行政院國家科學委員會專題研究計畫 期中進度報告

流體與結構物互制之整合數模研究(2/3)
期中進度報告(精簡版)

計 畫 類 別 ： 個別型
計 畫 編 號 ： NSC 95-2221-E-002-406-
執 行 期 間 ： 95 年 08 月 01 日至 96 年 07 月 31 日
執 行 單 位 ： 國立臺灣大學土木工程學系暨研究所

計 畫 主 持 人 ： 楊德良

處 理 方 式 ： 期中報告不提供公開查詢

中 華 民 國 96 年 05 月 29 日

流體與結構物互制之整合數模研究(2/3)

95-2221-E-002-406
2006/08/01~2007/07/31

摘要

本研究計畫之目的在於建立一套高效率的數值方法, 用於處理海洋工程學中之流體與結構物互制之工程應用問題。這種流體與結構物的動力分析將涉及結構物體與流體間之相互影響, 應用直接數值模擬方法來分析流體結構物互制作用在計算力學的實際應用上是一項極為重要的問題。任何的數值方法其目的是要能解決給定適當的條件, 而能求解各種流體與結構物互制作用的工程問題。在處理這一類的工程問題時, 通常使用質量、動量與能量守恆定理。因此, 流體與結構物互制作用的問題是需要考慮多方面的物理模式。為了維持守恆的關係, 所以如何處理動量、能量以及濃度更是格外重要。本文所採用的數值方法, 將結合各種數值方法的優點進行模擬, 其中包含有限差分法、有限元素法、有限體積法、基本解法以及微分積分法等傳統與無網格數值方法, 其目的在於處理流體與結構物耦合的問題。至於處理自由液面及物體移動, 可採用任意拉格朗日-尤拉法, 流體體積法、等位函數法等。計畫將分三年進行, 第一年的研究是為了建立二維模式來處理均質和密度層變流體與結構物互制作用, 使用此模式來模擬在海洋中流體和結構物互制作用的數值實驗。此外、第一年也將探討在某臨界流況中流體流過結構物而產生一系列孤立波的現象。在這一年度還要先驗證均質與密度層變流體與移動中物體的互制, 再者模擬包含自由液面且物體在流體中運動的流體結構物互制作用的二維模式。第二年的研究內容是推廣第一年的模式中之移動中的結構物之二維流體結構物問題且發展三維移動結構物之流體結構物耦合的數值模式。第三年將第一年之模式中之移動結構物之二維流體結構物問題及第二年之三維的移動結構物, 發展三維可移動結構物之流體結構物耦合數值模式, 全程計畫為分析含自由液面且物體在流體中運動的二維及三維之數值模式, 來求解各種流體結構物耦合的問題。

關鍵字: 流體結構物互制作用, 自由液面、直接數值模擬。

Abstract

The direct numerical simulation of coupling of a fluid-structure interaction is an important problem in computational mechanics. Any numerical analysis aims at finding engineering solution for the given conditions of interaction between the fluids and the structure. Such engineering problems have to be analyzed using the conservation principles of mass, momentum, energy and concentration. Thus the situations demand a multi-physic modeling of the fluid-structure interaction problems. In order to maintain conservation laws, other than the mass conservation, careful accounting of momentum, energy and concentration for the fluid-structure coupled system is very crucial to the numerical modeling. Furthermore, the effects of density stratification and the earth rotation can be easily incorporated too. As far as solutions of numerical methods are concerned, finite difference method, the finite element method, finite volume method, the method of fundamental solutions and differential quadrature method of both conventional and meshless numerical methods are adopted to analyze the fluid-structure interaction problems. The treatments of free surfaces and moving structures are proposed by Arbitrary Lagrangian-Eulerian (ALE), Volume of fluid (VOF) and level set methods.

The proposed research program will be achieved over a period of three years with a definite work allocation for each year. The first and second years of the research project will be devoted to the development of 2D numerical models for the fluid-structure system involving stationary solid structures with applications in the field of coastal engineering and oceanography with stratification and free surface. Furthermore, the free surface analysis for solitary waves generated by critical flow over a submerged solid structure will be carried out in this year. In the meantime, the 2D model developed will be applied to simulate flow over structures of various geometries such as square and cylinder in homogeneous and satisfied fluids. After validating the model, we will accomplish the acceleration and deceleration of structures moving in a still fluid, and vibration of structures induced by the vortex shedding. In the third year, using the models developed in the first and second years, the work will be extended to develop numerical models to simulate the flow field for the moving structures as coupled system in 2D and 3D geometries.

Keywords: fluid-structure interaction, free surface, direct numerical simulation.

1. Introduction

Numerical solution of incompressible Navier-Stokes equations is the prime subject in the field of computational fluid dynamics and many related fields in science and engineering. With the advent of high speed computing machines, obtaining numerical solutions of three-dimensional Navier-Stokes equations have become much easier compared to the previous decades. Though the numerical solution algorithms are well established for the Navier-Stokes equations, research efforts are still focused on the development of new solution schemes such as meshless methods because the applications of the Navier-Stokes equations are extended to many interdisciplinary fields such as MEMS, bio-medical technology, etc. other than the classical fields of hydrodynamics and fluid mechanics. Further the mathematical characteristics of the Navier-Stokes equations provide wide options for developing new solution algorithms due to very high nonlinear nature.

The analysis of free surface flows finds wide applications in the design of breakwaters and offshore structures. Many analytical methods have been developed to solve traveling and standing wave problems associated with incompressible and inviscid fluids as well as for the solution of linear and low order non-linear problems. Furthermore, the phenomena of solitary wave propagations in shallow water channel are the classical problems of flows with free surface. Theoretical studies of solitary waves and solitons have attracted many investigators to provide a series of work. Numerical techniques have allowed the solution of non-linear inviscid and viscous free surface motion. In order to comprehend the complexities of the phenomenon including the viscous effects, a more general approach to handle viscous free-surface flow is necessary. Lo and Young [1] developed a numerical model to obtain flow results for several free surface flow numerical examples including the interaction between two opposite solitary waves, seiche phenomenon in a rectangular reservoir, and the vortex formation during the passage of a solitary wave over a submerged rectangular structure. Most of the study cases are only 2D free surface flow examples. However, the present work intends to deal with fully 3D nonlinear free surface flows including the inviscid and the viscous effects.

The numerical treatment of free surface problems involves the tracking of the moving free surface boundary during the flow transients. The Lagrangian coordinate system is used to track the free surface position on the moving free surface at each time step. On the other hand the free surface is tracked by a coordinate system, which moves along with an identified fluid particle. As far as the interior computational domain is concerned, the arbitrary Lagrangian-Eulerian method was proposed to deal with the interior unknown variables. In this method, the nodal point can be arbitrarity controlled in order to get finer mesh distribution. In this study, the velocity-vorticity formulation is used to solve non-linear inviscid and viscous flows with free surface. A detailed literature survey indicated that only a very few studies have been cited on free surface flow analysis using the velocity-vorticity formulation except the work of Lo and Young [1]. This project will extend this concept to develop a model for 3D free surface flow for both inviscid and viscous flows based on the velocity-vorticity formulation. In the present work we propose a numerical method for dealing with fully non-linear free surface boundary conditions by adopting a coupled numerical solution procedure to obtain the flow variables at the free surface. The most important issues in the free surface flow simulation are to track the accurate position of the free surface elevation and compute the corresponding velocities. The equations for free surface kinematic boundary conditions (FSKBC) and free surface dynamic boundary conditions (FSDBC) are solved by a coupled scheme using the Galerkin's weighted residual finite element method to obtain the flow variables at the free surface of the computational domain.

One major target in this proposal is to apply the immersed boundary (IB) method with unstructured finite-volume method (FVM) and projection method to simulate flows over the complex structures by introducing a mass source or sink as well as a momentum forcing. Instead of using high-order interpolation scheme to satisfy the no-slip condition, in our study the unstructured grids will make the grid line coincide with the immersed boundary. By such a way, the numerical results should have an obvious improvement near the immersed boundary.

In the recent years, the meshless or meshfree or mesh reduction methods have received a considerable attention as alternative numerical schemes to the classical mesh-dependent numerical methods. Therefore another unified numerical study in this project is to introduce the meshless schemes to model the fluid-structure interaction problems. The comparisons between the classic and the new meshless methods will be made to evaluate the feasibility if the new developed meshless scheme. Most of the numerical schemes based on the boundary element method treats the time derivative term in the diffusion equation wither using the Laplace transform [2-4] or the finite difference scheme [5] to advance the solution in the time domain. However, the main drawback of the boundary element method is to determine the singular integrals at the boundaries, which require a great amount of computational effort especially for the case of three-dimensional problems.

The problem posed by the boundary element method can be alleviated by the use of the method of fundamental solutions. The method of fundamental solutions makes use of the fundamental solutions to get solution by satisfying the governing differential equation in the interior of the computational domain under consideration. By means of boundary collocation method, the boundary conditions for the problem are satisfied. This method is free from the singular integral evaluation problem as required by the boundary element method. Golberg [6] and Golberg & Chen [7] used the method of fundamental solutions to obtain numerical solution of the Poisson equation.

In this proposal, the time-dependent fundamental solution of the diffusion equation is directly used with the method of fundamental solutions without the need for the Laplace transform or the finite difference method to take care of the time-derivative term. Since any diffusion process is a time evolution process, which stars from a set of initial conditions, the transient parts of the diffusion process can be obtained in a number of time steps using the MFS.

2. Results

According to the proposal of this project, the objectives in this year are achieved completely. Some 23 good results related to this project were published in high-quality journals [1, 8-30]. In addition to those, some ongoing results will be submitted and published later.

3. References

1. Lo, D.C., Young, D.L., 2004, Arbitrary Lagrangian-Eulerian Finite Element Analysis of Free Surface Flow Using a Velocity-vorticity Formulation, Journal of Computational Physics, Vol. 195, pp. 175-201.

2. Zhu, S.P., 1998, Solving Transient Diffusion Problems: Time-dependent Fundamental Solution Approaches Versus LTDRM Approaches, Engineering Analysis with Boundary Elements, Vol. 21, pp. 87-90.

3. Zhu, S.P., Liu, H.W., Lu, X.P., 1998, A Combination of LTDRM and ATPS in Solving Diffusion Problems, Engineering Analysis with Boundary Elements, Vol. 21, pp. 285-289.

4. Sutradhar, A., Paulino, G.H., Gray, L.J., 2002, Transient Heat Conduction in Homogeneous and Non-homogeneous Materials by the Laplace Transform Galerkin Boundary Element Method, Engineering Analysis with Boundary

Elements, Vol. 26, pp. 119-132.

5. Bulgakov, V., Sarler, B., Kuhn, G., 1998, Iterative Solution of Systems of Equations in the Dual Reciprocity Boundary Element Method for the Diffusion Equation, International Journal for Numerical Methods in Fluids, Vol. 43, pp. 713-732.

6. Golberg, M.A., 1995, The Method of Fundamental Solutions for Poisson's Equation, Engineering Analysis with Boundary Elements, Vol. 16, pp. 205-213.

7. Golberg, M.A., Chen, C.S., 1998, The Method of Fundamental Solutions for Potential, Helmholtz and Diffusion Problems, In Boundary Integral Methods - Numerical and Mathematical Aspects, ed. M.A. Golberg, (Computational Mechanics Publications) pp. 103-176.

8. Young, D.L., Hu, S.P., Fan, C.M., Chen, C.W., 2006, Method of Fundamental Solutions and Cole-Hopf Transformation for Burgers' Equations, Computational Fluid Dynamics Journal, Vol. 14, No. 4, pp. 454-467. (invited paper)

9. Young, D.L., Jane, S.J., Fan, C.M., Murugesan, K., Tsai, C.C., 2006, The Method of Fundamental Solutions for 2D and 3D Stokes Problems, Journal of Computational Physics, Vol. 211, pp. 1-8. (SCI & EI)

10. Tsai, C.C., Young, D.L., Lo, D.C., Wong, T.K., 2006, Method of Fundamental Solutions for Three-dimensional Stokes Flow in Exterior Field, Journal of Engineering Mechanics, ASCE, Vol. 132, No. 3, pp. 317-326. (SCI & EI)

11. Young, D.L., Fan, C.M., Tsai, C.C., Chen, C.W., 2006, The Method of Fundamental Solutions and Domain Decomposition Method for Degenerate Seepage Flownet Problems, Journal of the Chinese Institute of Engineers, Vol. 29, No. 1, pp. 63-73. (SCI & EI)

12. Young, D.L., Chen, K.H., Lee, C.W., 2006, Singular Meshless Method Using Double Layer Potentials for Exterior Acoustics, The Journal of the Acoustical Society of America, Vol. 119, No. 1, pp. 96-107. (SCI & EI)

13. Young, D.L., Chen, C.W., Fan, C.M., Tsai, C.C., 2006, The Method of Fundamental Solutions with Eigenfunction Expansion Method for Nonhomogeneous Diffusion Equation, Numerical Methods for Partial Differential Equations, Vol. 22, No. 5, pp. 1173-1196. (SCI & EI)

14. Tsai, C.C., Young, D.L., Chen, C.W., Fan, C.M., 2006, The Method of Fundamental Solutions for Eigenproblems in Domains with and without Interior Holes, Proceedings of Royal Society, Series A, Vol. 462, pp. 1443-1466. (SCI & EI)

15. Tsai, C.C., Young, D.L., Chiang, J.H., Lo, D.C., 2006, The Method of

Fundamental Solutions for Solving Options Pricing Models, Applied Mathematics and Computation, Vol. 181, pp. 390-401. (SCI)

16. Young, D.L., Fan, C.M., Tsai, C.C., Chen, C.W., Murugesan, K., 2006, Eulerian-Lagrangian Method of Fundamental Solutions for Multi-dimensional Advection-diffusion Equation, International Mathematical Forum, Vol. 1, pp. 687-706. (invited paper)

17. Young, D.L., Chiu, C.L., Fan, C.M., 2006, Non-Singular Boundary Integral Equation for Two-Dimensional Electromagnetic Scattering Problems, Microwave and Optical Technology Letters, Vol. 48, No. 4, pp. 760-765. (SCI & EI)

18. Young, D.L., Chiu, C.L., Fan, C.M., Tsai, C.C., Lin, Y.C., 2006, Method of Fundamental Solutions for Multidimensional Stokes Equations by the Dual-Potential Formulation, European Journal of Mechanics - B/Fluids, Vol. 25, pp. 877-893. (SCI & EI)

19. Tsai, C.C., Young, D.L., Fan, C.M., Chen, C.W., 2006, Time-dependent Fundamental Solutions for Unsteady Stokes Problems, Engineering Analysis with Boundary Elements, Vol. 30, pp. 897-908. (SCI & EI)

20. Chen, K.H., Kao, J.H., Chen, J.T., Young, D.L., Lu, M.C., 2006, Regularized Meshless Method for Multiply-connected-domain Laplace Problem, Engineering Analysis with Boundary Elements, Vol. 30, pp. 882-896. (SCI & EI)

21. Tsai, C.C., Young, D.L., Fan, C.M., 2006, Method of Fundamental Solutions for Plate Vibrations in Multiply Connected Domains, Journal of Mechanics, Vol. 22, No. 3, pp. 235-245. (SCI & EI)

22. Tsai, C.C., Lin, Y.C., Young, D.L., Atluri, S.N., 2006, Investigations on the Accuracy and Condition Number for the Method of Fundamental Solutions, CMES: Computer Modeling in Engineering and Science, Vol. 16, No. 2, pp. 103-114. (SCI & EI)

23. Lo, D.C., Young, D.L., Murugesan, K., Tsai, C.C., Gou, M.H., 2007, Velocity-vorticity Formulation for 3D Natural Convection in an Inclined Cavity by DQ Method, International Journal of Heat and Mass Transfer, Vol. 50, pp. 479-491. (SCI & EI)

24. Lo, D.C., Young, D.L., Tsai, C.C., 2007, High Resolution of 2D Natural Convection in a Cavity by the DQ method, Journal of Computational and Applied Mathematics, Vol. 203, No. 1, pp. 219-236. (SCI)

25. Lo, D.C., Tsung, L., Young, D.L., Guo, M.H., 2007, Velocity-vorticity Formulation for 2D Natural Convection in an Inclined Cavity by the DQ Method, Journal of Mechanics, (in press) (SCI & EI)

26. Chiu, C.L., Fan, C.M., Young, D.L., 2007, Meshless Numerical Solutions for Burgers' Equations by Multiquadrics and Cole-Hopf Transformation, Journal of Aeronautics, Astronautics and Aviation, Series A, Vol. 39, No. 1, pp. 1-8. (EI) (invited paper)

27. Young, D.L. 2007, Meshless Boundary Element Methods for the Analysis of Large Scale Engineering Problems, Engineering Science and Technology Bulletin, Vol. 91,(in press) (invited paper)

28. Wu, N.J., Tsay, T.K., Young, D.L., 2007, Computation of Nonlinear Free-surface Flows by a Meshless Numerical Method, Journal of Waterway, Port, Coastal and Ocean Engineering, ASCE, (SCI & EI) (tentatively accepted, first revision submitted) (SCI & EI)

29. Tsai, C.C., Young, D.L., Chiu, C.L., Fan, C.M., 2007, Numerical Analysis of Acoustical Modes Using the Linear Least Squares MFS, Journal of Sound and Vibration, (tentatively accepted, first revision submitted) (SCI & EI)

30. Young, D.L., Tsai, C.C., Fan, C.M., Chen, C.W., 2007, The Method of Fundamental Solutions and Condition Number Analysis for Inverse problems of Laplace Equation, Computers & Mathematics with Applications. (accepted) (SCI & EI)

Investigations on the Accuracy and Condition Number for the Method of Fundamental Solutions

C.C. Tsai[1], Y.C. Lin[2], D.L. Young[2,3] and S.N. Atluri[4]

Abstract: In the applications of the method of fundamental solutions, locations of sources are treated either as variables or *a priori* known constants. In which, the former results in a nonlinear optimization problem and the other has to face the problem of locating sources. Theoretically, farther sources results in worse conditioning and better accuracy. In this paper, a practical procedure is provided to locate the sources for various time-independent operators, including Laplacian, Helmholtz operator, modified Helmholtz operator, and biharmonic operator. Wherein, the procedure is developed through systematic numerical experiments for relations among the accuracy, condition number, and source positions in different shapes of computational domains. In these numerical experiments, it is found that in general very good accuracy is achieved when the condition number approaches the limit of equation solver, which is a number dependent on the solution scheme and the precision. The proposed procedure is verified for both Dirichlet and Neumann boundary conditions. The general characteristics in these numerical experiments demonstrate the capability of the proposed procedure for locating sources of the method of fundamental solutions for problems without exact solutions.

keyword: Method of fundamental solutions, Condition number, Location of sources, Laplacian, Helmholtz operator, Modified Helmholtz operator, Biharmonic operator

1 Introduction

In the recent years, the meshless or mesh-free methods have received a considerable attention as alternative numerical schemes to the classical mesh-dependent numerical methods, such as the finite difference method (FDM), the finite element method (FEM), the finite volume method (FVM), and the boundary element method (BEM). Roughly speaking, the meshless or mesh-free methods can be divided into two categories. The first one is domain-type methods in which both the differential equations and boundary conditions are approximated, such as the Kansa's method (or multiquadrics (MQ) method) [Kansa (1990A, 1990B), Li, Cheng and Chen (2003), Young, Jane, Lin, Chiu and Chen (2004), Young, Chen and Wong (2005)] as well as the meshless local Petrov-Galerkin method (MLPG) [Wordelman, Aluru and Ravaioli (2000), Lin and Atluri (2000), Kim and Atluri (2000), Atluri (2004), Han and Atluri (2004)]. The second one is boundary-type methods where only boundary conditions are collocated, such as the method of fundamental solutions (MFS) [Kupradze and Aleksidze (1964), Mathon and Johnston (1977), Lyngby (1981), Bogomolny (1985), Smyrlis and Karageorghis (2001), Tsai, Young and Cheng (2002), Smyrlis and Karageorghis (2004), Chen, Fan, Young, Murugesan and Tsai (2005), Hon and Wei (2005), Young and Ruan (2005), Young, Tsai, Lin and Chen (2006)] and the MLPG [Atluri (2004)]. In this paper, we only concentrate on how to locate the sources of the MFS.

The MFS is first proposed by Kupradze and Aleksidze (1964). Originally, the sources of MFS are considered as unknown variables and solved by nonlinear optimization [Mathon and Johnston (1977)]. Later, Bogomolny (1985) improved the theoretical fundamentals of the MFS in considering *a priori* known positions of sources. As a result, the MFS becomes easier and more efficient in practical implementations. However, the ill-conditioning and the locations of source points are numerically problematic. Smyrlis and Karageorghis (2001) have researched the issue for the standard MFS, with the same number of source and collocation points, for harmonic problems in a disk. They suggested rotation and normalization to

[1] Department of Information Technology, Toko University, Chia-Yi County, 61363, Taiwan
[2] Department of Civil Engineering and Hydrotech Research Institute, National Taiwan University, Taipei, 10617, Taiwan
[3] Corresponding author, Email: dlyoung@ntu.edu.tw
[4] Center for Aerospace Research and Education, University of California, Irvine, 5251 California Ave, Suite 140, Irvine, CA 92612, USA

Available online at www.sciencedirect.com

ScienceDirect

International Journal of Heat and Mass Transfer 50 (2007) 479–491

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Velocity–vorticity formulation for 3D natural convection in an inclined cavity by DQ method

D.C. Lo [a], D.L. Young [b,*], K. Murugesan [c], C.C. Tsai [d], M.H. Gou [b]

[a] Institute of Marine Science and Technology, National Kaohsiung Marine University, Kaohsiung, 81157, Taiwan, ROC
[b] Department of Civil Engineering and Hydrotech Research Institute, National Taiwan University, Taipei 10617, Taiwan, ROC
[c] Department of Mechanical and Industrial Engineering, IIT Roorkee 247667, India
[d] Department of Information Technology, Toko University, Chia-Yi County 61363, Taiwan, ROC

Received 13 September 2005
Available online 11 October 2006

Abstract

The present work proposes a novel numerical solution algorithm based on a differential quadrature (DQ) method to simulate natural convection in an inclined cubic cavity using velocity–vorticity form of the Navier–Stokes equations. Since the DQ method employs a higher-order polynomial to approximate any given differential operator, the vorticity values at the boundaries can be computed more accurately than the conventionally followed second-order accurate Taylor's series expansion scheme. The numerical capability of the present algorithm is demonstrated by the application to natural convection in an inclined cubic cavity. The velocity Poisson equations, the continuity equation, the vorticity transport equations and the energy equation are all solved as a coupled system of equations for the seven field variables consisting of three velocities, three vorticities and temperature. Thus coupling the velocity and the vorticity transport equations allows the determination of the vorticity boundary values implicitly without requiring the explicit specification of the vorticity boundary conditions. The present algorithm is proved to be an efficient method to resolve the non-linearity involved with the vorticity transport equations and the energy equation. Test results obtained for an inclined cubic cavity with different angle of inclinations for Rayleigh number equal to 10^3, 10^4, 10^5 and 10^6 indicate that the present coupled solution algorithm could predict the benchmark results for temperature and flow fields using a much coarse computational grid compared to other numerical schemes.

Keywords: Velocity–vorticity formulation; Natural convection; Inclined cubic cavity; Differential quadrature method

1. Introduction

Numerical solution of incompressible Navier–Stokes equations is an important area in CFD related fields in science and engineering. With the development of a wide range of numerical schemes and algorithms, obtaining numerical solution of the Navier–Stokes equations now has become much easier compared to the previous decades. However, there is a continuous research going on in the development of new numerical algorithms as the CFD is used as a modeling tool in other areas of science as well. The velocity–vorticity formulation, pioneered by Fasel [1], is considered to be an alternate form of the Navier–Stokes equations without involving the pressure term. An important issue in the velocity–vorticity form of the Navier–Stokes equations is the enforcement of the vorticity definition on the boundary to assure divergence free velocity field. In many practical CFD problems, vortex dynamics has dominated the study of turbulence flow field for design purpose instead of the primitive variable form of the Navier–Stokes equations. Daube [2] pointed out that the satisfaction of the continuity equation reduces to enforce the vorticity definition at the boundaries in terms of curl of the velocity field. Moreover, the Navier–Stokes

* Corresponding author. Tel./fax: +886 2 2362 6114.
E-mail addresses: loderg@ntu.edu.tw (D.C. Lo), dlyoung@ntu.edu.tw (D.L. Young).

0017-9310/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2006.07.025