtained by the type-curve graphical method are 55 m^2/day, 0.095×10^{-4} and 0.75 (Dawson and Istok, 1991). In a like manner, once the aquifer parameters (T, S and r/B) are determined, the estimated drawdown can be calculated by the Hantuch-Jacob solution, Eq. (12). In addition, the RRMSE of the estimated drawdown is computed according to Eq. (27). The estimated aquifer parameters and the RRMSE values for the RBFN approach and the type-curve graphical method are summarized in Table 5. The RRMSE values for the RBFN approach and the type-curve graphical method are 16.7% and 95.3%, respectively. The RBFN approach achieves a smaller value of RRMSE than the type-curve graphical method.

5. Discussion

During the training process, the BPN and the

Table 5 The estimated aquifer parameters and RRMSE values for the RBFN approach and the type-curve graphical method (Example 4)

Method	Aquifer parameter			RRMSE (%)
	T (m^2/day)	S (10^{-4})	r/B	
RBFN approach	112	1.552	0.30	16.7
Type-curve graphical method	55	0.095	0.75	95.3

RBFN are trained with the same number of training patterns. As shown in Tables 2 and 4, the RBFNs are trained more rapidly than the BPNs for both nonleaky-confined (Example 1) and leaky-confined (Example 3) aquifers. As compared to the BPN approach, the RBFN approach yields a training time reduction of 92% and 94% in Examples 1 and 3, respectively.

In the testing process, synthetic data are used to test the accuracy of the BPN and the RBFN approaches. As shown in Figs. 5, 6, 7, 8 and 9, the BPN approach is capable of accurately estimating aquifer parameters only over the trained range for both nonleaky-confined (Example 1) and leaky-confined (Example 3) aquifers. When the tested values fall outside the trained range, the BPN approach cannot produce the desired output. The performance of the BPN approach is largely determined by the selection of the trained range. Moreover, there is no established methodology for selecting an appropriate trained range, especially when there is no prior information on the aquifer parameters available. Hence, the BPN approach has the problem of selecting an appropriate trained range.

On the contrary, almost all the estimated aquifer parameters obtained by the RBFN approaches match the idealized aquifer parameters perfectly over the whole tested range (Figs. 5, 6, 7, 8 and 9). The RBFN approach can accurately estimate aquifer parameters over a wide tested range. It also can avoid the problem of selecting an appropriate trained range. Hence, the RBFN approach is a better method as compared to the BPN approach.

As to the comparison between the RBFN approach and the type-curve graphical method, applications to actual time-drawdown data show that the RBFN approach achieves a smaller value of RRMSE for both nonleaky-confined (Example 2) and leaky-confined (Example 4) aquifers (Tables 3 and 5). The RBFN approach determines the aquifer parameters more accurately than the type-curve graphical method. As compared to the type-curve graphical

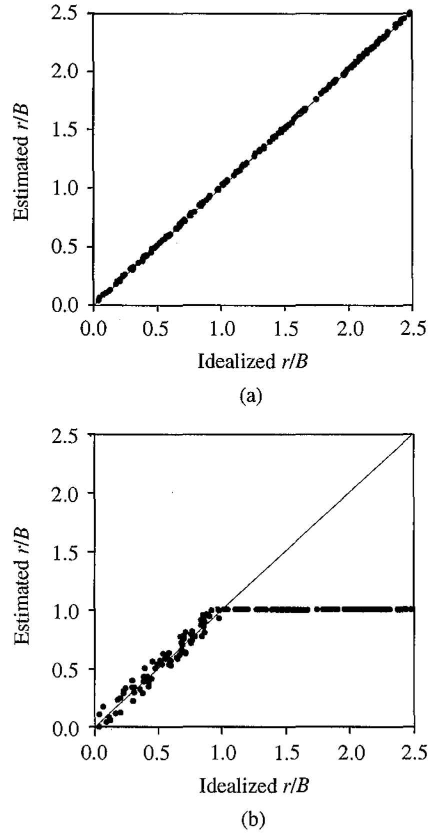

Fig. 9 Idealized r/B values versus estimated r/B values obtained by (a) the RBFN and (b) the BPN approaches (Example 3)

method, the RBFN approach yields an RRMSE reduction of 71% and 82% in Examples 2 and 4, respectively. Besides, the reliability of the type-curve graphical method is often doubted (Aziz and Wong, 1992; Balkhair, 2002). Hence another significant advantage of the RBFN approach is the reliable process of obtaining the aquifer parameters. The RBFN approach is a better method.

IV. SUMMARY AND CONCLUSIONS

ANNs provide a convenient and reliable approach to the determination of aquifer parameters from aquifer test data. But the existing ANN approach, which is a BPN approach, has a problem of selecting an appropriate trained range. In this paper, an alternative ANN approach, which is called the RBFN approach, has been proposed. Apart from using an RBFN instead of a BPN, another major difference between the RBFN and the BPN approaches is the design of the input and output components. Our design provides the RBFN approach with three advantages over the BPN approach. First, the RBFN approach avoids the problem of selecting an appropriate trained range. Second, the RBFN approach determines the aquifer parameter values more accurately. Third, the RBFN approach needs less training time.

Testing the BPN and the RBFN approaches by synthetic data also demonstrates these three advantages. As to the comparison between the RBFN approach and the type-curve graphical method, applications to actual time-drawdown data shows that the RBFN approach performs better than the type-curve graphical method. The RBFN approach is recommended as an alternative to the existing methods (the type-curve graphical method and the BPN approach) because of less required training time, higher accuracy, and more convenient implementation.

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