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ANALYSIS OF THE KINEMATIC STABILITY OF PYRAMIDAL BLOCKS

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Abstract: The planes of rock joints, weak planes and the slope plane comprise pyramidal blocks, which may slide according to their kinematic configuration, frictional properties and external forces. In order to identify whether these blocks are kinematically admissible of sliding, a new approach, the "intersection set method," is proposed in this research. In comparison to the block theory, the proposed method is relatively simple in application. Accordingly, the potential failure modes and the factor of safety of any particular pyramidal blocks prone to sliding can be identified. Besides, the rock blocks that have several parallel joints can also be analyzed by the proposed method. Instead of using the conventional method of equal-angle hemisphere projections, this paper developed a stereographic projection method on the slope plane (or the excavating plane), which facilitates identification of the block analyzed and its failure modes. Furthermore, the proposed method allows various possible distributions of static water pressure on the joint surface so that the influence of water pressure on the stability can be analyzed as well.

Keywords: Rock slope, landslide, pyramidal block, block theory, kinematic stability, intersection set method.

1. INTRODUCTION

Landslides of rock slopes may involve one set of joint plane (e.g., sliding of dip-slopes), two or three sets of joint planes (e.g., wedge failure), or more. When more than three sets of joint planes exist, which can often be found in the nature, the stability of such a rock slope is of concern. Researches on the stability of wedges, prisms or arbitrary rock blocks date from at least the1950's (John, 1968; Londe et al., 1969; Hoek et al., 1973; Warburton, 1981; Goodman, 1996; Mauldon et al., 1996; Kumsar et al., 2000; Yoon et al., 2002)

The stability of a rock slope with several sets joint planes can be analyzed by the block theory (Goodman and Shi, 1985). However, both the orientation and the location of each individual joint plane are required when conducting analysis using the block theory. Provision of the detailed information of the joint plane seems to be not feasible, especially when the excavation of a cut slope (or the inner-surfaces of a tunnel) has not yet been conducted. In such a case, only the orientation of joints is known through a field survey of outcrops; the exact location of the joint planes is unknown before an excavation. It is wished that, before the detailed information of the joint planes is available, stability analyses of the possible configuration of the free rock blocks could be conducted so that the subsequent assessments of the factor of safety can be performed. Only when such assessments are available, the precaution against the potential slide of the certain free blocks can be taken and the required remedial solutions, e.g. the application of rock bolts with proper orientation for the free blocks prone to sliding, can be prepared.

This research aims at developing a method in which only the orientations of joint planes are required. All the possible blocks formed by the existed joint planes can be systematically generated and their stability can be subsequently analyzed as well. Those rock blocks with a lower factor of safety can then be highlighted and a remedial solution can be accordingly developed.

Since the location of each individual joint plane is not requested, the free blocks considered are those comprised of some of the existed joints set with all the lines (of an intersection of two planes) emitting from one point (or co-intersected at on point). That is, the free rock blocks would have a pyramidal geometry. Accordingly, this paper focuses on how to: (1) identify the pyramidal blocks that are kinematically admissible of sliding; (2) systematically exam all possible forms of sliding (sliding on one plane, wedge failure and sliding of prism blocks) and the corresponding factor of safety; (3) to extend the analyzed blocks from pyramidal blocks to blocks with parallel planes; (4) to allow more patterns of water pressure distribution, which may happen in reality; (5) to develop a software which can accomplish the above-mentioned tasks. The developed software, named as SPBA (Slope of Pyramidal Block Analysis), can also analyze the improvement in stability after remedial methods (e.g., applying rock bolts or an increase in cementation) have been applied.

2. DEFINITIONS

Several planes, including the slope plane, the crest, the cut plane, free surfaces and joint planes, are to be mentioned in the following sections and these planes are defined and illustrated in Fig. 1 for the sake of clarification. The pyramidal block is enclosed by several joint planes, a slope plane and sometimes by one cut plane as well. The slope plane, the joint plane and the cut plane are expressed in terms of their plane normal vector as:

$$n_{xe}x + n_{ye}y + n_{ze}z = EPt \qquad (1)$$

where the magnitude of EPt determines how far the *Point O* is from the slope plane.

$$n_{xi}x + n_{yi}y + n_{zi}z = 0 \qquad (2)$$

$$n_{xt}x + n_{yt}y + n_{zt}z = TPt \qquad (3)$$

All the intersectional lines of the joint planes enclosing the pyramidal block have intersection lines meet at one remote point (marked as *Point O* in Fig. 1). The cut plane can be independent of the rest planes or, by selection of the user, be one of the joint planes. The planes exposed to daylight are also referred as "free surfaces." The distance between *Point O* and the slope plane determines the size of the pyramidal block.

The orientation and location of the cut plane can be arbitrary and not related to other planes; this enables the proposed analysis applicable to a much wider group of rock blocks. In fact, if the *Point O* is far away from the slope plane, pairs of parallel joint planes enclosing the rock block can be obtained. In conjunction with the selection of cut plane, rock blocks with parallel planes can be obtained and analyzed. As shown in Fig. 2, the types of rock blocks, in terms of their geometry, considered in this research include wedges, prisms and pyramidal blocks.

If analyses of blocks enclosed by arbitrary joint planes are desired, the existing block theory can serve this purpose; however, the location of each individual joint plane must be precisely defined.

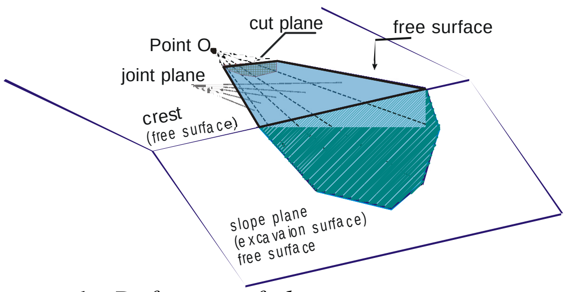

Figure 1. Definition of planes.

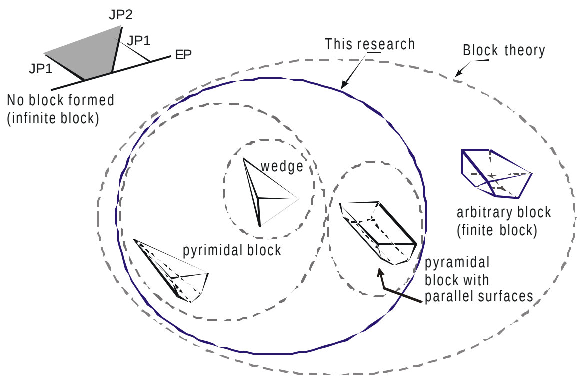

Figure 2. Types of rock blocks considered in this research and the block theory

3. DETERMINATION OF BLOCKS KINEMATICALLY ADMISIBLE OF SLIDING

3.1 All possible pyramidal blocks

A rock block, which is kinematically admissible of sliding out from the slope plane, should be enclosed by the slope plane. Considering a total of P_n planes (including the slope plane) in a slope, all the possible types of blocks, enclosed by all or some of the planes considered, can be expressed as:

$$N = C_3^{Pn-1} + C_4^{Pn-1} + \cdots + C_{Pn-1}^{Pn-1} \qquad (4)$$

where N is the number of combinations of blocks and $C_b^a = \dfrac{a!}{b! \times (b-a)!}$.

3.2 Slope-plane projection method

In order to determine whether a block will slide or not, or how it will slide, a stereo projection method was conventionally adopted. Considering the joint set summarized in Table 1, originally analysed by Hoek et al. (1973), the modes (no movement, slide along a plane, slide along a line, uplift, etc.) of movement can be indicated by the upper and the lower hemisphere projection, as shown in Fig. 3 and 4. If all the planes are projected toward the slope plane, it only takes one plot to indicate all the modes of movement, as shown in Fig. 5.

Table 1 Configuration of planes – Case A (Hoek et al., 1973)

No.	Plane	Orientation	Dip	c	ϕ
1	joint	105	45	23.95	20
2	joint	235	70	47.89	30
3	slope	185	65	0	0
4	crest	195	12	0	0
5	cut	165	70	0	0

Remarks:

- The height of the slope is 30.5 m; the distance between the cut plane (tensile crack) and the slope is 12.2 m.
- The unit weight of the rock is 25.1 kN/m^3. The unit for the orientation, dip angle and friction angle (f) is degree. The unit for cohesion (c) is kN/m^2.

3.3 Intersection set method

With all the possible block types found, those who cannot slide should be eliminated and the rest will be kinematically admissible of sliding. Projecting all planes on the slope plane not only possesses merits of presenting the modes of movement, as shown in Fig. 5 to Fig. 3 and 4, but it also facilitates the determination of blocks which are kinematically admissible of sliding.

Defining I_{ij} as the intersectional line of *Plane i* and *Plane j*, the stereo-projection of I_{ij} on the slope plane will be a point.

With all the sets of intersectional lines determined by Eqn. 1, the blocks that cannot slide can be eliminated by the following steps:

1. Setting the positive direction of I_{ij} – The positive direction of I_{ij} is set to point outward the slope plane. I_{ij} can be determined for the plane normal of plane i and plane j as:

$$\vec{I}_{ij} = \vec{n}_i \times \vec{n}_j \tag{5}$$

2. Putting all the combinations of I_{ij}'s determined by Eqn. 1 in individual sets. Now, considering the joint sets (P_1 to P_4) shown in Fig. 6 for example, the initial set of intersectional lines (I_{ij}) is $\{I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}\}$.

3. Starting from *Plane 1* (indicated as P_1), the I_{ij} in both sides of P_1 is set to be two individual sets. Two sets are accordingly determined as: $\{I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}\}$ and $\{I_{14}, I_{24}, I_{34}\}$.

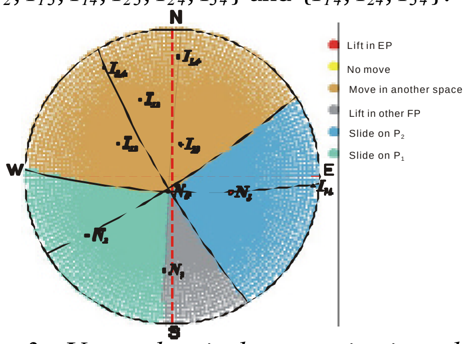

Figure 3. Upper hemisphere projection showing the modes of movement – Case A

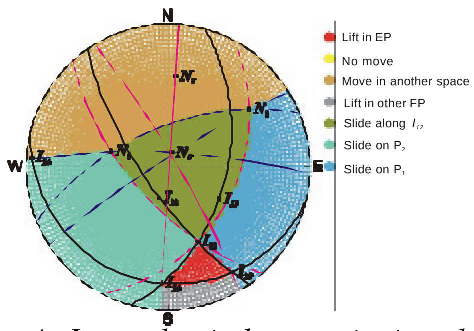

Figure 4. Lower hemisphere projection showing the modes of movement – Case A

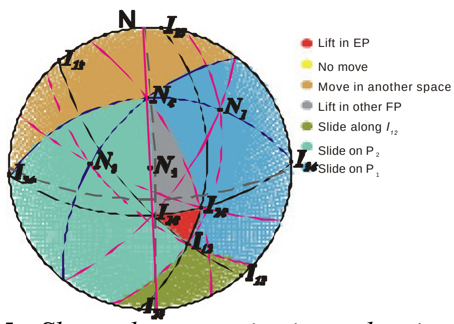

Figure 5. Slope-plane projection showing the modes of movement – Case A

4. Since the number of I_{ij} for a block, which is kinematically admissible of sliding, must be higher than the number of the enclosing planes, as illustrated in Fig. 7. The set, which has the

number of I_{ij} lower than the number of enclosing planes, is eliminated. Therefore, the set $\{I_{14}, I_{24}, I_{34}\}$ is eliminated for subsequent process and the set $\{I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}\}$ remains for subsequent process.

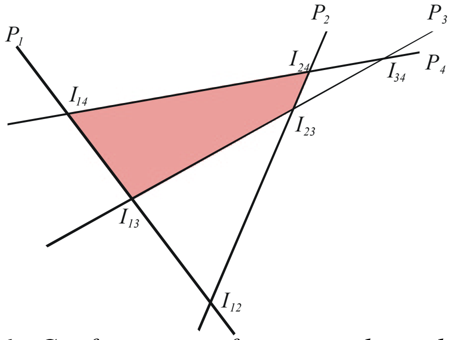

Figure 6. Configuration of joints on slope plane

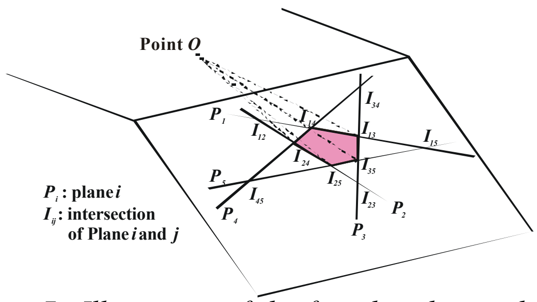

Figure 7. Illustration of the fact that the number of I_{ij} of a kinematically admissible block must greater than the number of planes.

Table 2 Process determining the pyramidal blocks enclosed by P_1 to P_4.

S	P	Set of intersectional lines	Set elimanated
1	1	$\{I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}\}$ $\{I_{14}, I_{24}, I_{34}\}$	$\{I_{14}, I_{24}, I_{34}\}$
2	2	$\{I_{14}, I_{24}, I_{13}, I_{23}, I_{12}\}$ $\{I_{12}, I_{24}, I_{23}, I_{34}\}$	None
3	3	$\{I_{14}, I_{24}, I_{13}, I_{23}\}$ $\{I_{13}, I_{12}, I_{23}\}$ $\{I_{12}, I_{23}, I_{34}\}$ $\{I_{24}, I_{23}, I_{34}\}$	$\{I_{13}, I_{12}, I_{23}\}$ $\{I_{12}, I_{23}, I_{34}\}$ $\{I_{24}, I_{23}, I_{34}\}$
4	4	$\{I_{14}, I_{24}, I_{13}, I_{23}\}$	

Remarks: S = Step number; P = Plane number.

5. By repeating steps 1 to 4 for each of the subsequent planes (P_2, P_3 and P_4), the final set is determined. As shown in Table 3, the final set (i.e., the pyramidal block, enclosed by four planes, kinematically admissible of sliding) is determined.
6. By repeating steps 1 to 5 for each of the configuration of blocks indicated by Eqn. 3, all the pyramidal blocks, which are kinematically admissible in sliding, are accordingly determined.

4. STABLILITY OF A SLIDING BLOCK

4.1 Determination of the volume

The magnitude of *EPt* can be determined in two ways, the length (*L*) or the height (*H*) of the trace of P_i on the slope plane (P_e) as:

$$EPt = L * \sqrt{\left(\vec{n_e} \cdot \vec{I_{ij}}\right)^{-2} + \left(\vec{n_e} \cdot \vec{I_{ik}}\right)^{-2} - 2 \times \frac{\overline{I_{ij}} \cdot \overline{I_{ik}}}{\vec{n_e} \cdot \vec{I_{ij}} + \vec{n_e} \cdot \vec{I_{ik}}}} \quad (6)$$

$$EPt = H \times \left(\frac{I_{z,ij}}{\overrightarrow{n_e} \cdot \overrightarrow{I_{ij}}} - \frac{I_{z,kl}}{\overrightarrow{n_e} \cdot \overrightarrow{I_{kl}}} \right) \quad (7)$$

The surface area of each joint plane and the excavation surface (of the block), A_i and A_e, can be determined as:

$$A_i = \frac{1}{2}\left|\vec{I}_{ij} \times \vec{I}_{ik}\right| \quad (8)$$

$$A_e = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{vmatrix} \quad (9)$$

Accordingly, the volume of the pyramidal block, *V*, can be expressed as:

$$V = \frac{1}{3} \times A_e \times |EPt| \quad (10)$$

4.2 Pyramidal block with parallel planes

The pair of parallel planes is simulated by the plane with almost parallel normal vectors, n_1 and n_2. The angle formed by this pair of vectors is q_g. That is,

$$n_1 \cdot n_2 = \cos q_g \quad (11)$$

The origin point of the pair is set to be far from the slope plane so that, when the pair of planes intersects the slope plane, they are almost parallel. Therefore, the three plane normals, n_1, n_2 and n_e, should be located on one plane, so that:

$$(n_1 \times n_e) \cdot n_2 = n_{bp} \cdot n_2 = 0 \quad (12)$$

Based on Eqn. 11 and Eqn. 12, the vector of n_2 can be determined by the following two equations:

$$\cos \boldsymbol{b} \cdot \sin \boldsymbol{a} \cdot n_{x1} + \sin \boldsymbol{b} \cdot \sin \boldsymbol{a} \cdot n_{y1} + \cos \boldsymbol{a} \cdot n_{z1} = \cos \boldsymbol{q}_g \quad (13)$$

$$\sin a\left(\cos b \cdot n_{x,bp} + \sin b \cdot n_{y,bp}\right) + \cos a \cdot n_{z,bp} = 0 \quad (14)$$

The plane normal of n_2 can be expressed in terms of *a* and *b* as:

$$(\cos b * \sin a, \sin b * \sin a, \cos a) \quad (15)$$

4.3 Factor of safety

The factor of safety of a block sliding on planes is the total sliding resistance of the block divided by the driving force, and can be expressed as:

$$F.S. = \frac{\sum_{i}^{2}(c_i \times A_i + Rn_i \times \tan f_i)}{Rs} \quad (16)$$

If the block slides only along one plane, then only the friction resistance of the one sliding plane is needed when applying Eqn. 16.

5. RESULTS OF THE PROPOSED ANALYSIS

5.1 Verification of the propsed method

A computer program, called "Slope Pyramidal Block Analysis (SPBA)" was developed to perform the abovementioned analysis. The program is able to analyzed wedges and pyramidal blocks with and without parallel planes. The output includes the configurations of all the kinematically admissible blocks, their factor of safety and the mode of sliding.

Case A (wedge) was analyzed by the proposed method, by Wedge Failure Analysis Module (by Kroeger) and by Swedge (Rockscience, Inc.) and identical results were obtained. The modes of sliding for the sliding wedge are shown in Fig. 5.

5.2 Analysis of a cut slope case

A slope system summarized in Table 3 has a natural slope plane (P_1) and three joint sets (P_2, P_3 and P_4). This natural slope is to be excavated to form a new slope plane (P_5).

After application of the intersection set method, it was found that only a block (enclosed by P_2, P_3 and P_4) is kinematically admissible of sliding. However, as shown in Fig. 8, the block can only move above the horizontal plane; therefore, it would not slide under gravity traction. As a result, it is confirmed that the slope system is stable before excavation.

After projecting on the excavated slope plane (P_5), as shown in Fig. 10, these blocks can move below the horizontal plane. The factor of safety, ranging from 1.08 to 1.81, for each block is summarized in Table 4.

Table 3 Configuration of a slope system – Case B

P	O	D	Type	C (kN/m^2)	ϕ (degree)
1	165	20	Ground surface	0	0
2	235	30	Joint plane	5	25
3	185	25	Joint plane	5	22
4	140	65	Joint plane	10	30
5	180	60	Excavation surface	0	0

Remarks: P = Number of plane; O = Orientation of plane (in degree clockwise from north); D = Dip angle (in degree)

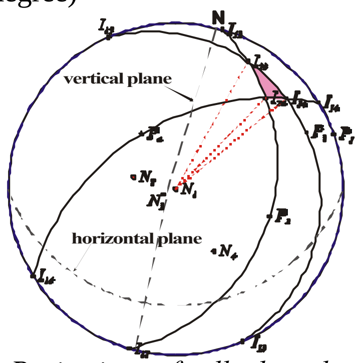

Figure 8. Projection of all the planes on the unexcavated slope plane

After excavation, a new slope plane is formed (P_5) and the original slope plane (P_1) becomes the crest of the slope. By applying intersection set method, four types of blocks prone to sliding were identified, as illustrated in Fig. 9.

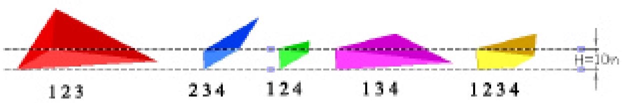

Figure 9. Kinematically admissible blocks identified.

If any improvement of the F_s of the blocks prone to sliding is desired, by applying rock bolt for instance, the optimum orientation of the rock bolt can be automatically found by first assuming a unit tensile force of the rock blot. After the optimum orientation of the rock bolt is found, the required tensile force of such rock bolt can be determined by specifying the desired new F_s (e.g., 1.5). Tables 5 summarized the orientations of rock bolts considered and the final factor of safety.

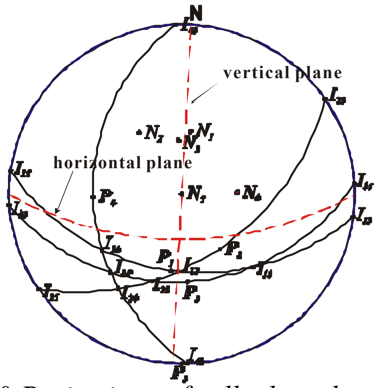

Figure 10.Projection of all the planes on the excavated slope plane.

Table 4 Identified sliding blocks and their F_s – Case B

P	Before application of rock bolt	
	Mode of sliding	F_s
1 2 3	Slide on P_3	1.10
1 2 4	Slide alone I_{24}	1.46
1 3 4	Slide on P_3	1.08
2 3 4	Slide alone I_{24}	1.81
1 2 3 4	Slide alone I_{23}	1.22

Remarks: P = planes which enclosing the block considered; F_s = factor of safety.

Table 5 Orientation and tensile force of rockblots

P	Optimum orientation			Designed orientation		
	O	D	T (kN/m^2)	O	D	T (kN/m^2)
1 2 3	5.0	10.0	35.0	5	-10	37.2
1 2 4	22.3	8.7	1.1	5	-10	1.2
1 3 4	5.0	10.0	20.7	5	-10	22.0
2 3 4						
1 2 3 4	24.7	8.7	13.4	5	-10	15.0

Remarks:

1. The requird factor of safety is 1.5.
2. P = planes which enclosing the block considered; O = Orientation of rock bolt (in degree); D = Dip of rock bolt (in degree); T = tensile force of the rock bolt.

6. CONCLUSION

Given that only the orientation of joints is known and in conjunction with the use of the proposed method and program, the analysis of the stability of blocks is now extended from a wedge (three planes) to a pyramidal block (more than three planes). Since the exact location of each joint is not needed for analyses, this development provides a handy and useful tool for assessing the stability of various types of blocks; stabilizing measures can be analyzed and prepared before excavation of the slope.

The proposed "intersection set method" and projection on slope plane enable convenience in determining the blocks which are kinematically admissible of sliding. All the blocks and all their possible sliding modes and associated factor of safety can be accordingly obtained and compared. This enables a systematic assessment on all the blocks prone to sliding, and a subsequent identification of the dominant types of sliding should be aware of.

In addition to the pyramidal blocks, the blocks with parallel planes and various water pressure distributions can also be analyzed.

7. REFERENCES

Goodman, R.E. & Shi, G.H. 1985. *Block Theory and its Application to Rock Engineering*, New Jersey, pp. 24-105.

Goodman, R.E. 1995. Block theory and its application. *Geotechnique* 4(3): pp. 383-423.

Hoek, E., Bray, J.W. & Boyd, J.M. 1973. The stability of a rock slope containing a wedge resting on two intersecting discontinuities. *Q. J. Engng Geol.* 6: pp.1-55.

John, K. W. 1968. Graphical Stability Analysis of Slopes in Jointed Rock. *J. of the Soil Mech. and Foundations Division* 94(SM2): pp. 497-526.

Kumsar, H., Aydan, O. & Ulusay, R. 2000. Dynamic and Static Stability Assessment of Rock Slopes Against Wedge Failure. *Rock Mechanics and Rock Engineering* 33(1): pp. 31-51.

Londe, P., Vigier, G. & Vormeringer, R. 1969. Stability of rock slopes–A Three-Dimensional study. *J. of the Soil Mech. and Foundations Division* 95(SM1): pp. 235-262.

Mauldon, M. & Ureta, J. 1996. Stability analysis of rock wedges with multiple sliding surfaces. *Geotech. & Geological Engrg.* 14: pp. 51-66.

Warburton, P.M. 1981. Vector Stability Analysis of an Arbitrary Polyhedral Rock Block with any Number of Free Faces. *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.* 18: pp. 415-427.

Yoon, W.S., Jeong, U.J. & Kim, J.H. 2002. Kinematic analysis for sliding Failure of Multi-faced rock slopes. *Engineering Geology*. 67: pp. 51-61.