行政院國家科學委員會補助專題研究計畫成果報告

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結構動力辛計算及控制之方法

Symplectic computation and control for structural dynamics

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一、摘要

本計畫在擴大的哈米爾頓架構下，處理多自由度結構之振動控制問題。主旨在利用李群理論之保群法則，建構合適之計算方法及控制方法，使得長時間計算得以準確穩定不發散，即時最佳控制得以實現。

本研究針對線性二次最佳控制法則所遭遇的困難提出解決之道，將外力對結構的累積效應納入控制律中。本研究得到縮辛群控制律，縮辛群即縮群與辛群的直積，其中的辛群保證了控制律的最佳化，縮群保證了控制律的穩定。換句話說，本研究已建立多自由度結構物受到外力歷時作用下之穩定最佳線性二次控制律。

關鍵詞：多自由度結構、振動、穩定化、辛群、縮群、線性二次控制、最佳控制、哈米爾頓系統

Abstract

The project studied the vibrations of the structures of multiple degrees of freedom (MDOF) under the extended Hamiltonian formalism. The emphasis was placed on constructing a Lie group of transformations with time being the parameter of the group so as to preserve as many as possible characteristic quantities as time elapses. The group was then used for computation and real-time optimal control.

The project examined the difficulties encountered by the linear quadratic (LQ) optimal control algorithm and took into consideration the accumulating effect of external disturbances. We established herein a control law based on a contractive symplectic group, which is the direct product of a contractive group and a symplectic group. The symplectic group ensures optimality of the control law, while the contractive group stabilizes the control law. In other words, we have achieved a stabilized optimal linear quadratic control law for MDOF structures subjected to external disturbances.

Keywords：MDOF structures, vibration, stabilization, symplectic group, contractive group, linear quadratic control, optimal control, Hamiltonian system.

二、緣由與目的

結構動力學除了牛頓體系外，尚有拉格朗日體系及哈米爾頓體系。這在學界是盡人皆知之事，但在土木結構工程界顯然很少拿來活用。多自由度結構的運動方程式如下：

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{w}(t) \tag{1}$$

若結構(1)加上控制力 $\mathbf{u}(t)$ ，要求最佳控制，這個問題如何在哈米爾頓體系下來做，這是本計畫要做的。我們的目的主要在於探討長時間準確穩定的算法，以及具有追蹤外力 $\mathbf{w}(t)$能力的真正最佳控制。

線性二次 (LQ) 最佳控制是目前比較完備的一種理論，應用十分廣泛，是現代控制理論中最重要的成果之一[1]。在土建工程的結構控制，受控對象往往有外力 $\mathbf{w}(t)$ 作用，譬如地震力或風力[2-10]。但應用線性二次最佳控制理論於受外力作用的結構時，對外力有非常嚴格的要求，具體言之，需事先知道外力的歷時，或假設外力是白噪音，或假設外力是線性濾波器在高斯白噪音為輸入下的輸出[3，11]。因此大部份文獻都是採用忽略外力作用的線性二次調節器 (LQR) 控制理論。基於上述的困難點，在各個方面皆考慮外力的線性二次最佳控制理論常被認為實際上不可行的，因為實時控制計算需要將來尚未發生的外力（載重歷時）的數據。經由我們以前的研究顯示，較佳的控制法則必須考慮內狀態變數，外力作用是完全不能忽略的[5]。本研究將針對此困難提出解決之道，基於包括外狀態及內狀態的全狀態觀念，提出全狀態反饋控制計算法則。 此一理論將外力對結構物的累積效應納入控制律中，因此可認為是線性二次最佳控制理論的徹底解決。

三、研究方法

Introduction

Recent decades witnessed many successful applications of linear quadratic (LQ) optimal control theory to active control of plants in various fields [1]. The theory provides the active control of plants with a two-point boundary value problem (TPBVP) formulation over a time interval $[t_0, t_f]$. Hence, to search for an optimal control, one has to solve the TPBVP. However, when the plants are engineering structures designed to withstand external disturbances (see, for example, [2-10]), solving the TPBVP encounters formidable obstacles since it has to be solved backward from the terminal time t_f , so that external disturbances which do not yet occur must be known in advance. Unless external disturbances are entirely absent or they are white noise stochastic processes or modeled as the output of a prescribed linear filter subjected to a Gaussian white noise input, the control laws currently being used do not achieve the goal of optimalizing the performance index that is set up [3, 4, 7, 11]. Unfortunately, most of external disturbances acting on engineering structures do not meet those restrictions. To overcome the obstacles we must trace back to the origin of the difficulties encountered, that is the TPBVP with the presence of external disturbances. In the next subsection we will show that with a correct construction we can obtain an initial value problem (IVP) formulation for the considered issue.

An Initial Value Problem Formulation

Consider a linear engineering structure with n degrees of freedom subjected to the external disturbance **w**(t) and the control force **u**(t). The equation of motion of the structure may be written as

$$\mathbf{M}\,\ddot{\mathbf{q}}(t)+\mathbf{C}\,\dot{\mathbf{q}}(t)+\mathbf{K}\,\mathbf{q}(t)=\mathbf{B}_S\mathbf{u}(t)+\mathbf{E}_S\mathbf{w}(t) \qquad (1)$$

in its lifetime $[t_b,\ t_d] \subset \mathrm{R}$ along with the values of $\mathbf{q}(t_0)$ and $\dot{\mathbf{q}}(t_0)$ prescribed at a certain time instant t_0. Here t is (the current) time, t_b and t_d are the birth and death times, respectively, of the structure, and t_0 is the initial time of a certain time interval $[t_0,\ t_f]$ in which we are interested. A superposed dot indicates time differentiation. $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the column matrices of relative displacements, velocities and accelerations, respectively. The symmetric matrices **M**, **C**, **K** are respectively, the mass, damping, and stiffness of the structure. To be an engineering structure **M** and **K** are positive definite and **C** is positive semidefinite.

This structural dynamical problem can be transformed to the following state space description:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{E}\mathbf{w}(t) \quad \forall t\in[t_b,t_d],\quad \mathbf{x}(t_0)=\mathbf{x}_0 \qquad (2)$$

where

$$\mathbf{x}(t)=\begin{bmatrix}\mathbf{q}(t)\\ \dot{\mathbf{q}}(t)\end{bmatrix},\quad \mathbf{x}_0=\begin{bmatrix}\mathbf{q}(t_0)\\ \dot{\mathbf{q}}(t_0)\end{bmatrix}, \qquad (3)$$

and

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{B}_s\end{bmatrix},\quad \mathbf{E}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{E}_s\end{bmatrix} \qquad (4)$$

As usual the superscript $^{-1}$ denotes the inverse and **0** and **I** denote the zero and identity matrices, respectively, of appropriate orders as implied in the context. In (2), the state $\mathbf{x} \in \mathrm{R}^{2n}$, the control $\mathbf{u} \in \mathrm{R}^{u}$, and the disturbance $\mathbf{w} \in \mathrm{R}^{w}$ are all functions of time; the initial state $\mathbf{x}_0 \in \mathrm{R}^{2n}$ is a constant column matrix; and $\mathbf{A} \in \mathrm{R}^{2n\times 2n}$, $\mathbf{B} \in \mathrm{R}^{2n\times u}$, $\mathbf{E} \in \mathrm{R}^{2n\times w}$ are constant matrices.

For a crucial consideration which will be made clear later let us introduce

$$\underline{\mathbf{x}}:=e^{at}\mathbf{x},\ \ \underline{\mathbf{u}}:=e^{at}\mathbf{u},\ \ \underline{\mathbf{w}}:=e^{at}\mathbf{w},\ \ \underline{\mathbf{x}}_0:=e^{at_0}\mathbf{x}_0 \qquad (5)$$

so that (2) becomes

$$\dot{\underline{\mathbf{x}}}(t)=(\mathbf{A}+a\mathbf{I})\underline{\mathbf{x}}(t)+\mathbf{B}\underline{\mathbf{u}}(t)+\mathbf{E}\underline{\mathbf{w}}(t) \quad \forall t\in[t_b,t_d],\quad \underline{\mathbf{x}}(t_0)=\underline{\mathbf{x}}_0. \qquad (6)$$

According to LQ optimal control theory, (6) is to be controlled during the fixed, finite time interval $[t_0,\ t_f] \subset [t_b,\ t_d] \subset \mathrm{R}$ such that the quadratic functional, called the performance index,

$$J=\frac{1}{2}\int_{t_0}^{t_f}[\underline{\mathbf{x}}'(t)\mathbf{Q}\underline{\mathbf{x}}(t)+\underline{\mathbf{u}}'(t)\mathbf{R}\underline{\mathbf{u}}(t)+2\underline{\mathbf{x}}'(t)\mathbf{T}\underline{\mathbf{u}}(t)]\,dt \qquad (7)$$

is minimized, where the symmetric weighting matrices

$$\begin{bmatrix}\mathbf{Q} & \mathbf{T}\\ \mathbf{T}' & \mathbf{R}\end{bmatrix}\in\mathrm{R}^{(2n+u)\times(2n+u)}$$

and $\mathbf{R} \in \mathrm{R}^{u\times u}$ are positive semidefinite and positive definite, respectively. The superscript $'$ stands for the transpose.

To minimize the performance index J subjected to the constraint (6), we adjoin the weighted residue of the IVP (6) to J,

$$L=J+\int_{t_b}^{t_d}\underline{\boldsymbol{\lambda}}'(t)[(\mathbf{A}+a\mathbf{I})\underline{\mathbf{x}}(t)+\mathbf{B}\underline{\mathbf{u}}(t)+\mathbf{E}\underline{\mathbf{w}}(t)-\dot{\underline{\mathbf{x}}}(t)]dt$$
$$+\underline{\boldsymbol{\lambda}}'(t_0)[\underline{\mathbf{x}}(t_0)-\underline{\mathbf{x}}_0], \qquad (8)$$

and then minimize L. The column matrix $\underline{\boldsymbol{\lambda}} \in \mathrm{R}^{2n}$ contains 2n Lagrange multipliers. Note that since the governing equation $(6)_1$ of the IVP (6) is valid throughout the lifetime $[t_b,\ t_d]$ of the engineering structure the constraint $(6)_1$ should be imposed for all the time t, $t_b \le t \le t_d$, as we have done in the above; this has led us to use the different time intervals $[t_0,\ t_f]$ and $[t_b,\ t_d]$ in the two integrals in (8) and this concept of nested intervals is crucial in obtaining an IVP formulation. Among all admissible varied functions $\underline{\mathbf{x}}(t)+\delta\underline{\mathbf{x}}(t),\underline{\mathbf{u}}(t)+\delta\underline{\mathbf{u}}(t),\underline{\boldsymbol{\lambda}}(t)+\delta\underline{\boldsymbol{\lambda}}(t)\ \forall t\in[t_b,t_d]$ which are otherwise arbitrary except that $\delta\underline{\mathbf{x}}(t)$ vanishes at the two ends t_b and t_d and that $\underline{\mathbf{u}}$(t) and hence $\delta\underline{\mathbf{u}}(t)$ are zero outside the control interval $[t_0,\ t_f]$, the necessary conditions for the optimal functions $\underline{\mathbf{x}}$(t), $\underline{\mathbf{u}}$(t), $\underline{\boldsymbol{\lambda}}$(t) that extremize L are found to be

$$\dot{\underline{\boldsymbol{\lambda}}}(t)=-(\mathbf{A}+a\mathbf{I})'\underline{\boldsymbol{\lambda}}(t)\quad \forall t\in[t_b,t_d], \qquad (9)$$

$$\dot{\underline{\boldsymbol{\lambda}}}(t)=-(\mathbf{A}+a\mathbf{I})'\underline{\boldsymbol{\lambda}}(t)-\mathbf{Q}\underline{\mathbf{x}}(t)-\mathbf{T}\underline{\mathbf{u}}(t) \quad \forall t\in[t_0,t_f],\ \underline{\boldsymbol{\lambda}}(t_0)=0, \qquad (10)$$

$$\dot{\underline{\boldsymbol{\lambda}}}(t)=-(\mathbf{A}+a\mathbf{I})'\underline{\boldsymbol{\lambda}}(t)\quad \forall t\in[t_f,t_d], \qquad (11)$$

and

$$\underline{\mathbf{u}}(t)=-\mathbf{R}^{-1}\mathbf{B}'\underline{\boldsymbol{\lambda}}(t)-\mathbf{R}^{-1}\mathbf{T}'\underline{\mathbf{x}}(t)\quad \forall t\in[t_0,t_f], \qquad (12)$$

as well as the IVP (6). All of these have resulted from extremizing the J of equation (8). In this way, as a result of the extremization, $(6)_1$ is valid for $t\in[t_b,\ t_d]$ as it should be. If in constructing L the constraint $(6)_1$ is imposed only for $[t_0,\ t_f]$ as was conventional, then through the extremization process $(6)_1$ will turn out to hold only for $[t_0,\ t_f]$, which is obviously absurd if we recall that an active control may last as short as $t_f - t_0$ = 50 seconds, say, but an engineering structure may stand for more than $t_d - t_b$ = 50 years, say. Notice that both equations (10) and (12) are valid only over the control interval $[t_0, t_f]$.

Let us now concentrate on the control interval$[t_0, t_f]$. Combining (6) and (10) and using (12), we have

$$\frac{d}{dt}\begin{bmatrix}\underline{\mathbf{x}}(t)\\ \underline{\lambda}(t)\end{bmatrix}=\mathbf{H}\begin{bmatrix}\underline{\mathbf{x}}(t)\\ \underline{\lambda}(t)\end{bmatrix}+\begin{bmatrix}\mathbf{E}\underline{\mathbf{w}}(t)\\ 0\end{bmatrix}\quad \forall t\in[t_0,t_f],\quad \begin{bmatrix}\underline{\mathbf{x}}(t_0)\\ \underline{\lambda}(t_0)\end{bmatrix}=\begin{bmatrix}\underline{\mathbf{x}}_0\\ 0\end{bmatrix} \tag{13}$$

where the matrix **H** is defined as

$$\mathbf{H}:=\begin{bmatrix}\mathbf{A}_1 & \mathbf{N}\\ -\mathbf{P} & -\mathbf{A}_1^t\end{bmatrix}, \tag{14}$$

in which

$$\mathbf{A}_1:=\mathbf{A}+a\mathbf{I}-\mathbf{B}\mathbf{R}^{-1}\mathbf{T}',\ \mathbf{N}:=-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}',\ \mathbf{P}:=\mathbf{Q}-\mathbf{T}\mathbf{R}^{-1}\mathbf{T}' \ . \tag{15}$$

Therefore, when the control is regulated by the optimal law (12), the optimal $\underline{x}(t)$ and $\underline{\lambda}(t)$ are governed by the IVP (13). It is easy to check that **N** and **P** are symmetrical and thus **H** is a Hamiltonian matrix, which is, by definition, a square matrix satisfying

$$(\mathbf{JH})^t=\mathbf{JH},\ \mathbf{J}:=\begin{bmatrix}0 & \mathbf{I}\\ -\mathbf{I} & 0\end{bmatrix},\quad \mathbf{J}^{-1}=\mathbf{J}^t=-\mathbf{J},\ \mathbf{J}^2=-\mathbf{I}\cdot \tag{16}$$

It can be shown that **N** is negative semidefinite. As such, the (13) is a constant coefficient linear Hamiltonian system defined in a symplectic space endowed with the canonical metric **J** defined in $(16)_2$.

Symplectic Group and Optimality

It is known that for any real Hamiltonian matrix $\mathbf{H} \in R^{4n\times 4n}$ there exists a symplectic matrix $\Psi \in C^{4n\times 4n}$, which is, by definition, a square matrix satisfying

$$\Psi'\mathbf{J}\Psi=\mathbf{J}\ , \tag{17}$$

or, equivalently

$$\Psi\mathbf{J}\Psi'=\mathbf{J}, \tag{18}$$

such that **H** is similar to a Hamiltonian matrix $\Omega \in C^{4n\times 4n}$, namely

$$\mathbf{H}\Psi=\Psi\Omega. \tag{19}$$

Equation (19) is called the relation of symplectic similarity, and equations (17) and (18) the relations of symplectic orthogonality. In fact, the Ψ is the fundamental solution of the IVP (13) and all Ψ's constitutes a symplectic group which characterizes the optimality of the active control.

Contractive Group and Stabilization

Although we have been able to get an IVP formulation such as (13) rather than a TPBVP formulation, the IVP (13) is still suffering from instability. To see this, let us examine the system matrix **H** of (13), which is a real Hamiltonian matrix. Recall that the eigenvalues of a real Hamiltonian matrix are of four types: (1) the quadruples of truly complex eigenvalues $\pm\gamma_1\pm\ \delta_1 i,\ \cdots,\ \pm\gamma_r\pm\delta_t i$, (2) the pairs of real eigenvalues$\pm\alpha_1\ \cdots\ \ \pm\alpha_r$, (3) the pairs of purely imaginary eigenvalues $\pm\beta_1 i,\ \cdots,\ \pm\beta_s i$, and (4) the eigenvalue 0. Hence, for the more frequently occurring types (1) and (2), half of eigenvalues have positive real parts, leading to unstable solutions of (13).

To stabilize the solutions let us now define

$$\nu:=\begin{bmatrix}\mathbf{x}\\ \lambda\end{bmatrix}, \tag{20}$$

where, similar to (5),

$$\underline{\lambda}=:e^{at}\lambda. \tag{21}$$

The ν may be called the complete state [5], consisting of the state **x** and the costate λ. From (13) together with (5) and (21), we find that the complete state ν of the controlled structure is governed by the IVP

$$\nu(t)=(\mathbf{H}-a\mathbf{I})\nu(t)+\mathbf{f}(t)\ \forall t\in[t_o,t_f],\nu(t_o)=\nu_o, \tag{22}$$

where

$$\mathbf{f}(t):=\begin{bmatrix}\mathbf{E}\mathbf{w}(t)\\ \mathbf{0}\end{bmatrix},\nu_0:=\begin{bmatrix}\mathbf{x}_0\\ \mathbf{0}\end{bmatrix},$$

the latter being the initial complete state. Now it is very easy to choose a value of a such that all the eigenvalues of $\mathbf{H}-a\mathbf{I}$ have negative real parts so as to stabilize the controlled structure. All the scalar matrices $a\mathbf{I}$ form a contractive group. The direct product of the contractive group and the symplective group characterizes the mathematical structure of the IVP formulation (22) of the optimally controlled engineering structure.

四、結論與討論

It has been shown that, with both the concepts of nested time intervals and exponentally weighted time functions, a stable initial value problem formulation (22) for the linear quadratic (LQ) optimal control of engineering structures subjected to external disturbance can be obtained. The formulation is markedly different from the conventional two-point boundary value problem formulation. The proposed new control theory has fully taken into account the effects of external disturbances. No essential restrictions are imposed on the nature of external disturbances; it can be deterministic or random, Gaussian or non-Gaussian. Although by itself sufficing to describe the linear structure, the state **x** must be supplemented by the costate λ for a complete state **v** description of the LQ system, which contains the linear structure and the quadratic-form optimal controller. The complete state control takes full advantage of the information of external disturbances but does not need to know the external disturbances in advance. It has also been shown that through an analysis of the eigenvalues of the Hamiltonian matrix **H**, which contains the information of the parameters of the controlled structures, one can easily choose an appropriate value of a such as to stabilize the optimally controlled structure.

五、參考文獻

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六、計畫成果自評

1. 工作項目

(1) 多自由度結構運動方程式轉成哈米爾頓體系
(2) 多自由度結構運動方程式轉成哈米爾頓體系比較，尤其阻尼項
(3) 多自由度結構運動方程式轉成哈米爾頓體系物理意義探討
(4) 多自由度結構之線性二次最佳控制之初始問題列式
(5) 辛正交關係以及特徵值問題
(6) 辛群之應用研究
(7) 多自由度結構之辛算法
(8) 多自由度結構在外力作用下之最佳化控制

自評：

項目（1）~（8）大致完成，尤其核心部分已超越進度並全數完成，現在不僅能做到最佳化而且是穩定最佳化，這是一個極大的突破。

項目（1） 多自由度結構運動方程式轉成哈米爾頓體系已完成。

項目（2） 多自由度結構運動方程式轉成哈米爾頓體系，不能含阻尼項；如果要含阻尼項，必須擴大哈米爾頓體系。

項目（3） 多自由度結構運動方程式轉成哈米爾頓體系物理意義探討，將於第二期計畫中繼續探討。

項目（4） 多自由度結構之線性二次最佳控制之初始問題列式已全數完成。

項目（5） 辛正交關係以及特徵值問題已全數完成。

項目（6） 辛群之應用研究已有相當深入之了解。

項目（7） 多自由度結構之辛算法。我們高度懷疑辛算法是穩定的，但是文獻中一直未正面提及這個問題。對此，本計畫有突破性的成果，已完成縮辛群法則，並由此發展縮辛群控制法則，如本精簡報告所述；此外亦可由此發展縮辛算法，這樣才會穩定。

項目（8） 多自由度結構在外力作用下之最佳化控制已全數完成。在此我們特別強調，本計畫發展的控制律，可實現為實時控制，所需要的外力是只到現在為止，並不需要尚未發生的將來外力。

2. 著作發表

已發表研討會論文[10] H-K. Hong, 'Stabilized optimal linear quadratic control for structures under external disturbances,' *Proceedings of the First International Conference on Structural Stability and Dynamics (ICSSD 2000)*, December 7-9, 2000, Taipei, Taiwan, pp.453-458.

預計修改論文稿[8] H.-K. Hong, C.-S. Liu and D.-Y. Liou, 'An IVP formulation for LQ optimal control of structures against earthquakes and its symplectic properties,' unpublished manuscript, 1999. 與論文稿[9] H.-K. Hong, C.-S. Liu and D.-Y. Liou, 'Linear quadratic controllers for buildings against earthquakes,' unpublished manuscript, 1999. 使其中的最佳化成為穩定最佳化，並發表於學術期刊。

3. 學生畢業論文

陳冠中，'以單自由度問題探討線性二次最佳控制的辛性質'，國立台灣大學土木工程學研究所碩士論文，民國八十九年六月，台北。