行政院國家科學委員會專題研究計畫成果報告

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計畫主持人： 黃 良 雄

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行政院國家科學委員會專題研究計畫成果報告

波、流平行於多孔介質牆之弔詭的研究(I)

A study on the paradox of wave and current with a parallel porous wall (I)

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Uniform Laminar Flow past a Porous Spherical Shell

H. J. Hsu[1], L. H. Huang[2], P. C. Hsieh[3]

Abstract

By applying Darcy's law as governing equation, Jones (1973) investigated the flow field of a uniform laminar flow with low Reynolds number past a porous spherical shell. Due to the potential characteristics of Darcy's law, it is impossible to imply viscous effect in the porous medium. And due to the improperly extension from the results of Beavers and Joseph (1967) and Saffman (1971) for spherical coordinate to fulfill the tangential boundary conditions, the continuity conditions of tangential velocity and shear stress are not satisfied on boundaries. Therefore the result obtained by Jones (1973) has the necessity to be modified.

A new solution of Jones' (1973) problem is reinvestigated based on Song and Huang's (2000) theory of laminar poroelastic media flow in the present work. It is observed that the new analytical solution can sufficiently delineate the viscous effect on boundaries and satisfy the boundary conditions of continuity of tangential velocity and shear stress. It is also found that as the porosity approaches to 1, the flow field retrogrades to the uniform laminar flow; and as the porosity approaches to 0, the well-known Stokes' solutions of a uniform laminar flow with low Reynolds number past a porous spherical shell are obtained.

均勻層流通過多孔介質球殼之流場解析

徐浩仁[1] 黃良雄[2] 謝平城[3]

摘　要

Jones (1973)以達西定律(Darcy's law)為控制方程式，並以 Beavers and Joseph (1967)與 Saffman (1971)之邊界條件處理多孔介質流，探討低雷諾數均勻層流通過多孔介質球殼之流場。然而達西定律無法展現多孔介質邊界層之黏性效應，且邊界上均未滿足切線速度及剪應力之連續條件。

本文中，對於多孔介質的部分，係採用 Song and Huang (2000)所提出之多孔介質彈性理論的控制方程式與邊界條件，重新分析低雷諾數均勻層流通過多孔介質球殼之流場。其結果可充分展現邊界之黏滯效應，同時亦滿足切線速度連續與剪應力連續

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之邊界條件。

本文推導出完整的解析解，且進一步探討介質孔隙率之極限情形，即當孔隙率趨近於 1 時，流場可退化為均勻層流；而當孔隙率趨近於 0 時，可獲得史托克斯(Stokes)著名的低雷諾數均勻層流通過圓球之解析解。

1. Introduction

In early researches for the flow interacting with a porous medium, Darcy's law was often used to dominate the flow field in the porous region. Although the results were not bad in agreement with experiments, and a great deal of achievements had been obtained in use of this potential model [e.g., Huang and Song (1993)], slip phenomena at interfaces could not be avoided. Because Darcy's law only poses the form of a first-order differentiation, the surplus boundary conditions with respect to continuous tangential velocity and shear stress are overdetermined. It brings about some perplexity, especially when the no slip condition is applied for revealing physical mechanism on the boundaries.

A plausible but empirical tangential boundary condition proposed by Beavers and Joseph (1967) and Saffman (1971) for porous plane boundaries when viscous effect is not negligible is

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k_p}}(u - q), \qquad (1)$$

where y is the coordinate normal to the surface, u is the mean velocity parallel to the surface, q is the tangential fluid velocity inside the porous medium at $y = 0$, k_p is the specific permeability, and α is a dimensionless parameter depending only upon the properties of the medium. For laminar flow passing over plane porous bed of infinite thickness, Huang and Chiang (1997) proved that equation (1) is appropriate.

By applying Darcy's law as governing equation, Jones (1973) investigated the flow field of a uniform flow with low Reynolds number past a porous spherical shell. Because of the spherical geometry of boundaries, not only boundary conditions in normal direction are necessary [see Huang, Hsieh, and Chang (1993) for normal boundary conditions], but also those in tangential direction are required. In order to solve the problem with spherical coordinate, Jones (1973) hence proposed a tangential boundary condition, which was improperly extended from Beavers and Joseph (1967) and Saffman (1971) as

$$e_{r\theta} = \frac{\alpha}{\sqrt{k_p}}(u_\theta - q_\theta), \qquad (2)$$

where $e_{r\theta}$ is the rate of strain, in application for problems with spherical boundaries.

We will find later that due to the improper extension of the partial- slip boundary condition proposed by Beavers and Joseph (1967) and Saffman (1971), the occurrence of slip situation at interfaces is inevitable. It will yield handicaps while the continuity of tangential velocity and shear stress are absolutely essential and have a great effect upon actual circumstances. Furthermore, the potential characteristic of Darcy's model means that the solutions cannot demonstrate viscous effect of the flow inside the porous medium, as well as at interfaces.

Song and Huang (2000) proposed complete governing equations and boundary conditions in the poroelastic media theory. By applying the governing equations and boundary conditions proposed by Song and Huang (2000), it is noted that both the normal and tangential boundary conditions are applied in this study. The result can properly represent viscous effect in the porous medium and satisfy the continuity of tangential velocity and shear stress on boundaries. In addition, the verifications of limiting cases ascertain that the flow field retrogrades to the uniform laminar flow as the porosity approaches to 1, and retrogrades to the well-known Stokes flow as it approaches to 0.

2. Jones' Solutions

Jones (1973) investigated the flow field of a uniform flow passing a porous spherical shell. The spherical shell of external radius a and internal radius b is immersed in a uniform stream of velocity U. The external region, the porous region, and the cavity are denoted as regions I, II, and III respectively, shown in Figure 1. He derived and obtained complete analytic solutions by using Darcy's law in control of the porous medium, region II, and boundary conditions extended from Beavers and Joseph (1967) and Saffman (1971).

In regions I and III, equations governing the flow are the Stokes equation

$$\nabla^2 \underline{u} = \frac{1}{\mu}\nabla p, \tag{3}$$

where $\underline{u}$ and p are the velocity vector and pressure of flow, and μ is the fluid viscosity. And the continuity equation is

$$\nabla \cdot \underline{u} = 0. \tag{4}$$

In the porous medium, region II, the flow is governed by Darcy's law

$$\underline{q} = -\frac{k_p}{\mu}\nabla p, \tag{5}$$

where $\underline{q}$ is the velocity vector inside the porous medium. And the continuity equation is

$$\nabla \cdot \underline{q} = 0. \tag{6}$$

Note that Darcy's law is only a first-order differentiation, so it cannot demonstrate the viscous effect in region II, and by connecting with regions I and III dominated by the second-order Stokes equation, the continuity conditions with respect to tangential properties are overdetermined.

Boundary conditions at interfaces of the spherical shells (i.e., r = a, b) are continuity of pressure,

$$p_1\big|_{r=a} = p_2\big|_{r=a},\ p_2\big|_{r=b} = p_3\big|_{r=b}, \tag{7}$$

continuity of normal fluid flux,

$$u_{r1}\big|_{r=a} = q_{r2}\big|_{r=a},\ q_{r2}\big|_{r=b} = u_{r3}\big|_{r=b}, \tag{8}$$

and empirical tangential boundary conditions, which are improperly extended from Beavers and Joseph (1967) and Saffman (1971), proposed by Jones (1973),

$$e_{r\theta 1}\big|_{r=a} = \beta\left(u_{\theta 1}\big|_{r=a} - q_{\theta 2}\big|_{r=a}\right),$$

$$e_{r\theta 3}\big|_{r=b} = -\beta\left(u_{\theta 3}\big|_{r=b} - q_{\theta 2}\big|_{r=b}\right), \tag{9}$$

where $\frac{\alpha}{\sqrt{k_p}}$ is denoted as β and the rate of strain $e_{r\theta}$ is given as

$$e_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}. \tag{10}$$

By solving equations (3)~(6) with boundary conditions (7)~(9), analytic solutions are obtained.

In region I,

$$p_1 = \mu U\left(\frac{C_2}{r^2}\right)\cos\theta, \tag{11}$$

$$u_{r1} = U\left(1 + \frac{C_2}{r} + \frac{C_3}{r^3}\right)\cos\theta, \tag{12}$$

$$u_{\theta 1} = -U\left(1 + \frac{C_2}{2r} - \frac{C_3}{2r^3}\right)\sin\theta. \tag{13}$$

In region II,

$$p_2 = U\left(\frac{C_5}{r^2} + C_6 r\right)\cos\theta, \tag{14}$$

$$q_{r2} = \frac{k_p U}{\mu}\left(\frac{2C_5}{r^3} - C_6\right)\cos\theta, \tag{15}$$

$$q_{\theta 2} = \frac{k_p U}{\mu}\left(\frac{C_5}{r^3} + C_6\right)\sin\theta. \tag{16}$$

In region III,

$$p_3 = \mu U(10C_4 r)\cos\theta, \quad (17)$$

$$u_{r3} = U(C_1 + C_4 r^2)\cos\theta, \quad (18)$$

$$u_{\theta 3} = -U(C_1 + 2C_4 r^2)\sin\theta, \quad (19)$$

where $C_1 \sim C_6$ can be determined by solving simultaneous boundary conditions (7)~(9).

3. Present Study

In dealing anew with the problem of a uniform flow passing a porous spherical shell, it is required to reset equations that will appropriately delineate the viscous effect of fluid in the porous medium. Besides, the equations of continuity of tangential velocity and shear stress are to be satisfied at interfaces, too.

Song and Huang (2000) proposed complete governing equations and boundary conditions in the poroelastic media theory. For the porous medium, region Ⅱ, we adopt the governing equations and boundary conditions from the result of Song and Huang (2000) to reanalyze the flow field.

3.1 Governing Equations

In regions I and Ⅲ, governing equations are taken to be identical to Jones' (1973) study, i.e., equations (3) and (4).

When we analyze the flow phenomena inside the porous medium, region Ⅱ, the fluid viscosity is considered. Hence instead of Darcy's flow, the governing equation proposed by Song and Huang (2000) is employed to dominate the flow field in region Ⅱ,

$$\nabla^2 \underline{q} - \frac{n}{k_p}\underline{q} = \frac{n}{\mu}\nabla p, \quad (20)$$

where n is the porosity of porous medium. And the equation of continuity is still

$$\nabla \cdot \underline{q} = 0 \quad (21)$$

as usual. Because equation (20) is a second-order differential equation, different from equation (5), it will completely handle the overdetermined tangential boundary conditions provided that proper boundary conditions are founded. Besides, the added viscous term of equation (20), compared with equation (5), indicates that the governing equation will be able to represent properly the viscous effect of the porous medium, region Ⅱ.

3.2 Boundary Conditions

Since we have adopted equation (20) to govern the porous medium flow in region Ⅱ, the correction of tangential velocity and shear stress at interfaces are probably available to be concerned about. For this reason, it is necessary to find out suitable boundary conditions that satisfy the continuity of above physically properties.

Song and Huang (2000) derived and proposed eight physical variables that should be continuous at porous interfaces. In our work, we just use four of them to construct proper boundary conditions. They are normal fluid stress, normal fluid flux, tangential fluid stress, and tangential relative flow velocity.

The boundary conditions at interfaces of the porous shells r = a, b according to the continuity of above four physical variables are listed below:

1. Continuity of normal fluid stress:

$$2\mu\frac{\partial u_{r1}}{\partial r}\Big|_{r=a} - p_1\Big|_{r=a} = \frac{2\mu}{n}\frac{\partial q_{r2}}{\partial r}\Big|_{r=a} - p_2\Big|_{r=a}, \quad (22)$$

$$2\mu\frac{\partial u_{r3}}{\partial r}\Big|_{r=b} - p_3\Big|_{r=b} = \frac{2\mu}{n}\frac{\partial q_{r2}}{\partial r}\Big|_{r=b} - p_2\Big|_{r=b}. \quad (23)$$

Because equation (20) is the differentiation of second order, we no longer merely use the continuity of pressure, i.e., equation (7), but can sufficiently employ more generalized normal fluid stress as the continuity condition.

2. Continuity of normal fluid flux:

$$u_{r1}\Big|_{r=a} = q_{r2}\Big|_{r=a}, \quad (24)$$

$$u_{r3}\Big|_{r=b} = q_{r2}\Big|_{r=b}, \quad (25)$$

which are identical to equation (8).

3. Continuity of tangential fluid stress:

$$\frac{\partial u_{\theta 1}}{\partial r}\Big|_{r=a} - \frac{u_{\theta 1}}{r}\Big|_{r=a} + \frac{1}{r}\frac{\partial u_{r1}}{\partial \theta}\Big|_{r=a}$$

$$= \frac{1}{n}\frac{\partial q_{\theta 2}}{\partial r}\Big|_{r=a} - \frac{1}{n}\frac{q_{\theta 2}}{r}\Big|_{r=a} + \frac{1}{n}\frac{1}{r}\frac{\partial q_{r2}}{\partial \theta}\Big|_{r=a}, \quad (26)$$

$$\frac{\partial u_{\theta 3}}{\partial r}\Big|_{r=b} - \frac{u_{\theta 3}}{r}\Big|_{r=b} + \frac{1}{r}\frac{\partial u_{r3}}{\partial \theta}\Big|_{r=b}$$

$$= \frac{1}{n}\frac{\partial q_{\theta 2}}{\partial r}\Big|_{r=b} - \frac{1}{n}\frac{q_{\theta 2}}{r}\Big|_{r=b} + \frac{1}{n}\frac{1}{r}\frac{\partial q_{r2}}{\partial \theta}\Big|_{r=b}. \quad (27)$$

4. Continuity of tangential relative fluid velocity:

$$u_{\theta 1}\big|_{r=a} = q_{\theta 2}\big|_{r=a}, \quad (28)$$

$$u_{\theta 3}\big|_{r=b} = q_{\theta 2}\big|_{r=b}. \quad (29)$$

Equations (26)~(29) are now adopted to replace equation (9). Equation (9) is an improper extension from Beavers and Joseph (1967) and Saffman (1971), proposed by Jones (1973). We now realize that because of the property of first-order differentiation of equation (5), it is neither possible to imply viscous effect in the porous medium, nor to introduce equations (26)~(29) as boundary conditions for Jones' (1973) solutions. Therefore, Jones' solutions do not satisfy the continuity of tangential velocity and shear stress.

3.3 Solutions

The method of separation of variables is applied to solve the governing equations (3)(4) and (20)(21) with the boundary conditions (22)~(29). By taking some manipulation, the final forms of solutions are obtained.

In region Ⅰ,

$$p_1 = \mu U\left(\frac{C_2}{r^2}\right)\cos\theta, \quad (30)$$

$$u_{r1} = U\left(1 + \frac{C_2}{r} + \frac{C_3}{r^3}\right)\cos\theta, \quad (31)$$

$$u_{\theta 1} = -U\left(1 + \frac{C_2}{2r} - \frac{C_3}{2r^3}\right)\sin\theta. \quad (32)$$

In region Ⅱ,

$$p_2 = \frac{\mu U}{n}\left(\frac{1}{2r^2}C_7 - m^2 r C_8\right)\cos\theta, \quad (33)$$

$$q_{r2} = U\Big[\,\frac{mr\sinh mr - \cosh mr}{r^3}C_5 + \frac{mr\cosh mr - \sinh mr}{r^3}C_6 + \frac{1}{m^2 r^3}C_7 + C_8\,\Big]\cos\theta, \quad (34)$$

$$q_{\theta 2} = U\Big[\,\frac{mr\sinh mr - (1+m^2r^2)\cosh mr}{2r^3}C_5 + \frac{mr\cosh mr - (1+m^2r^2)\sinh mr}{2r^3}C_6 + \frac{1}{2m^2r^3}C_7 - C_8\,\Big]\sin\theta, \quad (35)$$

where $\sqrt{\frac{n}{k_p}}$ is denoted as m.

In region Ⅲ,

$$p_3 = \mu U(10C_4 r)\cos\theta, \quad (36)$$

$$u_{r3} = U(C_1 + C_4 r^2)\cos\theta, \quad (37)$$

$$u_{\theta 3} = -U(C_1 + 2C_4 r^2)\sin\theta. \quad (38)$$

The boundary conditions (22)~(29) provide eight simultaneous equations to determine eight unknown constants, $C_1 \sim C_8$.

4. Discussion

Since we have employed the continuity of tangential relative fluid velocity and tangential fluid stress as boundary conditions, tangential relative fluid velocity and tangential fluid stress are continuous on the boundaries. We make clear our point of view in Figures 2 to 5. Figure 2 shows the tangential relative fluid velocity profiles obtained according to analytical solutions (30)~(38) in two directional angles from the spherical origin outward ($\theta = 45°, 90°$); Figure 3 shows the profiles according to Jones' (1973) solutions (11)~(19) for comparison. We find that our curves are continuous at both inner and outer surfaces of spherical shell ($r = a, b$), but Jones' are not. The same circumstances are shown in Figure 4 and 5 for the tangential fluid stress. Therefore, in the present analytical solutions the slipping condition due to the potential Darcy's model is eliminated at interfaces, and Jones' handicap hence disappears.

Another verification of our analytical solutions is to investigate special cases such as how the limiting values of porosity n lead to. We note that $m=\sqrt{n/k_p}$ is composed of the porosity n and the permeability k_p that always interact mutually. By introducing Kozeny-Karman equation of soil mechanics [e.g. see Lambe and Whitman (1969)]

$$k_p = \frac{n^3}{K_0 S^2 (1-n)^2}, \tag{39}$$

where K_0 is the shape factor and S is the specific surface area. We express k_p in terms of n and then transform the solutions into polynomial terms in preparation for discussing limiting values of porosity.

First, we study the limiting case of n approaching to 0. It is found that the coefficient C_2 converges to $-\frac{3}{2}a$, C_3 converges to $\frac{a^3}{2}$, and the other coefficients vanish. Consequently, only the flow field of region I still exists, and the solutions turn into

$$p_1 = \mu U \left(-\frac{3}{2a} \right) \cos\theta, \tag{40}$$

$$u_{r1} = U \left(1 - \frac{3a}{2r} + \frac{a^3}{2r^3} \right) \cos\theta, \tag{41}$$

$$u_{\theta 1} = -U \left(1 - \frac{3a}{4r} - \frac{a^3}{4r^3} \right) \sin\theta, \tag{42}$$

which are just the well-known Stokes solutions of a uniform flow passing a rigid impermeable sphere, shown in Figure 6. Thus it is recognized that the flow in the limiting condition of porosity approaching to 0 indeed conforms physical expectation.

Second, let n approach to 1 and ascertain how the flow field becomes. The solutions turn into

$$p_1 = p_2 = p_3 = 0, \tag{43}$$

$$u_{r1} = u_{r2} = u_{r3} = U \cos\theta, \tag{44}$$

$$u_{\theta 1} = u_{\theta 2} = u_{\theta 3} = -U \sin\theta, \tag{45}$$

which are just the uniform flow driven by a far field velocity U as if the porous spherical shell didn't exist, shown in Figure 7. (Note that Jones (1973) could not recover this limiting case.) Therefore, it can be identified that the flow in the limiting condition of porosity approaching to 1 will retrograde to a simple uniform flow field, which also coincides with physical expectation.

The general range of the porosity of soils are known as 0.1~0.5, so we select a medium value of 0.3 and assume that the permeability is $0.5 m^2$. The flow state of the analytical solutions (30)~(38), and coefficients in substitution for the specified values are known in Figure 8. We can see clearly the flow variates and penetrates through the porous shell.

5. Conclusion

In this study, in order to remedy handicaps of Jones' (1973) solution of a uniform laminar flow with low Reynolds number passing a porous spherical shell, we reinvestigate the flow field by adopting the second-order governing equation and sufficient boundaries from the laminar poroelastic flow model proposed by Song and Huang (2000). The solutions imply apparently the viscous effect in the porous region that the Darcy's model can not provide. Furthermore, the continuity conditions of tangential velocity and shear stress are properly satisfied at interfaces.

It is also proved in the present study that the flow field will retrograde to Stokes flow while the porosity approaches to 0, and retrograde to a simple uniform flow while the porosity approaches to 1. Both of the limiting cases are in accord with actual physical expectation and verify the correctness of the present study.

Acknowledgement

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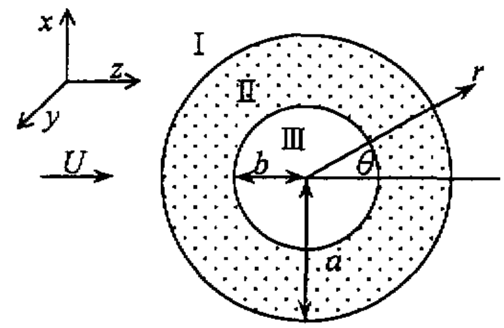

Figure 1. The physical situation and coordinate system

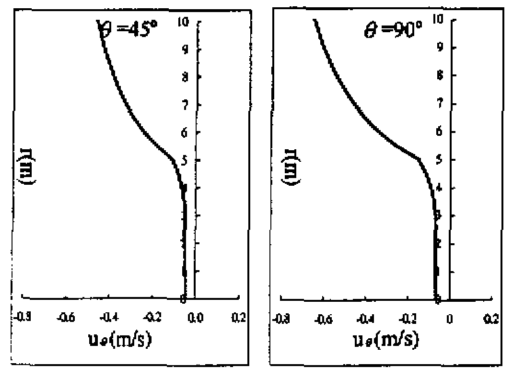

Figure 2. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m^2, U=1.0m/s, a=5m, b=3m in the present solutions

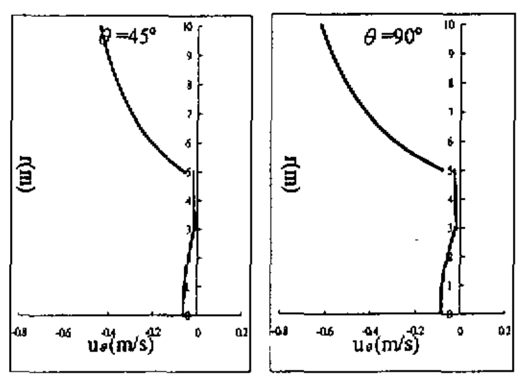

Figure 3. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m^2, U=1.0m/s, a=5m, b=3m in Jones' solutions

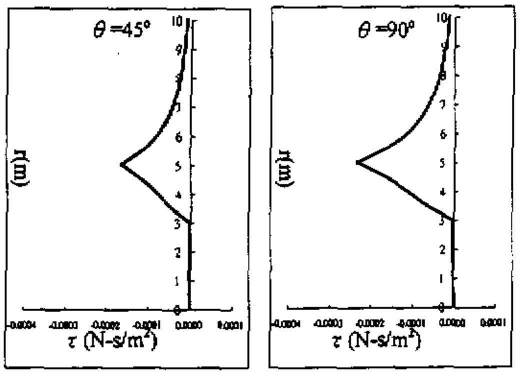

Figure 4. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ=0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in the present solutions

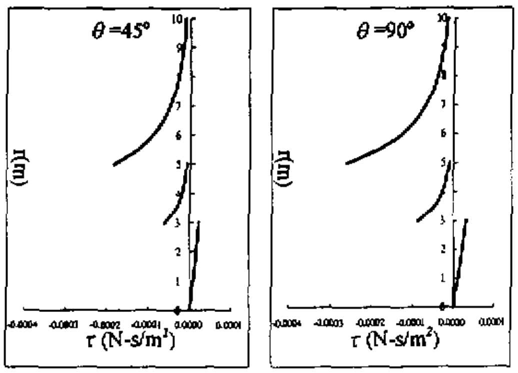

Figure 5. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ=0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in Jones' solutions

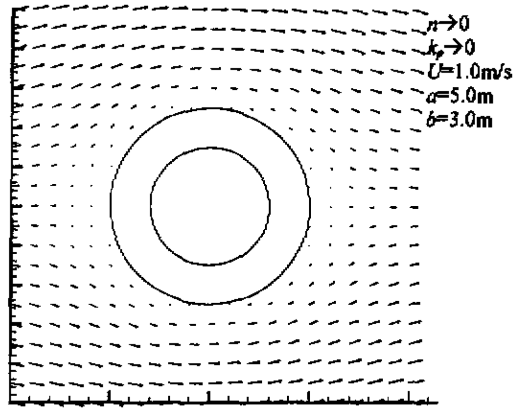

Figure 6. Flow field as n approaches to 0

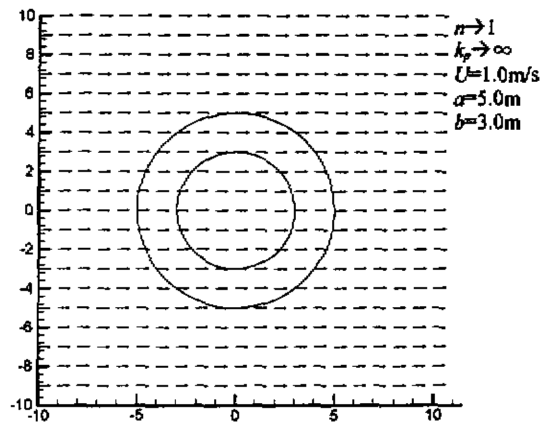

Figure 7. Flow field as n approaches to 1

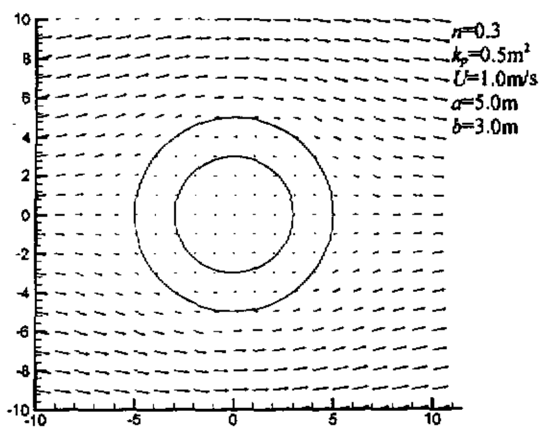

Figure 8. Flow field of a general case: n=0.3, k_p=0.5 m^2

A REINVESTIGATION OF UNIFORM LAMINAR FLOW PAST A POROUS SPHERICAL SHELL

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By applying Darcy's law as the governing equation, Jones (1973) investigated the flow field of a uniform laminar flow with low Reynolds number past a porous spherical shell. Due to the potential characteristics of Darcy's law, it is impossible to imply viscous effect in the porous medium of Jones' work. And due to the improperly extension from the results of Beavers & Joseph (1967) and Saffman (1971) for spherical coordinate to fulfill the tangential boundary conditions, the continuity conditions of tangential velocity and shear stress were not satisfied on boundaries. Therefore the result obtained by Jones (1973) has the necessity to be modified.

A new solution of Jones' (1973) problem is investigated based on Song & Huang's (2000) theory of laminar poroelastic media flow in the present work. It is observed

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that the new analytical solution can sufficiently delineate the viscous effect on boundaries and satisfy the boundary conditions of continuity of tangential velocity and shear stress. It is also found that as the porosity approaches to 1, the flow field retrogrades to the uniform laminar flow; and as the porosity approaches to 0, the well-known Stokes' solutions of a uniform laminar flow with low Reynolds number past a porous spherical shell are obtained.

1. INTRODUCTION

In early researches for the flow interacting with a porous medium, Darcy's law was often used to dominate the flow field in the porous region. Although the results were not bad in agreement with experiments, and a great deal of achievements had been obtained in use of this potential model [e.g., Huang & Song (1993)], slip phenomena at interfaces could not be avoided. Because Darcy's law only poses the form of first-order differentiation, the surplus boundary conditions with respect to continuous tangential velocity and shear stress are overdetermined. It brings about some perplexity, especially when the no slip condition is applied for revealing physical mechanism on the boundaries.

A plausible but empirical tangential boundary condition proposed by Beavers & Joseph (1967) and Saffman (1971) for porous plane boundaries when viscous effect is not negligible is

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k_p}}(u - q), \tag{1}$$

where y is the coordinate normal to the surface, u is the mean velocity parallel to the surface, q is the tangential fluid velocity inside the porous medium at $y = 0$, k_p is the specific permeability, and α is a dimensionless parameter depending only upon the properties of the medium. For laminar flow passing over plane porous bed of infinite thickness, Huang & Chiang (1997) proved that equation (1) is appropriate, and $\alpha = 1/n$, where n is the porosity.

By applying Darcy's law as the governing equation, Jones (1973) investigated the flow field of a uniform flow with low Reynolds number past a porous spherical shell. Because of the spherical geometry of boundaries, not only boundary conditions in normal direction [see Huang et al. (1993) for normal boundary conditions] but also those in tangential direction are required. In order to solve the problem with spherical coordinate, Jones (1973) hence proposed a tangential boundary condition, which was improperly extended from Beavers & Joseph (1967) and Saffman (1971) as

$$e_{r\theta} = \frac{\alpha}{\sqrt{k_p}}(u_\theta - q_\theta), \tag{2}$$

where $e_{r\theta}$ is the rate of strain, in application for problems with spherical boundaries.

We will find later that due to the improper extension of the partial-slip boundary condition proposed by Beavers & Joseph (1967) and Saffman (1971), the occurrence of slip situation at interfaces is inevitable. It will yield handicaps while the continuities of tangential velocity and shear stress are absolutely essential and have a great effect upon actual circumstances. Furthermore, the potential characteristic of Darcy's model means that the solutions cannot demonstrate viscous effect of the flow inside the porous medium, as well as at the interfaces.

Song & Huang (2000) proposed complete governing equations and boundary conditions by the poroelastic media theory. According to their results, both normal and tangential boundary conditions are applied in this study and thus the continuities of tangential velocity and shear stress on boundaries of the porous medium are satisfied in consideration of viscous effect. In addition, the verifications of limiting cases ascertain that the flow field retrogrades to the uniform laminar flow as the porosity approaches to 1, and retrogrades to the well-known Stokes flow as it approaches to 0.

2. JONES' SOLUTIONS

Jones (1973) investigated the flow field of a uniform flow passing a porous spherical shell. The spherical shell of external radius a and internal radius b is immersed in a uniform stream of velocity U. The external region, the porous region, and the cavity are denoted as regions I, II, and III respectively as shown in Figure 1. He established the complete analytic solutions by using Darcy's law in control of the porous medium, region Ⅱ, and adopting the boundary conditions extended from the works of Beavers & Joseph (1967) and Saffman (1971).

In regions I and Ⅲ, equations governing the flow are Stokes equation

$$\nabla^2 \underline{u} = \frac{1}{\mu} \nabla p \tag{3}$$

and continuity equation

$$\nabla \cdot \underline{u} = 0. \tag{4}$$

where $\underline{u}$ and p are flow velocity and pressure respectively, and μ is the fluid viscosity.

In the porous medium, region Ⅱ, the flow is governed by Darcy's law

$$\underline{q} = -\frac{k_p}{\mu} \bar{\nabla} p, \tag{5}$$

where $\underline{q}$ is the flow velocity inside the porous medium. And continuity equation is

$$\nabla \cdot \underline{q} = 0. \tag{6}$$

Note that Darcy's law has only first-order differentiation, so it cannot demonstrate the viscous effect in region Ⅱ, and the continuity conditions connected to regions I and Ⅲ, which are dominated by the second-order Stokes equation, are overdetermined in tangential properties.

Boundary conditions at the interfaces of spherical shells (i.e., r = a, b) are continuity of pressure

$$p_1\big|_{r=a} = p_2\big|_{r=a},\ p_2\big|_{r=b} = p_3\big|_{r=b}, \tag{7}$$

continuity of normal fluid flux

$$u_{r1}\big|_{r=a} = q_{r2}\big|_{r=a},\ q_{r2}\big|_{r=b} = u_{r3}\big|_{r=b}, \tag{8}$$

and empirical tangential boundary conditions, which are improperly extended from Beavers & Joseph (1967) and Saffman (1971), proposed by Jones (1973),

$$e_{r\theta 1}\big|_{r=a} = \beta\left(u_{\theta 1}\big|_{r=a} - q_{\theta 2}\big|_{r=a}\right),\ e_{r\theta 3}\big|_{r=b} = -\beta\left(u_{\theta 3}\big|_{r=b} - q_{\theta 2}\big|_{r=b}\right), \tag{9}$$

where $\alpha/\sqrt{k_p}$ is denoted as β and the rate of strain $e_{r\theta}$ is given as

$$e_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}. \tag{10}$$

By solving equations (3)–(6) with boundary conditions (7)–(9), analytic solutions are obtained.

In region I ,

$$p_1 = \mu U\left(\frac{C_2}{r^2}\right)\cos\theta, \tag{11}$$

$$u_{r1} = U\left(1 + \frac{C_2}{r} + \frac{C_3}{r^3}\right)\cos\theta, \tag{12}$$

$$u_{\theta 1} = -U\left(1 + \frac{C_2}{2r} - \frac{C_3}{2r^3}\right)\sin\theta. \tag{13}$$

In region II,

$$p_2 = U\left(\frac{C_5}{r^2} + C_6 r\right)\cos\theta, \tag{14}$$

$$q_{r2} = \frac{k_p U}{\mu}\left(\frac{2C_5}{r^3} - C_6\right)\cos\theta, \tag{15}$$

$$q_{\theta 2} = \frac{k_p U}{\mu}\left(\frac{C_5}{r^3} + C_6\right)\sin\theta. \tag{16}$$

In region III,

$$p_3 = \mu U\left(10C_4 r\right)\cos\theta, \tag{17}$$

$$u_{r3} = U\left(C_1 + C_4 r^2\right)\cos\theta, \tag{18}$$

$$u_{\theta 3} = -U\left(C_1 + 2C_4 r^2\right)\sin\theta, \tag{19}$$

where

$$C_1 = -\frac{6k_p}{a^2}\left(1+\frac{a\beta}{2}\right)\left[\frac{9}{40a\beta}+\frac{3b}{20a}+\frac{3k_p}{4ab}\right]\Big/\Gamma, \tag{20}$$

$$C_2 = 3a\left(1+\frac{a\beta}{2}\right)\left[\frac{3k_p}{2ab}+\frac{3}{20a\beta}+\frac{b}{20a}-\frac{3b^3}{20a^4\beta}-\frac{b^4}{20a^4}\right]\Big/\Gamma, \tag{21}$$

$$C_3 = 2a^4\beta\left[\left(\frac{1}{4}-\frac{3k_p}{2a^2}\right)\left(\frac{3b^3}{20a^4\beta}+\frac{b^4}{20a^4}\right)-\frac{1}{4}\left(\frac{3k_p}{2ab}+\frac{3}{20a\beta}+\frac{b}{20a}\right)\right]\Big/\Gamma, \tag{22}$$

$$C_4 = \frac{9k_p}{20a^3b}\left[1+\frac{a\beta}{2}\right]\Big/\Gamma, \tag{23}$$

$$C_5 = -3\mu a\left(1+\frac{a\beta}{2}\right)\left[\frac{3b^3}{20a^4\beta}+\frac{b^4}{20a^4}\right]\Big/\Gamma, \tag{24}$$

$$C_6 = \frac{3\mu}{a^2}\left(1+\frac{a\beta}{2}\right)\left[\frac{3k_p}{2ab}+\frac{3}{20a\beta}+\frac{b}{20a}\right]\Big/\Gamma, \tag{25}$$

$$\Gamma = \left(3+a\beta-\frac{6k_p}{a^2}\right)\left(\frac{3b^3}{20a^4\beta}+\frac{b^4}{20a^4}\right)$$

$$-\left(3+a\beta+\frac{3k_p}{a^2}+\frac{3k_p\beta}{2a}\right)\left(\frac{3k_p}{2ab}+\frac{3}{20a\beta}+\frac{b}{20a}\right). \tag{26}$$

3. PRESENT STUDY

In dealing anew with the problem of a uniform flow passing a porous spherical shell, it is required to reset equations that will appropriately delineate the viscous effect of fluid in the porous medium. Besides, the equations of continuity of tangential velocity and shear stress ought to be satisfied at the interfaces, too.

In the present study, we adopt the governing equations and boundary conditions proposed by Song & Huang (2000) to reanalyze the flow field inside the porous medium, region . II

3.1 Governing Equations

In regions I and III, the governing equations are identical to Jones' (1973) study, i.e. equations (3) and (4).

While in region II, to consider the viscosity of fluid, the governing equation proposed by Song & Huang (2000), instead of Darcy's flow, is employed as follows.

$$\nabla^2 \underline{q} - \frac{n}{k_p}\underline{q} = \frac{n}{\mu}\nabla p, \tag{27}$$

where n is the porosity of porous medium. And the equation of continuity is still

$$\nabla \cdot \underline{q} = 0 \tag{28}$$

as usual. Because equation (27) is a second–order differential equation, different from equation (5), it will completely handle the overdetermined tangential boundary conditions provided that proper boundary conditions are founded. Besides, the added viscous term of equation (27), compared with equation (5), indicates that this governing equation will be able to delineate the viscous effect of the porous medium, region II properly.

3.2 Boundary Conditions

Since equation (27) are adopted to govern the porous medium flow, the boundary conditions at the interfaces need to be corrected to satisfy the continuities of tangential velocity and shear stress.

Song & Huang (2000) proposed eight physical quantities that should be continuous at porous interfaces. In this work, only four of them are used to construct proper boundary conditions. They are normal fluid stress, normal fluid flux, tangential fluid stress, and tangential relative flow velocity.

The continuities at the interfaces of the porous shell (r = a, b) are listed below:

1. Continuity of normal fluid stress:

$$2\mu \frac{\partial u_{r1}}{\partial r}\Big|_{r=a} - p_1\Big|_{r=a} = \frac{2\mu}{n}\frac{\partial q_{r2}}{\partial r}\Big|_{r=a} - p_2\Big|_{r=a}, \tag{29}$$

$$2\mu \frac{\partial u_{r3}}{\partial r}\Big|_{r=b} - p_3\Big|_{r=b} = \frac{2\mu}{n}\frac{\partial q_{r2}}{\partial r}\Big|_{r=b} - p_2\Big|_{r=b}. \tag{30}$$

Because equation (27) has second-order differentiation, we no longer merely use the continuity of pressure, i.e., equation (7), but can sufficiently employ more generalized normal fluid stress as the continuity condition.

2. Continuity of normal fluid flux:

$$u_{r1}\Big|_{r=a} = q_{r2}\Big|_{r=a}, \tag{31}$$

$$u_{r3}\Big|_{r=b} = q_{r2}\Big|_{r=b}, \tag{32}$$

which are identical to equation (8).

3. Continuity of tangential fluid stress:

$$\frac{\partial u_{\theta 1}}{\partial r}\Big|_{r=a} - \frac{u_{\theta 1}}{r}\Big|_{r=a} + \frac{1}{r}\frac{\partial u_{r1}}{\partial \theta}\Big|_{r=a} = \frac{1}{n}\frac{\partial q_{\theta 2}}{\partial r}\Big|_{r=a} - \frac{1}{n}\frac{q_{\theta 2}}{r}\Big|_{r=a} + \frac{1}{n}\frac{1}{r}\frac{\partial q_{r2}}{\partial \theta}\Big|_{r=a}, \tag{33}$$

$$\frac{\partial u_{\theta 3}}{\partial r}\Big|_{r=b} - \frac{u_{\theta 3}}{r}\Big|_{r=b} + \frac{1}{r}\frac{\partial u_{r3}}{\partial \theta}\Big|_{r=b} = \frac{1}{n}\frac{\partial q_{\theta 2}}{\partial r}\Big|_{r=b} - \frac{1}{n}\frac{q_{\theta 2}}{r}\Big|_{r=b} + \frac{1}{n}\frac{1}{r}\frac{\partial q_{r2}}{\partial \theta}\Big|_{r=b}. \tag{34}$$

4. Continuity of tangential relative fluid velocity:

$$u_{\theta 1}\big|_{r=a} = q_{\theta 2}\big|_{r=a}, \tag{35}$$

$$u_{\theta 3}\big|_{r=b} = q_{\theta 2}\big|_{r=b}. \tag{36}$$

Equations (33) to (36) are now adopted to replace equation (9) which is an improper extension from Beavers & Joseph (1967) and Saffman (1971), proposed by Jones (1973). Referring to equation (10), we find that equations (33) and (34) will never become equation (9).

We now realize that because of the property of first-order differentiation of equation (5), it is neither possible to imply viscous effect in the porous medium, nor to introduce equations (33) to (36) as boundary conditions for Jones' (1973) solutions Since Jones' solutions do not satisfy the continuity of tangential velocity and shear stress.

4. SOLUTIONS

The method of separation of variables is applied to solve the governing equations (3), (4) and (27), (28) with the boundary conditions (29)–(36). After some manipulation, the final forms of solutions are obtained as follows.

In region I,

$$p_1 = \mu U\left(\frac{C_2}{r^2}\right)\cos\theta, \quad (37)$$

$$u_{r1} = U\left(1+\frac{C_2}{r}+\frac{C_3}{r^3}\right)\cos\theta, \quad (38)$$

$$u_{\theta 1} = -U\left(1+\frac{C_2}{2r}-\frac{C_3}{2r^3}\right)\sin\theta. \quad (39)$$

In region II,

$$p_2 = \frac{\mu U}{n}\left(\frac{1}{2r^2}C_7 - m^2 r C_8\right)\cos\theta, \quad (40)$$

$$q_{r2} = U\Big[\frac{mr\sinh mr - \cosh mr}{r^3}C_5 + \frac{mr\cosh mr - \sinh mr}{r^3}C_6 + \frac{1}{m^2r^3}C_7 + C_8\Big]\cos\theta, \quad (41)$$

$$q_{\theta 2} = U\Big[\frac{mr\sinh mr - \left(1+m^2r^2\right)\cosh mr}{2r^3}C_5 + \frac{mr\cosh mr - \left(1+m^2r^2\right)\sinh mr}{2r^3}C_6 + \frac{1}{2m^2r^3}C_7 - C_8\Big]\sin\theta, \quad (42)$$

where $\sqrt{n/k_p}$ is denoted as m.

In region III,

$$p_3 = \mu U\left(10C_4 r\right)\cos\theta, \quad (43)$$

$$u_{r3} = U\left(C_1 + C_4 r^2\right)\cos\theta, \quad (44)$$

$$u_{\theta 3} = -U\left(C_1 + 2C_4 r^2\right)\sin\theta. \quad (45)$$

The boundary conditions at the interfaces provide eight simultaneous equations to determine eight unknown coefficients, which are found to be:

$$C_1 = \frac{\alpha_1}{\lambda}, C_2 = \frac{\alpha_2}{\lambda}, C_3 = \frac{\alpha_3}{\lambda}, C_4 = \frac{\alpha_4}{\lambda}, C_5 = \frac{\alpha_5}{\lambda}, C_6 = \frac{\alpha_6}{\lambda}, C_7 = \frac{\alpha_7}{\lambda}, C_8 = \frac{\alpha_8}{\lambda}, \quad (46)$$

where

$$\begin{aligned} \lambda &= K_1\{P_1(C_1M_1 + B_1O_1) - Q_1(C_1L_1 + A_1O_1) + R_1(B_1L_1 - A_1M_1)\} \\ &- D_1\{P_1(J_1M_1 - H_1O_1) - Q_1(J_1L_1 - G_1O_1) + R_1(H_1L_1 - G_1M_1)\} \\ &+ U_1\{\ C_1[H_1(L_1 - P_1) - G_1(M_1 - Q_1)] + B_1[J_1(L_1 - P_1) - G_1(O_1 - R_1)] \\ &- A_1[J_1(M_1 - Q_1) - H_1(O_1 - R_1)]\ \} \end{aligned} \quad (47)$$

$$\alpha_1 = -\frac{1}{2b^3}\left[\ (5D - 5mbC + m^2b^2D)\alpha_5 + (5C - 5mbD + m^2b^2C)\alpha_6 - \frac{5}{m^2}\alpha_7 - 2b^3\alpha_8\ \right] \quad (48)$$

$$\alpha_2 = -\frac{1}{2a^2}\left[-m^2a^2B\alpha_5 - m^2a^2A\alpha_6 - 3a^3\alpha_8 + 3a^3\right] \quad (49)$$

$$\alpha_3 = -\frac{1}{6n}\left[\ (6B + 3m^2a^2B - 6maA - m^3a^3A)\alpha_5 + (6A + 3m^2a^2A - 6maB - m^3a^3B)\alpha_6 - \frac{6}{m^2}\alpha_7\ \right] \quad (50)$$

$$\alpha_4 = -\frac{1}{6na^5}\left[\ (6D + 3m^2b^2D - 6mbC - m^3b^3C)\alpha_5 + (6C + 3m^2b^2C - 6mbD - m^3b^3D)\alpha_6 - \frac{6}{m^2}\alpha_7\ \right] \quad (51)$$

$$\alpha_5 = -F_1\{K_1(O_1Q_1 + M_1R_1) + U_1[J_1(M_1 - Q_1) - G_1(O_1 + R_1)]\}, \quad (52)$$

$$\alpha_6 = F_1\{K_1(O_1P_1 + L_1R_1) + U_1[J_1(L_1 - P_1) - G_1(O_1 + R_1)]\}, \quad (53)$$

$$\alpha_7 = F_1\{K_1(M_1P_1 - L_1Q_1) + U_1[H_1(L_1 - P_1) - G_1(M_1 + Q_1)]\}, \quad (54)$$

$$\alpha_8 = -F_1\{P_1(J_1M_1 - H_1O_1) - Q_1(J_1L_1 - G_1O_1) - R_1(H_1L_1 - G_1M_1)\}, \quad (55)$$

$$A_1 = (3n - 2)m^2a^2B + 2m^3a^3A, \quad (56)$$

$$B_1 = (3n - 2)m^2a^2A + 2m^3b^3B, \quad (57)$$

$$C_1 = -a^2, \quad (58)$$

$$D_1 = 9na^3 + 2m^2a^5, \tag{59}$$

$$F_1 = 9na^3, \tag{60}$$

$$G_1 = 18(n-1)(B - maA) + (6n-7)(m^2a^2B) + m^3a^3A, \tag{61}$$

$$H_1 = 18(n-1)(A - maB) + (6n-7)(m^2a^2A) + m^3a^3B, \tag{62}$$

$$J_1 = \frac{18}{m^2}(n-1) - a^2, \tag{63}$$

$$K_1 = 2m^2a^5, \tag{64}$$

$$L_1 = 3(3n+2)(D - mbC) + (3n+4)(m^2b^2D) - 2m^3b^3C, \tag{65}$$

$$M_1 = 3(3n+2)(C - mbD) + (3n+4)(m^2b^2C) - 2m^3b^3D, \tag{66}$$

$$P_1 = m^2b^2(D - mbC), \tag{67}$$

$$Q_1 = m^2b^2(C - mbD), \tag{68}$$

$$R_1 = \frac{b^2}{2}, \tag{69}$$

$$U_1 = m^2b^5, \tag{70}$$

$$A = \sinh ma, \tag{71}$$

$$B = \cosh ma, \tag{72}$$

$$C = \sinh mb, \tag{73}$$

$$D = \cosh mb. \tag{74}$$

5. DISCUSSION

Since we have employed the continuity of tangential relative fluid velocity and tangential fluid stress as boundary conditions, tangential relative fluid velocity and tangential fluid stress are continuous on the boundaries. We make clear our point of view in Figures 2 to 5. Figure 2 shows the tangential relative fluid velocity profiles obtained according to analytical solutions (37)-(45) in three directional angles from the spherical origin outward ($\theta = 45^\circ, 90^\circ, 135^\circ$). Figure 3 shows the profiles according to Jones' (1973) solutions (11)-(19) for comparison. We find that the curves of the present study are continuous at both outer and inner surfaces of spherical shell ($r = a, b$), but Jones' are not. The same circumstances are shown in Figure 4 and 5 for the tangential fluid stress. Therefore, in the present analytical solutions the slipping condition due to the potential Darcy's model is eliminated at the interfaces, and Jones' handicap hence disappears.

Another verification of our analytical solutions is to investigate special cases such as how the limiting values of porosity n lead to. Note that $m = \sqrt{n / k_p}$ is composed of the porosity n and the permeability k_p that always interact mutually. By introducing Kozeny-Karman equation of soil mechanics [e.g. see Lambe & Whitman (1969)]

$$k_p = \frac{n^3}{K_0 S^2 (1-n)^2}, \tag{75}$$

where K_0 is the shape factor and S is the specific surface area. We express k_p in terms of n and then transform the solutions into polynomial terms in preparation for discussing limiting values of porosity. First, in the limiting case of n approaching to

0, it is found that the coefficient C_2 converges to $-\frac{3}{2}a$, C_3 converges to $\frac{a^3}{2}$, and the other coefficients vanish. Consequently, only the flow field of region I still exists, and the solutions turn into

$$p_1 = \mu U\left(-\frac{3}{2a}\right)\cos\theta, \tag{76}$$

$$u_{r1} = U\left(1-\frac{3a}{2r}+\frac{a^3}{2r^3}\right)\cos\theta, \tag{77}$$

$$u_{\theta 1} = -U\left(1-\frac{3a}{4r}-\frac{a^3}{4r^3}\right)\sin\theta, \tag{78}$$

which are just the well-known Stokes solutions of a uniform flow passing a rigid impermeable sphere of radius a, shown in Figure 6. Thus it is recognized that the flow in the limiting condition of porosity approaching to 0 indeed conforms physical expectation.

Second, let n approach to 1 and ascertain how the flow field becomes. The present solutions turn into

$$p_1 = p_2 = p_3 = 0, \tag{79}$$

$$u_{r1} = u_{r2} = u_{r3} = U\cos\theta, \tag{80}$$

$$u_{\theta 1} = u_{\theta 2} = u_{\theta 3} = -U\sin\theta, \tag{81}$$

which are just the uniform flow driven by a far field velocity U as if the porous spherical shell didn't exist, shown in Figure 7. (Note that Jones (1973) could not recover this limiting case.) Therefore, it can be identified that the flow in the limiting condition of porosity approaching to 1 will retrograde to a simple uniform flow field, which also coincides with physical expectation.

The general range of the porosity of soils are known as 0.1~0.5, so we select a medium value of 0.3 and assume that the permeability is $0.5m^2$. The flow field of the analytical solutions (37)-(45), and coefficients in substitution for the specified values are shown in Figure 8. We can see clearly how the flow varies and penetrates through the porous shell.

6. CONCLUSIONS

The use of the boundary conditions of Beavers & Joseph (1967) and Saffman (1971) should be very careful. In the present study, it is shown that the extension of the boundary conditions with spherical coordinate proposed by Jones (1973) is incorrect.

In order to remedy the handicaps of Jones' (1973) solution of a uniform laminar flow with low Reynolds number passing a porous spherical shell, we reinvestigate the flow field by adopting the second-order differential governing equation and adequate boundary conditions from the laminar poroelastic flow model proposed by Song & Huang (2000). The solutions imply apparently the viscous effect in the porous region that the Darcy's model cannot provide. Furthermore, the continuity conditions of tangential velocity and shear stress are properly satisfied at the interfaces of the porous spherical shell.

In the present study, it is also proved that the flow field will retrograde to Stokes flow while the porosity approaches to 0, and retrograde to a simple uniform flow while the porosity approaches to 1. Both of the limiting cases are in accord with actual physical expectation and thus verify the correctness of the present study.

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NOTATIONS

The following symbols are used in this paper:

a = external radius of the spherical shell;

b = internal radius of the spherical shell;

$e_{r\theta}$ = rate of strain;

k_p = specific permeability of the porous medium;

n = porosity of the porous medium;

p = pressure of flow field;

q = tangential fluid velocity inside the porous medium at $y = 0$;

$\underline{q}$ = velocity vector inside the porous medium;

U = velocity of the uniform flow;

u = mean velocity parellel to the plane surface;

$\underline{u}$ = velocity vector of flow;

α = dimensionless parameter depending upon the properties of the porous medium;

μ = viscosity of fluid.

CAPTION OF FIGURES

Figure 1. Schematic figure of the problem

Figure 2. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m^2, U=1.0m/s, a=5m, b=3m in the present solutions

Figure 3. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m^2, U=1.0m/s, a=5m, b=3m in Jones' (1973) solutions

Figure 4. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ =0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in the present solutions

Figure 5. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ =0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in Jones' (1973) solutions

Figure 6. Flow field as n approaches to 0

Figure 7. Flow field as n approaches to 1

Figure 8. Flow field of a general case: n=0.3, k_p=0.5 m^2

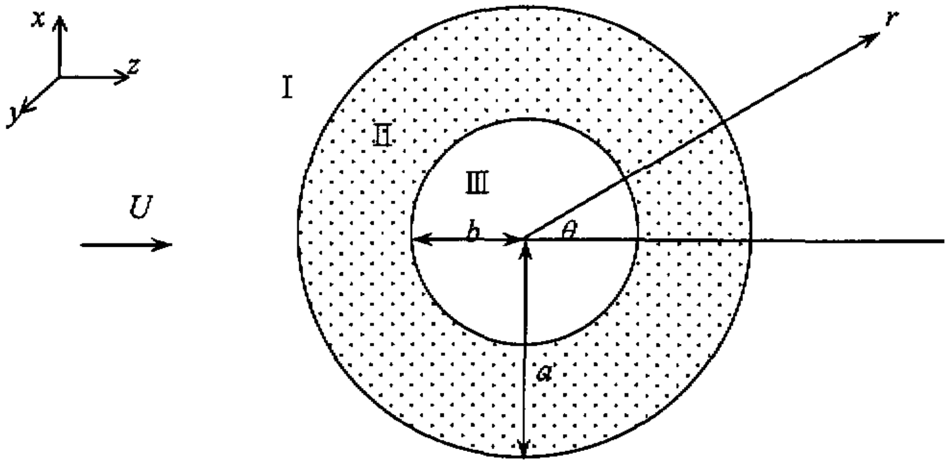

Figure 1. Schematic figure of the problem

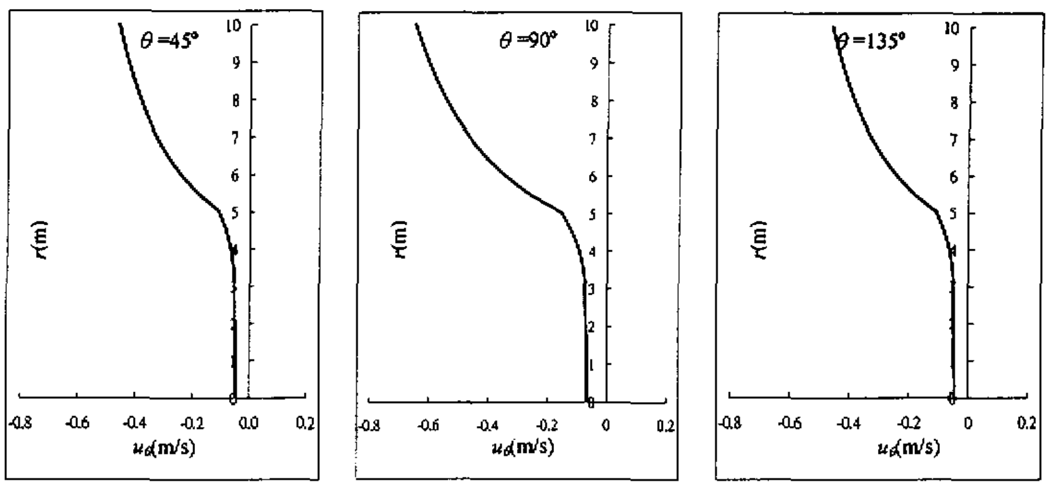

Figure 2. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m², U=1.0m/s, a=5m, b=3m in the present solutions

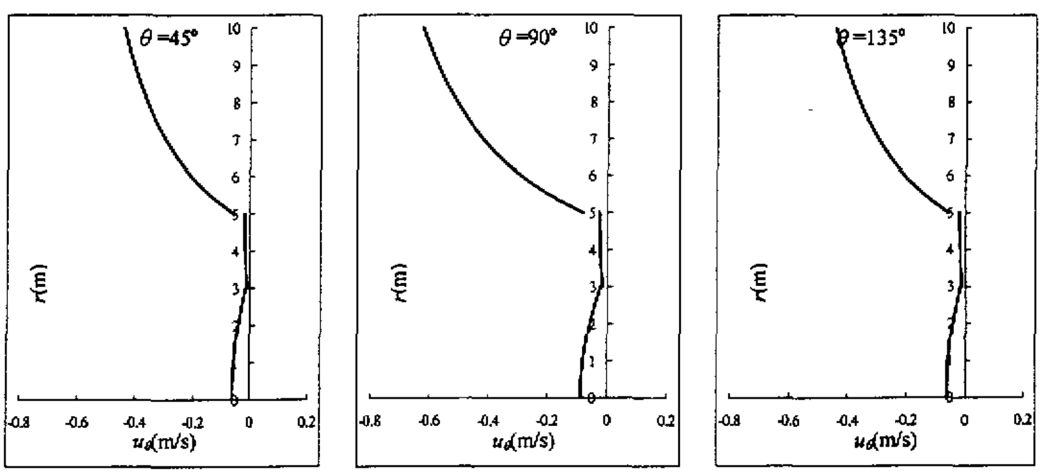

Figure 3. Distribution of tangential velocity for different θ for n=0.3, k_p=0.5m², U=1.0m/s, a=5m, b=3m in Jones' (1973) solutions

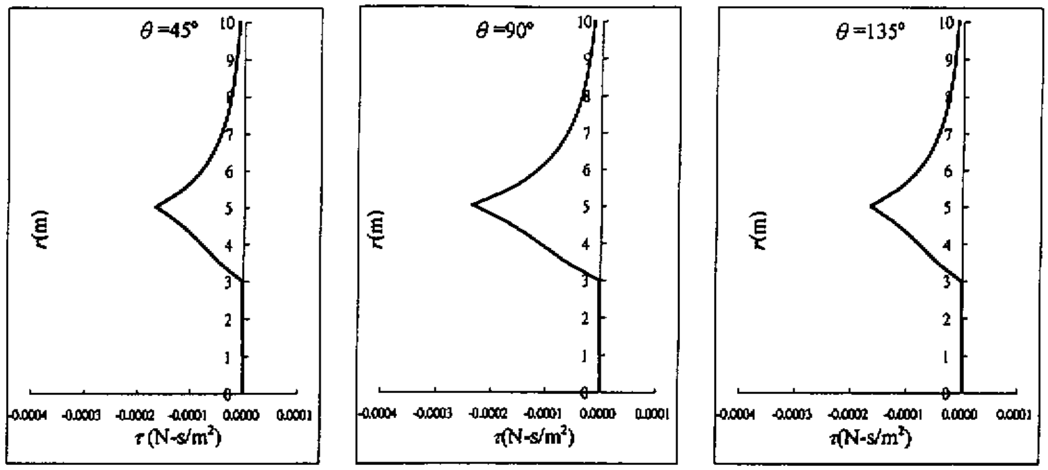

Figure 4. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ=0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in the present solutions

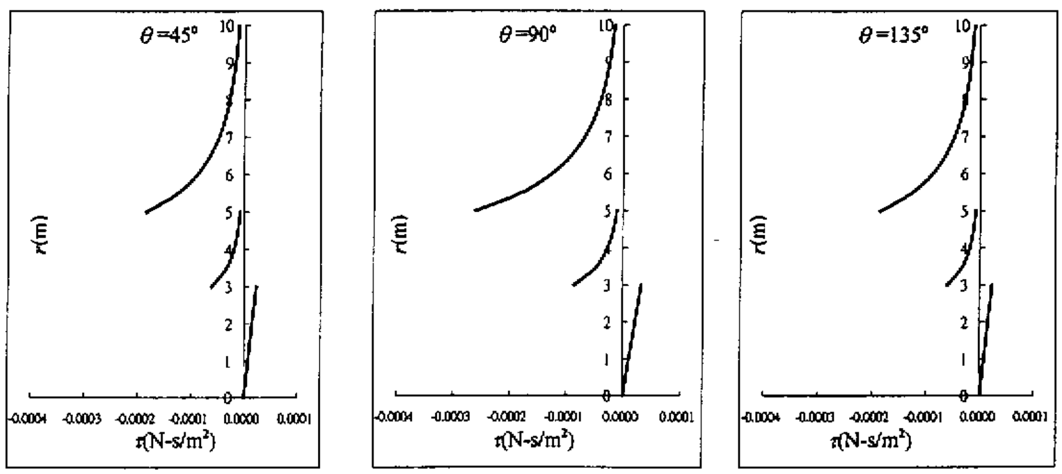

Figure 5. Distribution of shear stress for different θ for n=0.3, k_p=0.5m^2, μ=0.001N-s/m^2, U=1.0m/s, a=5m, b=3m in Jones' (1973) solutions

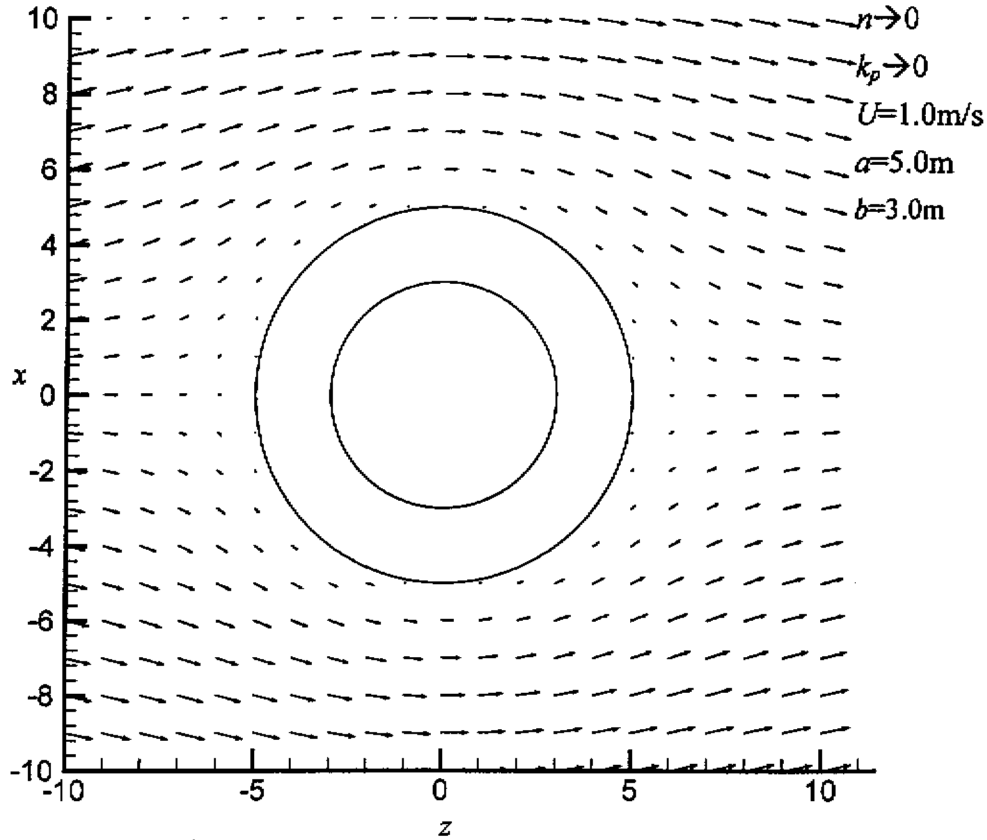

Figure 6. Flow field as n approaches to 0

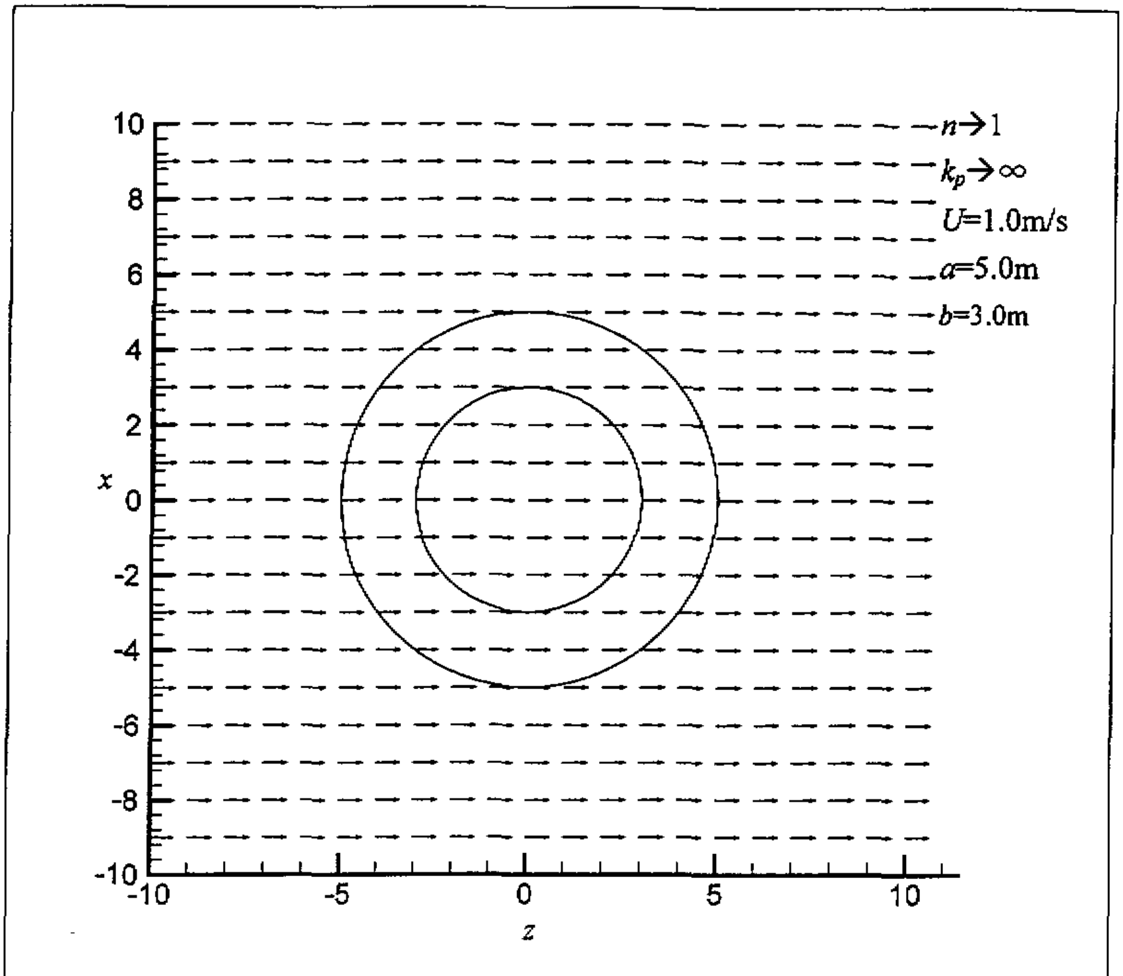

Figure 7. Flow field as n approaches to 1

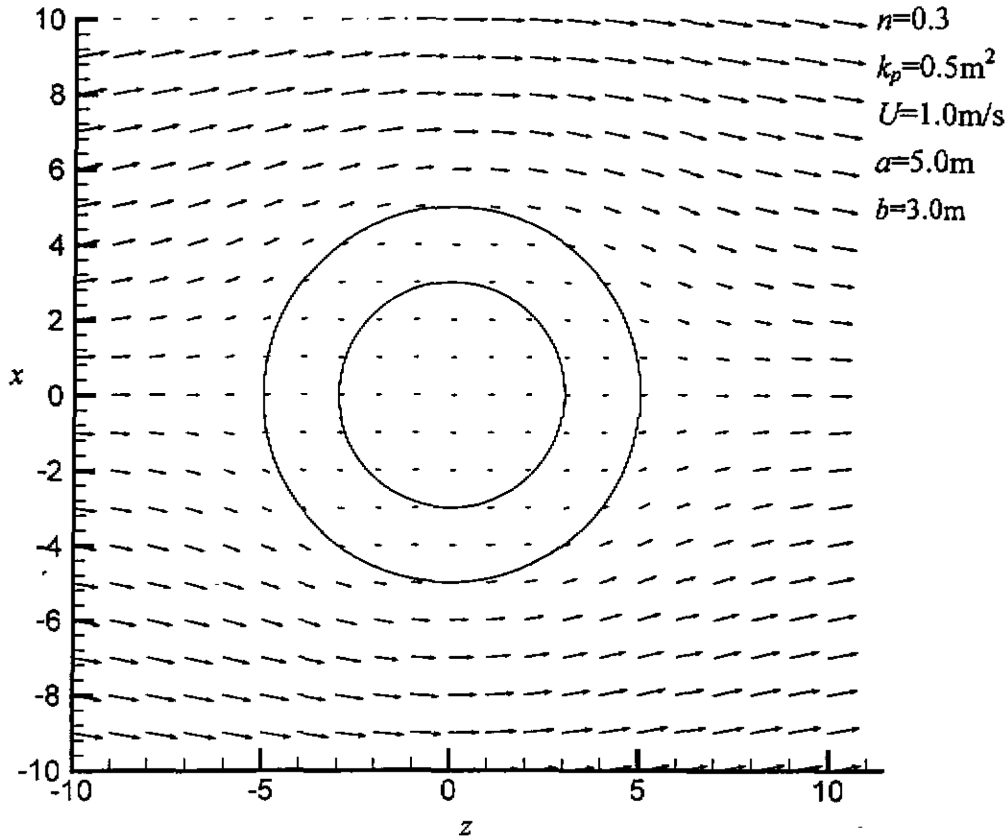

Figure 8. Flow field of a general case: n=0.3, k_p=0.5 m^2