Invited Paper

WAVE EQUATIONS OF ELASTIC TYPE-II SUPERCONDUCTORS

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Dedicated to Professor Y.H. Pao on the occasion of his seventieth birthday

ABSTRACT

It is shown that when the thermodynamic fluxes are included as independent thermodynamic state variables of the generalized entropy density, the constitutive equations for the conserved state variables and the evolution equations for the nonconserved state variables of elastic type-II superconductors can be derived systematically. In particular, the transport equations generalizing the Fourier law for heat transport and the time-dependent Ginzburg-Landau equation for the relaxation of superelectron are proposed in this paper.

Keywords : Type-II superconductor, Thermodynamics, Generalized entropy.

1. INTRODUCTION

Study of electrodynamic behaviors of elastic type-II superconductors is of interest due to the increasing number of application of superconductive devices after the discovery of high temperature superconductors in 1986. In the theory of elastic type-II superconductors, the macroscopic approach, of benefit to avoid handling the complicated microscopic quantum phenomenon, has been provided by some researchers, especially in the framework of continuum mechanics. Several continuum theories for elastic superconductors have been proposed and one of the earliest descriptions was the macroscopic theory of Zhou and Miya [1,2]. Motivated by the concept of internal variable, Yeh and Chen [3,4] developed a phenomenological theory of deformable superconductors. Among other contribu-tions to this field, we refer the readers to the works of Ghaleb [5], van de Ven [6] and Maugin [7]. As for the further continuum mechanics discussion of type-II superconductors, Zhou [8], with the aid of Coffey-Clem approach, and Yeh and Chen [9,10], based upon the work of Mathieu and Simon, have independently proposed the macroscopic theories describing the response of elastic type-II superconductors. Recently, Zhou [11] has discussed electrodynamic behaviors of moving superconductors at the magneto-quasi-static approximation, the issue of superelectron inertia and flux-quantization, and shown that superconductors not only exhibit the features of the zero-dc resistance, the Meissner effect and the macroscopic quantum behavior but also have a unique feature on their macroscopic electromagnetic response to the non-uniform motion and local dynamic deformation of the superconducting media.

Of these theories, what is most concerned is their theoretical background, namely, the thermodynamic theory. More and more experimental evidences reveal that the classical formulation of non-equilibrium thermodynamics, in which the parabolic type of transport equations are obtained and the signal speeds are infinite, is not enough to delineate the physical world but requires some modifications. Many researchers have achieved the formulations of thermodynamics going beyond the classical paradigm by enlarging the space of basic independent variables through the introduction of non-equilibrium variables such as the dissipative fluxes. We refer the readers to some excellent references [12~16]. In this paper we examine the theory of elastic type-II superconductors in the framework of extended irreversible thermodynamics (EIT). This study is necessary since in the world of low temperature the thermal impulse will propagate as a wave and the traditional thermodynamic approach can not be used to analyze this phenomenon (the same situation occurs in the theory of liquid helium II [17] and elastic semiconductors [18]).

In Section 2 we outline some basic equations of elastic type-II superconductors we proposed previously. The EIT approach for this material is characterized in Section 3. Section 4 presents the generalized complementary constitutive equations and the wave types of evolution equations are obtained. Finally a short conclusion is given in Section 5.

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2. BASIC EQUATIONS OF ELASTIC TYPE-II SUPERCONDUCTORS

Before a thorough analysis, it is necessary to outline our approach as follows. Firstly we adopt the complex internal variable and the vector potential as the independent variables to capture the physical response of superconductors [3]. Secondly in view of the gauge problem, the independent variables — the complex internal variable and the vector potential — are replaced by the amplitude of complex internal variable and the microscopic velocity of superelectron [4]. Thirdly by the method of macroscopic average for the macroscopic velocity of superelectron is adopted in place of the microscopic velocity of superelectron [9]. Recently, Fabrizio [19] has derived an alternative gauge-invariant form of the Ginzburg-Landau equations. This work supports our approach and is completely consistent with our previous work [4].

For the development of the theory, some basic balance and field equations of elastic type-II superconductors should be outlined here. On the basis of the material description for the Lorentz model is adopted and let x_i and ξ_α be the positions of a material point in the spatial and material frame, respectively. The balance equations concerning mass, linear momentum and energy can be written as

$$\frac{\partial \rho^f}{\partial t} + (\rho^f \upsilon_j)_{,j} = 0 \tag{1}$$

$$(\tau_{\alpha\beta}\, x_{i,\beta})_{,\alpha} + (\tau^M_{\alpha\beta}\, x_{i,\beta})_{,\alpha} = \rho^0\, \ddot{x}_i \tag{2}$$

$$\rho^0 \frac{dW}{dt} = \frac{1}{2}\tau_{\alpha\beta}\,\dot{C}_{\alpha\beta} - Q_{\alpha,\alpha} + J^N_\alpha E_\alpha + J^S_\alpha E_\alpha \tag{3}$$

and the field equations of electromagnetism are

$$\begin{aligned} &B_{\alpha,\alpha} = 0 \;, \quad e_{\alpha\beta\gamma} E_{\gamma,\beta} + \dot{B}_\alpha = 0 \\ &D_{\alpha,\alpha} = 0 \;, \quad e_{\alpha\beta\gamma} H_{\gamma,\beta} - \dot{D}_\alpha = J_\alpha \end{aligned} \tag{4}$$

with the transformation of field quantities between the material description and the spatial description

$$\begin{aligned} &E^e_i = E_\alpha\, \xi_{\alpha,i} \;, && B_i = J^{f^{-1}} B_\alpha\, x_{i,\alpha} \\ &H^e_i = H_\alpha\, \xi_{\alpha,i} \;, && D^L_i = J^{f^{-1}} D_\alpha\, x_{i,\alpha} \\ &J_i = J^{f^{-1}} J_\alpha\, x_{i,\alpha} \;, && D^L_i = \varepsilon_0 E_i \\ &E^e_i = E_i + e_{ijk}\,\upsilon_j B_k \;, && H^e_i = \frac{1}{\mu_0} B_i - e_{ijk}\,\upsilon_j D^L_k \end{aligned} \tag{5}$$

where $_{,i} \equiv \partial/\partial x_i$ and $_{,\alpha} \equiv \partial/\partial \xi_\alpha$; the dot designates the material time derivative; e_{ijk} denotes alternating tensor; B_i, D_i^L, E_i^e, H_i^e, J_i, ε_0, and μ_0 denote the magnetic induction, electric displacement, effective electric field, effective magnetic field, free current, permittivity and permeability in vacuum, respectively; $J^{f^{-1}}$ is the inverse of $J^f \equiv \det |x_{i,\alpha}|$, which is the Jacobian determinant of the motion. In the Eq. (2), $\tau_{\alpha\beta}$ and $\tau^M_{\alpha\beta}$ are the second Piola-Kirchhoff and Maxwell stress tensors, respectively; ρ^f, υ_i, W, Q_α, $C_{\alpha\beta}$, J_α^N, and J_α^S are the mass density, velocity of the material point, internal energy, heat flux, right Cauchy-Green tensor, normal current density, and supercurrent density, respectively. Note that the mechanical body force term and the radiation source term can be given a priori in EIT approach [12] and are not included in the following derivation.

In order to characterize the behavior of elastic type-II superconductors we have proposed two additional field equations describing the motions of superelectron and the vortex state as

$$\frac{m}{e} C_{\alpha\beta}\, \dot{V}^S_\beta = E_\alpha + \Lambda_\alpha - \frac{m}{e} V^S_\beta\, x_{i,\alpha}\, \dot{x}_{i,\beta} \tag{6}$$

$$B_\alpha + \frac{m}{e} e_{\alpha\beta\gamma} (C_{\gamma\delta,\beta} V^S_\delta + C_{\gamma\delta} V^S_{\delta,\beta}) = \Omega_\alpha \tag{7}$$

where Λ_α is the friction field and Ω_α characterizes the flux line density, i.e., the number of flux line per unit area.

3. EXTENDED IRREVERSIBLE THERMODYNAMICS OF ELASTIC TYPE-II SUPERCONDUCTORS

Aside from the traditional state variables, EIT uses dissipative fluxes as independent fields such that the assumption of local equilibrium can be abandoned. In the thermodynamics theory of elastic type-II superconductors, we propose that the generalized entropy density be in the form of

$$\rho^0 \eta = \hat{\eta}\,(\hat{W}, E_{\alpha\beta}, |\Psi|, |\Psi|_{,\alpha}, V^S_\alpha, \Omega_\alpha; Q_\alpha, J^N_\alpha, \overline{\Lambda}_\alpha, |\dot{\Psi}|) \tag{8}$$

From the second to the sixth variables are the traditional state variables used in [9] and they represent the Lagrangian strain $E_{\alpha\beta}$, the amplitude of the complex internal variable $|\Psi|$, the space gradient of the amplitude of the complex internal variable $|\Psi|_{,\alpha}$, the macroscopic relative velocity of electron V_α^s, and the vector of magnetic flux line Ω_α, respectively. $\hat{W}$ is the internal energy per unit volume. The last four variables are the dissipative fluxes Q_α, which stand for the heat flux, the normal current density J_α^N, the generalized friction field $\overline{\Lambda}_\alpha$ defined in [9], and the time derivative of the amplitude of the complex internal variable $|\dot{\Psi}|$, respectively. The expression of (8) implies the generalized Gibbs equation as

$$\begin{aligned} \rho^0 \frac{d\eta}{dt} = &\frac{\partial\hat{\eta}}{\partial\hat{W}}\frac{d\hat{W}}{dt} + \frac{\partial\hat{\eta}}{\partial E_{\alpha\beta}}\frac{dE_{\alpha\beta}}{dt} + \frac{\partial\hat{\eta}}{\partial|\Psi|}\frac{d|\Psi|}{dt} \\ &+ \frac{\partial\hat{\eta}}{\partial|\Psi|_{,\alpha}}\frac{d|\Psi|_{,\alpha}}{dt} + \frac{\partial\hat{\eta}}{\partial V^S_\alpha}\frac{dV^S_\alpha}{dt} + \frac{\partial\hat{\eta}}{\partial\Omega_\alpha}\frac{d\Omega_\alpha}{dt} + \frac{\partial\hat{\eta}}{\partial Q_\alpha}\frac{dQ_\alpha}{dt} \\ &+ \frac{\partial\hat{\eta}}{\partial J^N_\alpha}\frac{dJ^N_\alpha}{dt} + \frac{\partial\hat{\eta}}{\partial\overline{\Lambda}_\alpha}\frac{d\overline{\Lambda}_\alpha}{dt} + \frac{\partial\hat{\eta}}{\partial|\dot{\Psi}|}\frac{d|\dot{\Psi}|}{dt} \end{aligned} \tag{9}$$

Disregarding the higher order terms in the relationship between the non-equilibrium quantities and their equilibrium counterparts, we can define the equilibrium temperature θ, the equilibrium elastic stress $\tau^E_{\alpha\beta}$, the equilibrium vortex line potential $\in_\alpha$, and the equilibrium supercurrent $J_\alpha^{\,S}$ as

$$\theta^{-1} = \frac{\partial\hat{\eta}}{\partial\hat{W}} \tag{10}$$

$$\theta^{-1}\,\tau^E_{\alpha\beta} = -\frac{\partial\hat{\eta}}{\partial E_{\alpha\beta}} \tag{11}$$

$$\theta^{-1} \in_\alpha = -\frac{\partial\hat{\eta}}{\partial \Omega_\alpha} \tag{12}$$

$$\theta^{-1}\,\frac{m}{e} C_{\alpha\beta}\, J^S_\beta = -\frac{\partial\hat{\eta}}{\partial V^S_\alpha} \tag{13}$$

Then the substitutions of the balance Eqs. (1) ~ (3), the additional field Eqs. (6), (7), and the definitions of equilibrium variables (10) ~ (13) into (9) lead to

$$\begin{aligned}
\rho^0 \frac{d\eta}{dt} &= \theta^{-1}\left[\tau_{\alpha\beta} - \tau^E_{\alpha\beta} + \frac{m}{e}(J^S_\alpha V^S_\beta + J^S_\beta V^S_\alpha)\right]\dot{E}_{\alpha\beta} \\
&\quad + \theta^{-1} J^N_\alpha E_\alpha - \theta^{-1}(J^S_\alpha + e_{\alpha\beta\gamma} \in_{\gamma,\beta})\overline{\Lambda}_\alpha - \theta^{-1} Q_{\alpha,\alpha} \\
&\quad - \theta^{-1} e_{\alpha\beta\gamma}(\overline{\Lambda}_\beta \in_\gamma)_{,\alpha} + \frac{\delta\hat{\eta}}{\delta|\Psi|}|\dot{\Psi}| + \left(\frac{\partial\hat{\eta}}{\partial|\Psi|_{,\alpha}}|\dot{\Psi}|\right)_{,\alpha} \\
&\quad + \frac{\partial\hat{\eta}}{\partial Q_\alpha}\dot{Q}_\alpha + \frac{\partial\hat{\eta}}{\partial J^N_\alpha}\dot{J}^N_\alpha + \frac{\partial\hat{\eta}}{\partial\overline{\Lambda}_\alpha}\dot{\overline{\Lambda}}_\alpha + \frac{\partial\hat{\eta}}{\partial|\dot{\Psi}|}|\ddot{\Psi}|
\end{aligned} \tag{14}$$

where

$$\frac{\delta\hat{\eta}}{\delta|\Psi|} \equiv \frac{\partial\hat{\eta}}{\partial|\Psi|} - \left(\frac{\partial\hat{\eta}}{\partial|\Psi|_{,\alpha}}\right)_{,\alpha} \tag{15}$$

is the Euler-Lagrange derivative with respect to $|\Psi|$. It is well known that the elastic stress cannot contribute to dissipation, so from (14) it yields

$$\tau_{\alpha\beta} = -\theta\frac{\partial\hat{\eta}}{\partial E_{\alpha\beta}} - \frac{m}{e}(J^S_\alpha V^S_\beta + J^S_\beta V^S_\alpha) \tag{16}$$

Moreover, the derivatives of the generalized entropy with respect to the dissipative fluxes can be defined as

$$\frac{\partial\hat{\eta}}{\partial Q_\alpha} = -\frac{1}{\theta}(A_{11} Q_\alpha + A_{12} J^N_\alpha + A_{13}\overline{\Lambda}_\alpha) \tag{17}$$

$$\frac{\partial\hat{\eta}}{\partial J^N_\alpha} = -\frac{1}{\theta}(A_{21} Q_\alpha + A_{22} J^N_\alpha + A_{23}\overline{\Lambda}_\alpha) \tag{18}$$

$$\frac{\partial\hat{\eta}}{\partial\overline{\Lambda}_\alpha} = -\frac{1}{\theta}(A_{31} Q_\alpha + A_{32} J^N_\alpha + A_{33}\overline{\Lambda}_\alpha) \tag{19}$$

$$\frac{\partial\hat{\eta}}{\partial|\dot{\Psi}|_\alpha} = -\frac{1}{\theta}A_4\,|\dot{\Psi}| \tag{20}$$

with the symmetrical conditions $A_{ij} = A_{ji}$.

Finally we have the balance law for entropy

$$\rho^0\,\theta\frac{d\eta}{dt} + \theta\, S_{\alpha,\alpha} = \theta\,\sigma^S \geq 0 \tag{21}$$

with S_α the entropy flux and σ^S the non-negative entropy production. In this study the following form of the entropy flux, deviated from the traditional form of heat flux over temperature Q_α/θ, is adopted:

$$S_\alpha = \frac{1}{\theta}Q_\alpha + \frac{1}{\theta}e_{\alpha\beta\gamma}\overline{\Lambda}_\beta \in_\gamma - \frac{\partial\hat{\eta}}{\partial|\Psi|_{,\alpha}}|\dot{\Psi}| \tag{22}$$

The last two terms of the right-hand side of (22) is called the extra entropy flux and has an equivalent expression as that used in [9].

After the rearrangement of (14) and (16) ~ (22), we obtain the final form of entropy inequality as

$$\theta\,\sigma^S = Q_\alpha X^Q_\alpha + J^N_\alpha X^J_\alpha + \overline{\Lambda}_\alpha X^\Lambda_\alpha + |\Psi|\, X^\Psi \geq 0 \tag{23}$$

where Q_α, $J_\alpha^{\,N}$, $\overline{\Lambda}_\alpha$ and $|\dot{\Psi}|$ can be viewed as the generalized fluxes, and their conjugated generalized thermodynamic forces are derived as

$$X^Q_\alpha = -\frac{1}{\theta}\theta_{,\alpha} - A_{11}\dot{Q}_\alpha - A_{21}\dot{J}^N_\alpha - A_{31}\dot{\overline{\Lambda}}_\alpha \tag{24}$$

$$X^J_\alpha = E_\alpha - A_{12}\dot{Q}_\alpha - A_{22}\dot{J}^N_\alpha - A_{32}\dot{\overline{\Lambda}}_\alpha \tag{25}$$

$$\begin{aligned}
X^\Lambda_\alpha &= -J^S_\alpha - e_{\alpha\beta\gamma} \in_{\gamma,\beta} + \frac{1}{\theta}e_{\alpha\beta\gamma} \in_\gamma \theta_{,\beta} \\
&\quad - A_{13}\dot{Q}_\alpha - A_{23}\dot{J}^N_\alpha - A_{33}\dot{\overline{\Lambda}}_\alpha
\end{aligned} \tag{26}$$

$$X^\Psi = \theta\frac{\delta\hat{\eta}}{\delta|\Psi|} - A_4\,|\ddot{\Psi}| \tag{27}$$

If the generalized entropy density is not a function of these fluxes, which implies that all of the coefficients A_{ij} vanish, then the entropy inequality will be reduced to that obtained in [9] except that the position of the free energy $\hat{F}$ in [9] should be replaced by the generalized entropy $\hat{\eta}$. Moreover, compared with (2.29) in [9], an additional term $(1/\theta)e_{\alpha\beta\gamma}\in_\gamma\theta_{,\beta}$ in (26) appears as a force to drive the flux flow. However, a careful manipulation of the second and third terms of the right-hand side in (26) can give

$$-e_{\alpha\beta\gamma} \in_{\gamma,\beta} + \frac{1}{\theta}e_{\alpha\beta\gamma} \in_\gamma \theta_{,\beta} = e_{\alpha\beta\gamma}\left(\frac{\partial\hat{\eta}}{\partial\Omega_\gamma}\right)_{,\beta} \tag{28}$$

which shares a similar feature with the form

$$e_{\alpha\beta\gamma} \in_{\gamma,\beta} = e_{\alpha\beta\gamma}\left(\frac{\partial\hat{F}}{\partial\Omega_\gamma}\right)_{,\beta} \tag{29}$$

of (2.29) used in [9].

4. GENERALIZED WAVE EQUATIONS

The explicit relationships between the generalized fluxes and the generalized forces are needed to study the evolution processes of the fluxes. In the simplest linear version we adopt these equations:

$$Q_\alpha = \mu^{QQ} X^Q_\alpha + \mu^{QJ} X^J_\alpha + \mu^{Q\Lambda} X^\Lambda_\alpha \tag{30}$$

$$J^N_\alpha = \mu^{JQ} X^Q_\alpha + \mu^{JJ} X^J_\alpha + \mu^{J\Lambda} X^\Lambda_\alpha \tag{31}$$

$$\overline{\Lambda}_\alpha = \mu^{\Lambda Q} X^Q_\alpha + \mu^{\Lambda J} X^J_\alpha + \mu^{\Lambda\Lambda} X^\Lambda_\alpha \tag{32}$$

$$|\dot{\Psi}| = \mu^\Psi X^\Psi \tag{33}$$

The number of these coefficient μ^{ij} can be reduced by using the Onsager relations: $\mu^{ij} = \mu^{ji}$. Then the substitutions of (30) ~ (33) into (23) lead to

$$\theta\,\sigma^S = \mu^{QQ} X^Q_\alpha X^Q_\alpha + \mu^{JJ} X^J_\alpha X^J_\alpha + \mu^{\Lambda\Lambda} X^\Lambda_\alpha X^\Lambda_\alpha + \mu^\Psi X^\Psi X^\Psi + 2\mu^{QJ} X^Q_\alpha X^J_\alpha + 2\mu^{Q\Lambda} X^Q_\alpha X^\Lambda_\alpha + 2\mu^{J\Lambda} X^J_\alpha X^\Lambda_\alpha \geq 0 \tag{34}$$

The condition of non-negative entropy production provides some constraints on these coefficients, i.e.,

$$\mu^{QQ} \geq 0 \;,\quad \mu^{JJ} \geq 0 \;,\quad \mu^{\Lambda\Lambda} \geq 0 \;,\quad \mu^{\Psi\Psi} \geq 0$$
$$\mu^{QQ}\mu^{JJ} \geq (\mu^{QJ})^2 \;,\quad \mu^{QQ}\mu^{\Lambda\Lambda} \geq (\mu^{Q\Lambda})^2 \;,\quad \mu^{\Lambda\Lambda}\mu^{JJ} \geq (\mu^{\Lambda J})^2 \tag{35}$$

Finally the combination of (24) ~ (27) with (30) ~ (33) yields the evolution equations for the fluxes of the system

$$\tau^{QQ}\dot{Q}_\alpha + Q_\alpha = -\frac{1}{\theta}\mu^{QQ}\theta_{,\alpha} + \mu^{QJ} E_\alpha - \mu^{Q\Lambda}\left(J^S_\alpha + e_{\alpha\beta\gamma}\,\epsilon_{\gamma,\beta} + \frac{1}{\theta} e_{\alpha\beta\gamma}\,\epsilon_\beta\,\theta_{,\gamma}\right) - \tau^{QJ}\dot{J}^N_\alpha - \tau^{Q\Lambda}\dot{\overline{\Lambda}}_\alpha \tag{36}$$

$$\tau^{JJ}\dot{J}^N_\alpha + J^N_\alpha = -\frac{1}{\theta}\mu^{JQ}\theta_{,\alpha} + \mu^{JJ} E_\alpha - \mu^{J\Lambda}\left(J^S_\alpha + e_{\alpha\beta\gamma}\,\epsilon_{\gamma,\beta} + \frac{1}{\theta} e_{\alpha\beta\gamma}\,\epsilon_\beta\,\theta_{,\gamma}\right) - \tau^{JQ}\dot{Q}_\alpha - \tau^{J\Lambda}\dot{\overline{\Lambda}}_\alpha \tag{37}$$

$$\tau^{\Lambda\Lambda}\dot{\overline{\Lambda}}_\alpha + \overline{\Lambda}_\alpha = -\frac{1}{\theta}\mu^{\Lambda Q}\theta_{,\alpha} + \mu^{\Lambda J} E_\alpha - \mu^{\Lambda\Lambda}\left(J^S_\alpha + e_{\alpha\beta\gamma}\,\epsilon_{\gamma,\beta} + \frac{1}{\theta} e_{\alpha\beta\gamma}\,\epsilon_\beta\,\theta_{,\gamma}\right) - \tau^{\Lambda Q}\dot{Q}_\alpha - \tau^{\Lambda J}\dot{J}^N_\alpha \tag{38}$$

$$\tau^\Psi|\ddot{\Psi}| + |\dot{\Psi}| = \mu^\Psi\,\theta\frac{\delta\hat{\eta}}{\delta|\Psi|} \tag{39}$$

where the characteristic times τ^{ij} are defined by

$$\tau^{QQ} = A_{11}\mu^{QQ} + A_{12}\mu^{QJ} + A_{13}\mu^{Q\Lambda} \tag{40}$$

$$\tau^{QJ} = A_{21}\mu^{QQ} + A_{22}\mu^{QJ} + A_{23}\mu^{Q\Lambda} \tag{41}$$

$$\tau^{Q\Lambda} = A_{31}\mu^{QQ} + A_{32}\mu^{QJ} + A_{33}\mu^{Q\Lambda} \tag{42}$$

$$\tau^{JQ} = A_{11}\mu^{JQ} + A_{12}\mu^{JJ} + A_{13}\mu^{J\Lambda} \tag{43}$$

$$\tau^{JJ} = A_{21}\mu^{JQ} + A_{22}\mu^{JJ} + A_{23}\mu^{J\Lambda} \tag{44}$$

$$\tau^{J\Lambda} = A_{31}\mu^{JQ} + A_{32}\mu^{JJ} + A_{33}\mu^{J\Lambda} \tag{45}$$

$$\tau^{\Lambda Q} = A_{11}\mu^{\Lambda Q} + A_{12}\mu^{\Lambda J} + A_{13}\mu^{\Lambda\Lambda} \tag{46}$$

$$\tau^{\Lambda J} = A_{21}\mu^{\Lambda Q} + A_{22}\mu^{\Lambda J} + A_{23}\mu^{\Lambda\Lambda} \tag{47}$$

$$\tau^{\Lambda\Lambda} = A_{31}\mu^{\Lambda Q} + A_{32}\mu^{\Lambda J} + A_{33}\mu^{\Lambda\Lambda} \tag{48}$$

$$\tau^\Psi = A_4\,\mu^\Psi \tag{49}$$

The Eqs. (36) ~ (39) can be used to describe the response of elastic type-II superconductors in a time scale in which the fluxes have not relaxed. They can be solved for giving initial and boundary conditions as long as the coefficients τ^{ij} and μ^{ij} are given. However, it's still not enough to describe the actual response of this material due to the lack of anisotropic consideration between fluxes and forces. This linear approximation could be extended such that the tensor-dependent coefficients $\mu^{ij}_{\alpha\beta}$ are involved, i.e.,

$$Q_\alpha = \mu^{QQ}_{\alpha\beta} X^Q_\beta + \mu^{QJ}_{\alpha\beta} X^J_\beta + \mu^{Q\Lambda}_{\alpha\beta} X^\Lambda_\beta \tag{50}$$

$$J^N_\alpha = \mu^{JQ}_{\alpha\beta} X^Q_\beta + \mu^{JJ}_{\alpha\beta} X^J_\beta + \mu^{J\Lambda}_{\alpha\beta} X^\Lambda_\beta \tag{51}$$

$$\overline{\Lambda}_\alpha = \mu^{\Lambda Q}_{\alpha\beta} X^Q_\beta + \mu^{\Lambda J}_{\alpha\beta} X^J_\beta + \mu^{\Lambda\Lambda}_{\alpha\beta} X^\Lambda_\beta \tag{52}$$

and the coefficients $\mu^{ij}_{\alpha\beta}$ are functions of the conserved variables $(\hat{W}, E_{\alpha\beta}, |\Psi|, |\Psi|_{,\alpha}, V^S_\alpha, \Omega_\alpha)$.

Back to the Eqs. (36) ~ (38), we find that these three equations can be solved for three unknowns $\dot{Q}_\alpha$, $\dot{J}^N_\alpha$ and $\dot{\overline{\Lambda}}_\alpha$. After some manipulations we arrive at

$$\dot{Q}_\alpha = \frac{1}{\Delta}\left(-\xi^{QQ} Q_\alpha - \xi^{QJ} J^N_\alpha - \xi^{Q\Lambda}\overline{\Lambda}_\alpha - \overline{\mu}^{QQ}\frac{\theta_{,\alpha}}{\theta} + \overline{\mu}^{QJ} E_\alpha - \overline{\mu}^{Q\Lambda}\bar{J}^S_\alpha\right) \tag{53}$$

$$\dot{J}^N_\alpha = \frac{1}{\Delta}\left(-\xi^{JQ} Q_\alpha - \xi^{JJ} J^N_\alpha - \xi^{J\Lambda}\overline{\Lambda}_\alpha - \overline{\mu}^{JQ}\frac{\theta_{,\alpha}}{\theta} + \overline{\mu}^{JJ} E_\alpha - \overline{\mu}^{J\Lambda}\bar{J}^S_\alpha\right) \tag{54}$$

$$\dot{\overline{\Lambda}}_\alpha = \frac{1}{\Delta}\left(-\xi^{\Lambda Q} Q_\alpha - \xi^{\Lambda J} J^N_\alpha - \xi^{\Lambda\Lambda}\overline{\Lambda}_\alpha - \overline{\mu}^{\Lambda Q}\frac{\theta_{,\alpha}}{\theta} + \overline{\mu}^{\Lambda J} E_\alpha - \overline{\mu}^{\Lambda\Lambda}\bar{J}^S_\alpha\right) \tag{55}$$

where

$$\bar{J}^S_\alpha = J^S_\alpha + e_{\alpha\beta\gamma}\,\epsilon_{\gamma,\beta} + \frac{1}{\theta} e_{\alpha\beta\gamma}\,\epsilon_\beta\,\theta_{,\gamma} \tag{56}$$

and

$$\Delta = \begin{vmatrix} \tau^{QQ} & \tau^{QJ} & \tau^{Q\Lambda} \\ \tau^{JQ} & \tau^{JJ} & \tau^{J\Lambda} \\ \tau^{\Lambda Q} & \tau^{\Lambda J} & \tau^{\Lambda\Lambda} \end{vmatrix} \tag{57}$$

In addition, two sets of new coefficients ξ^{ij} and $\bar{\mu}^{ij}$ are taken as

$$\xi^{QQ} = \tau^{JJ}\,\tau^{\Lambda\Lambda} - \tau^{J\Lambda}\,\tau^{\Lambda J} \tag{58}$$

$$\xi^{QJ} = \tau^{\Lambda J}\,\tau^{Q\Lambda} - \tau^{\Lambda\Lambda}\,\tau^{QJ} \tag{59}$$

$$\xi^{Q\Lambda} = \tau^{QJ}\,\tau^{J\Lambda} - \tau^{Q\Lambda}\,\tau^{JJ} \tag{60}$$

$$\xi^{JQ} = \tau^{J\Lambda}\,\tau^{\Lambda Q} - \tau^{\Lambda\Lambda}\,\tau^{JQ} \tag{61}$$

$$\xi^{JJ} = \tau^{QQ}\,\tau^{\Lambda\Lambda} - \tau^{\Lambda Q}\,\tau^{Q\Lambda} \tag{62}$$

$$\xi^{J\Lambda} = \tau^{JQ}\,\tau^{Q\Lambda} - \tau^{QQ}\,\tau^{J\Lambda} \tag{63}$$

$$\xi^{\Lambda Q} = \tau^{\Lambda J}\,\tau^{JQ} - \tau^{\Lambda Q}\,\tau^{JJ} \tag{64}$$

$$\xi^{\Lambda J} = \tau^{\Lambda Q}\,\tau^{QJ} - \tau^{QQ}\,\tau^{\Lambda J} \tag{65}$$

$$\xi^{\Lambda\Lambda} = \tau^{QQ}\,\tau^{JJ} - \tau^{JQ}\,\tau^{QJ} \tag{66}$$

$$\bar{\mu}^{QQ} = \xi^{QQ}\,\mu^{QQ} + \xi^{QJ}\,\mu^{JQ} + \xi^{Q\Lambda}\,\mu^{\Lambda Q} \tag{67}$$

$$\bar{\mu}^{QJ} = \xi^{QQ}\,\mu^{QJ} + \xi^{QJ}\,\mu^{JJ} + \xi^{Q\Lambda}\,\mu^{\Lambda J} \tag{68}$$

$$\bar{\mu}^{Q\Lambda} = \xi^{QQ}\,\mu^{Q\Lambda} + \xi^{QJ}\,\mu^{J\Lambda} + \xi^{Q\Lambda}\,\mu^{\Lambda\Lambda} \tag{69}$$

$$\bar{\mu}^{JQ} = \xi^{JQ}\,\mu^{QQ} + \xi^{JJ}\,\mu^{JQ} + \xi^{J\Lambda}\,\mu^{\Lambda Q} \tag{70}$$

$$\bar{\mu}^{JJ} = \xi^{JQ}\,\mu^{QJ} + \xi^{JJ}\,\mu^{JJ} + \xi^{J\Lambda}\,\mu^{\Lambda J} \tag{71}$$

$$\bar{\mu}^{J\Lambda} = \xi^{JQ}\,\mu^{Q\Lambda} + \xi^{JJ}\,\mu^{J\Lambda} + \xi^{J\Lambda}\,\mu^{\Lambda\Lambda} \tag{72}$$

$$\bar{\mu}^{\Lambda Q} = \xi^{\Lambda Q}\,\mu^{QQ} + \xi^{\Lambda J}\,\mu^{JQ} + \xi^{\Lambda\Lambda}\,\mu^{\Lambda Q} \tag{73}$$

$$\bar{\mu}^{\Lambda J} = \xi^{\Lambda Q}\,\mu^{QJ} + \xi^{\Lambda J}\,\mu^{JJ} + \xi^{\Lambda\Lambda}\,\mu^{\Lambda J} \tag{74}$$

$$\bar{\mu}^{\Lambda\Lambda} = \xi^{\Lambda Q}\,\mu^{Q\Lambda} + \xi^{\Lambda J}\,\mu^{J\Lambda} + \xi^{\Lambda\Lambda}\,\mu^{\Lambda\Lambda} \tag{75}$$

From the preceding discussions, we mention two wave-type equations about the internal variable $|\Psi|$ and the temperature θ. The Eq. (39) characterizes the wave propagation of the internal variable $|\Psi|$, which is the representation of superelectron distribution and the coefficient τ^{Ψ} is directly related to this wave speed. In the equilibrium state we have the equation for the equilibrium distribution of superelectron, i.e., $\delta\hat{\eta}/\delta|\Psi| = 0$ and this equation resembles the famous Ginzburg-Landau equation. Moreover, in the absence of cross effect, the Eq. (36) can be reduced to

$$\tau^{QQ}\,\dot{Q}_{\alpha} + Q_{\alpha} = -\frac{1}{\theta}\,\mu^{QQ}\,\theta_{,\alpha} \tag{76}$$

and obviously the traditional Fourier law is lacking in the consideration of the first term. Combining (76) with the energy balance equation, we obtain

$$\tau^{QQ}\,c\,\ddot{\theta} + c\,\dot{\theta} = \bar{\mu}\,\theta_{,\alpha\alpha} \tag{77}$$

to which corresponds a finite velocity given by

$$\upsilon = \left(\frac{\bar{\mu}^{QQ}}{c\,\tau^{QQ}}\right)^{\frac{1}{2}} \tag{78}$$

where the assumptions (i) $\hat{W} = c\,\dot{\theta}$ and (ii) $\bar{\mu}^{QQ} = \mu^{QQ}/\theta = \text{const.}$, are adopted. The wave for the temperature is the second sound and is of primary importance in low-temperature physics.

5. CONCLUSION

In this paper we have derived the constitutive equations for the conserved state variables and the generalized evolution equations for the nonconserved state variables of elastic type-II superconductors within the framework of EIT approach. All of the relevant quantities are derived from the differentiations of the generalized entropy function, which is a function of the traditional state variables and the dissipative fluxes as well. The merit of this approach is to extend our scope beyond the equilibrium state, i.e., nonequilibrium state, in which the physical processes always occur. The derived evolution equations for fluxes are consistent with the actual physical transport phenomena (wave-type instead of parabolic-type signal propagation). Finally, in special cases in which cross effects are not taken into account, the wave equations for the temperature and superelectron density are obtained.

ACKNOWLEDGEMENT

The authors thank the financial support granted by the National Science Council, R.O.C. (NSC-89-2211-E-260-003).

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(Manuscript received Feb. 8, 2000,
Accepted for publication Feb. 8, 2000.)